

# Carrier Phase Adjustment for Multiple Access Communication Systems with Multi-Packet Reception Capability

Wooyeol Choi\*, Taewoon Kim\*, Heung-No Lee\*, and Hyuk Lim\*<sup>†</sup>

\*Department of Information and Communications

<sup>†</sup>Department of Nanobio Materials and Electronics

Gwangju Institute of Science and Technology (GIST), Korea

Email: {wychoi, taewoonkim, heungno, hlim}@gist.ac.kr

**Abstract**—Driven by advances in signal processing and multiuser detection (MUD) technologies, it has become possible for a wireless node to simultaneously receive multiple signals from other transmitters. In order to take full advantage of MUD in multi-packet reception (MPR) capable wireless networks, it is highly desirable to make the compounded signal from multiple transmitters more separable on its constellation at the receiver by coordinating the carrier phase offsets of the transmitters. In this paper, we propose a feedback-based carrier phase adjustment scheme that estimates the carrier phase offset for each user's received signal, computes the optimal phase shift to maximize the minimum Euclidean distance between the constellation points, and finally feeds the phase shift information back to the transmitters. We then evaluate the performance of the proposed carrier phase adjustment scheme and compare it to that of the no carrier phase adjustment case for QPSK and 8PSK with 2–4 users, and subsequently show that the proposed scheme significantly reduces the error probability in a multiuser communication system having MPR capability.

**Index Terms**—Carrier phase adjustment, phase estimation, wireless MAC protocol, multi-packet reception.

## I. INTRODUCTION

In conventional wireless networks, each receiver is only capable of decoding signals from one transmitter at a time; referred to as single-user detection (SUD). In SUD, when a mixed signal from multiple transmitters is sensed, the receiver discards the signal and treats it as a collision. However, signal processing technology has rapidly evolved, and compounded signals from multiple users has become decodable at the receiver side; referred to as multiuser detection (MUD). Since MUD technology permits simultaneous packet reception from multiple sources, the mixed signal, which was previously treated as a collision event on conventional wireless networks, is now preferred for its ability to enhance the achievable throughput performance [1]–[4]. However, how to take advantage of the MUD technique and how to adjust its tunable parameters in designing the medium access control (MAC) for multi-packet reception (MPR) capable wireless networks in order to maximize the achievable throughput has yet to be sufficiently studied.

Considering the error-prone nature of the wireless medium, the symbol separation and decoding of a mixed signal are

primarily influenced by channel conditions and characteristics. To this end, several studies have attempted to overcome channel effects by means of carrier phase error correction [5]–[7]. Steendam *et al.* [5] investigated the effects of carrier phase offsets on a low-density parity-check (LDPC) coded system, and then proposed a maximum likelihood (ML)-based carrier phase synchronization algorithm that exploits the posterior probabilities of the data symbols. Similarly, Zhang *et al.* [6] proposed an a priori probability aided carrier phase estimation for turbo decoding. They showed that the technique provides a reliable carrier phase estimation that approaches the Cramer-Rao bounds at a very low signal-to-noise ratio (SNR). Harshan *et al.* [7] identified the problem of maximizing the capacity region between two users for Gaussian Multiple Access Channel (GMAC). By performing rotation on one of the sets in a way that the error probability is minimized, the capacity gain can be maximized. Compared with the Harshan's work, our work is applicable to a general and complex condition.

In this paper, we propose a MAC/PHY cross-layer approach for enhancing the separation and decoding performance of compounded signals on an additive white gaussian noise (AWGN) channel having fading effects. A receiver with MPR capability performs multiuser detection and then estimates the carrier phase offset of each transmitter's signal from the mixed signal. Next, the receiver piggybacks the carrier phase shift information, which is the difference between the estimated carrier phase offset and an optimal carrier phase offset, to the corresponding transmitters so that they can adjust their carrier phase offset to the optimal value when transmitting signals. To determine the optimal phase shift, we formulate an optimization problem in order to maximize the minimum Euclidean distance between the constellation coordinates of the compounded signal. We subsequently evaluate the performance of the proposed carrier phase adjustment scheme and compare it against that of the no carrier phase adjustment case for QPSK and 8PSK with 2–4 users. The simulation results show that the proposed scheme significantly reduces the error probability for all the cases investigated in our simulation scenarios.

The remainder of this paper is organized as follows. Section

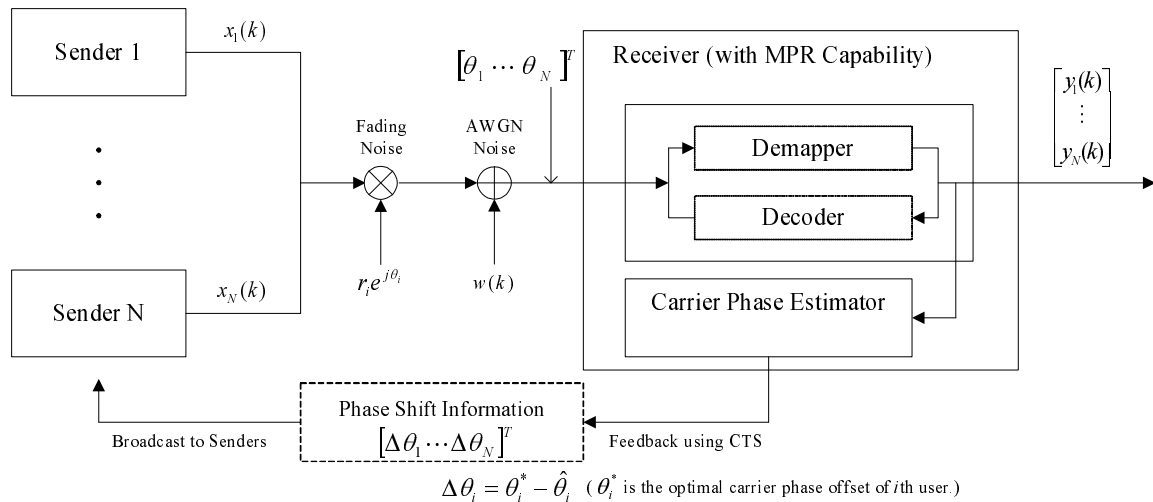


Fig. 1. The carrier phase adjustment scheme for a multiple access communication system with multi-packet reception capability.

II describes the system model on which the proposed scheme is based, and the motivation that initiates this study. Section III then explains the mathematical basis and detailed procedures of the proposed scheme. The performance evaluation is carried out in Section IV, and we finally conclude this paper in Section V.

## II. SYSTEM MODEL AND MOTIVATION

We consider a simple MAC protocol for uplink a synchronous single-cell system that is coordinated by a base station (BS) having MPR capability; the BS decodes a compounded signal from multiple senders via multiple antenna arrays and MUD. Here, the maximum number of simultaneous receptions is denoted by  $N_{mpr}$ .

In this MPR communication system, all nodes that want to send a data frame are required to transmit a request-to-send (RTS) frame to the intended receiver, which is responsible for coordinating the packet transmissions among competing transmitters. On receiving multiple RTS frames, the receiver broadcasts a clear-to-send (CTS) frame, which includes the set of transmitters that are permitted to transmit. We will use this CTS frame to inform the sender nodes of the optimal carrier phase shift offsets that were calculated by the proposed carrier phase adjustment scheme.<sup>1</sup>

Fig. 1 shows the block diagram for the proposed carrier phase adjustment scheme. Each sender transmits a signal  $x(k)$  to its receiver, and due to the AWGN and fading, the compounded signal from  $N$  senders at the receiver is given by

$$r(k) = \sum_{i=1}^N x_i(k) r_i e^{j\theta_i} + w(k), \quad (1)$$

where  $N$  is the number of senders,  $x_i(k)$  is the sequence of independent identically distributed equiprobable  $i$ th user data

<sup>1</sup>The overhead for providing the carrier phase shift information in the CTS frame is negligible, since it occupies only a small number of bits.

symbols,  $r_i$  is the real-valued average energy per symbol for the  $i$ th user,  $\theta_i$  is the carrier phase distortion of the  $i$ th user, and  $w(k)$  is the complex valued AWGN channel noise. The signals received in this manner can be successfully detected and decoded at the receiver using the MUD technique.

For the signal received in Eq. (1), the constellation has a number of densely distributed points for the signals received from multiple senders. For example, Fig. 2 shows the two-user constellation at 8PSK modulation when  $r_i = 1$  and  $\theta_i = 0$  for  $i = 1, 2$ . In Fig. 2, only 33 out of 64 constellation points are visible as the other 31 points overlap and are canceled out. In this case, the receiver cannot correctly separate each signal from the compounded signal due to the overlapped constellation points, which are then identified as decoding errors; thus, network capacity in the multiuser communication system significantly degrades.

To overcome this problem, the distance between constellation points at the receiver side should be kept as large as possible. Specifically, the constellation should be constructed such that the minimum Euclidean distance between the constellation points is maximized in order to decrease the error probability in multiple signal decoding for MPR communication systems.

## III. PROPOSED CARRIER PHASE ADJUSTMENT SCHEME

In this section, we propose a carrier phase adjustment scheme that controls the carrier phase offset in order to fully exploit the MPR channel capacity. The proposed carrier phase adjustment scheme has two steps. The first is *phase estimation*, which estimates the carrier phase offset incurred by channel noises, such as AWGN and fading. The second is *carrier phase adjustment*, which computes the difference between the estimated carrier phase offset and an optimal phase offset, and feeds the carrier phase offset difference back to the senders. The optimal carrier phase offsets are obtained for a given modulation scheme and the MPR capacity  $N_{mpr}$  based on the placement of constellation points that maximize the minimum Euclidean distance between the points.

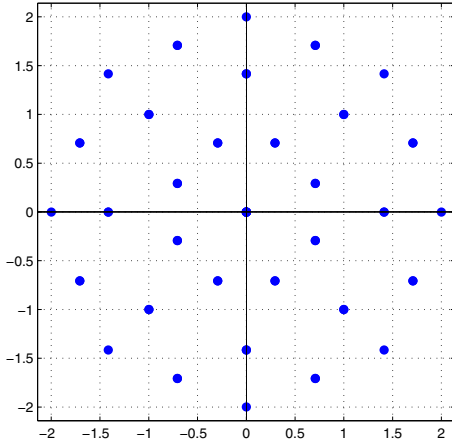


Fig. 2. A two-user constellation for an 8PSK signal in which 31 constellation points are overlapped with other points.

### A. Feedback Scheme

As briefly addressed in Section II, the nodes waiting to send packets are required to transmit an RTS frame to the receiver; the receiver can then detect and decode the signals received from the multiple senders. Concurrently, Fig. 1 the receiver estimates the carrier phase offset that occurs when a signal is transmitted on the wireless medium, computes the phase shift so as to reduce the error probability, and returns the phase shift information. The carrier phase shift offset is the difference between the estimated carrier phase offset of the received signal and the calculated optimal carrier phase offset. This feedback-based carrier phase adjustment mechanism makes it possible for the receiver to separate the constellations of multiple signals, significantly reducing the error probability at the receiver side.

In other words, the receiver estimates the carrier phase distortion through the RTS frame, and broadcasts a CTS frame that includes the set of transmitters that are permitted to transmit and the phase shift information, to achieve low transmission error probability and high reliability.

### B. Phase Estimation

As effects of the carrier phase error are significant in multi-phase schemes, accurate carrier phase estimation is crucial in achieving performance gains in the carrier phase adjustment. In this paper, we adopt the maximum likelihood (ML) carrier phase estimation method from among various carrier phase estimation schemes. Note that the ML estimation does not need a preamble to acquire the carrier phase offset [8].

The ML carrier estimator maximizes the log-likelihood function  $\log L(\theta')$ , where  $\theta'$  is the trial phase within  $(0, 2\pi)$ . This log-likelihood function [8]–[11] is given by

$$\log L(\theta') = \sum_{k=1}^K \log L(k; \theta'), \quad (2)$$

and

$$L(k; \theta') = E_c \left[ \exp \left( -\frac{E_s}{N_0} \left( |x(k)|^2 - 2\text{Re}[x^*(k)p(k)e^{-j\theta'}] \right) \right) \right],$$

where  $E_c[\cdot]$  is the averaging function for all the modulation symbols in a block,  $K$  is the number of complex data symbols, and  $p(k)$  is the sample of the matched filter output. In the  $2\pi/M$ -rotationally symmetric constellations, the carrier phase offset estimated by the ML estimation method in [8] is given by

$$\hat{\theta} = \frac{1}{M} \arg \left( \sum_{k=1}^K E[x^M(k)]p^M(k) \right). \quad (3)$$

### C. Optimal Carrier Phase Offset for Multiple Users

We propose an optimization-based approach for deriving the optimal carrier phase offsets for multiple users.

As studied in Section II, the minimum Euclidean distances between multiple constellations have a critical effect on the decoding performance of the multiuser wireless communication system. To minimize the bit error rate on the iterative decoding process, the minimum Euclidean distances between the received multiple constellation points should be maximized so that the receiver can successfully separate the each users signal from the original superimposed signal. In this section, we derive the optimal carrier phase offsets between multiple users in order that the resulting carrier phase information can be used for carrier phase adjustment on each transmitter side.

In this case, let  $c_i^u$  denote the coordinate of the  $i$ th constellation point for the  $u$ th user, and  $d(c_i^u, c_j^v)$  denote the Euclidean distance between the  $i$ th constellation coordinate of the  $u$ th user and the  $j$ th constellation coordinate of the  $v$ th user. Then, the minimum Euclidean distance ( $d_{min}$ ) between the signals of the  $u$ th and  $v$ th users is represented by

$$d_{min}(u, v) = \min_{1 \leq i, j \leq M} d(c_i^u, c_j^v), \quad (4)$$

where  $M$  is the number of constellations and is determined by the modulation scheme.

As stated above, optimal carrier phase offsets are obtained by the placement of constellation points when the minimum Euclidean distance between the points are maximized. Therefore, we formulate the following optimization problem to determine the optimal carrier phase offsets:

$$\begin{aligned} & \max_{\theta_1, \dots, \theta_N} \left( \min_{\substack{1 \leq u \leq N \\ u < v \leq N \\ 1 \leq i, j \leq M}} d(c_i^u, c_j^v) \right) \\ & \text{s.t.} \quad \forall \theta_n \in \left[ 0, \frac{\pi}{M} \right], \quad n = 1, \dots, N. \end{aligned}$$

The above optimization maximizes the minimum  $d_{min}$  for all pairs of users.

We then numerically solve the optimization for 2–4 users for QPSK and 8PSK signals using MATLAB, the results of which are listed in Table I. As an illustrative example, the two-user constellation for 8PSK is shown in Fig. 3. In this case, all

TABLE I  
OPTIMAL CARRIER PHASE OFFSETS

# of Users	Modulation	Carrier Phase Offsets			
		$\theta_1^*$	$\theta_2^*$	$\theta_3^*$	$\theta_4^*$
2	QPSK	0	0.524	-	-
	8PSK	0	0.286	-	-
3	QPSK	0	0.628	0.286	-
	8PSK	0	0.314	0.121	-
4	QPSK	0	0.628	0.286	0.143
	8PSK	0	0.349	0.314	0.121

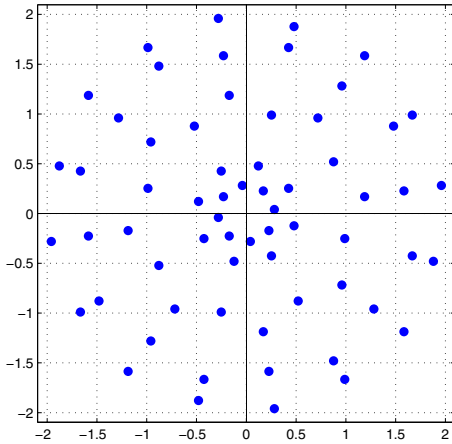


Fig. 3. Two-user constellation for 8PSK signal under the proposed carrier phase adjustment scheme.

64 constellation points are non-overlapped with the minimum Euclidean distance maximized, unlike the constellation shown in Fig. 2.

D. Example: Two-user case for QPSK signal

In a simple case such as for a two-user signal, it is possible to readily derive the optimal carrier phase offsets without numerically solving the optimization in Eq. (5). To this end, Fig. 4 shows the two-user constellation for the QPSK signal set. Here, we assume that the carrier phase offset for the first user ( $\theta_1$ ) is zero, and that the second user ( $\theta_2$ ) is in the range of 0 and  $\frac{\pi}{4}$ . With respect to  $\theta_2$ , the minimum Euclidean distance ( $d_{min}$ ) is found to be either  $\sqrt{a^2 + b^2}$  or  $\sqrt{c^2 + d^2}$  (denoted by  $d_1$  and  $d_2$ , respectively), and the distances between other points in the constellation are greater than or equal to  $d_1$  and  $d_2$ . The distances  $a$ ,  $b$ ,  $c$ , and  $d$  are given by

$$\begin{aligned} a &= 2 - \sin \theta_2 - \cos \theta_2, \\ b &= \cos \theta_2 - \sin \theta_2, \\ c &= \cos \theta_2 - \sin \theta_2, \\ d &= 1 + \sin \theta_2 - \cos \theta_2, \end{aligned}$$

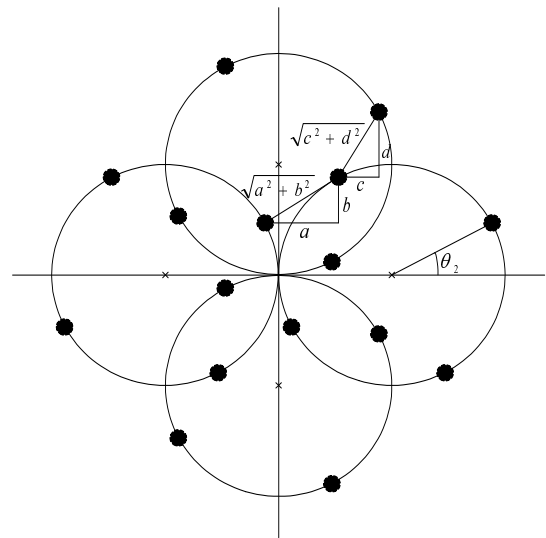


Fig. 4. The two-user constellation for the QPSK signal set.

and then

$$\begin{aligned} d_{min} &= \min(d_1, d_2), \\ d_1 &= \sqrt{6 - 4(\sin \theta_2 + \cos \theta_2)}, \\ d_2 &= \sqrt{3 - 4 \sin \theta_2 \cos \theta_2 + 2(\sin \theta_2 - \cos \theta_2)}. \end{aligned}$$

For  $0 \leq \theta_2 \leq \frac{\pi}{4}$ ,  $d_{min}$  is maximized when  $d_1$  and  $d_2$  are the same; by setting  $d_1 = d_2$ , we have  $(\sin \theta_2 = 1/2)$ .

As a result, when the second user has a carrier phase offset of  $\frac{\pi}{6}$ , the minimum distance among the points in the constellation for the two-user signal set is maximized. Note that the obtained value is equal to the numerically obtained value for the two-user case shown in Table I.

IV. PERFORMANCE EVALUATION

We conducted a performance evaluation for the proposed carrier phase adjustment scheme through a comparison with the unmodified PSK modulation scheme. We implemented the carrier phase adjustment system for MPR depicted in Fig. 1 on MATLAB (version 7.6.0), which includes the LDPC coding scheme, the mapper and demapper, modulation, iterative decoding, the carrier phase estimator, and the carrier phase feedback scheme. The LDPC [12] coding scheme has a rate of  $R = \frac{1}{2}$  and a block length of  $n = 1024$ . Two modulation schemes (QPSK and 8PSK) are evaluated, and the number of transmitters was varied from 2 to 4. Note also that the carrier phase errors are modeled to follow Gaussian distribution and the the bit error rate (BER) performance is evaluated with respect to SNR on the AWGN channel.

A. Carrier phase estimation results

First, simulations were conducted to verify the performance of the implemented carrier phase estimator. Fig. 5 shows the average estimated value  $E[\hat{\theta}]$  with respect to the real carrier phase offsets for 8PSK modulation. These simulations were performed for two cases; 50 and 1000 intervals (symbols).

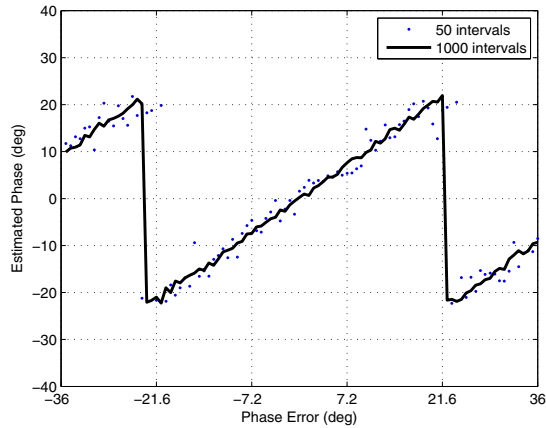


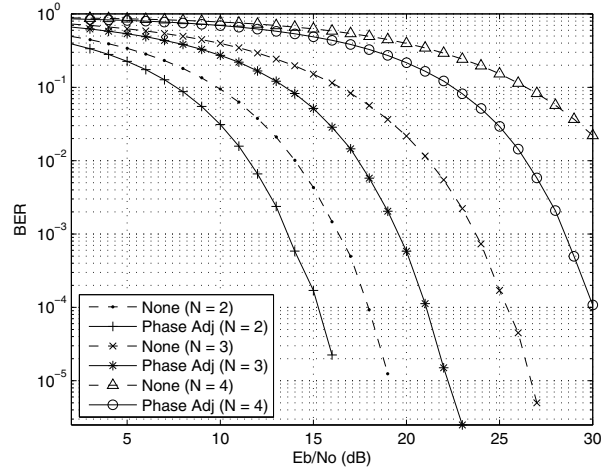
Fig. 5. Phase estimation performance for 8PSK modulation.

The figure shows that in the range of  $-\pi/8 \leq \theta \leq \pi/8$ , the estimated carrier phase linearly increases, and the estimated values are quite accurate. We can also see that the estimation accuracy becomes better as the number of symbols used in the estimation increases.

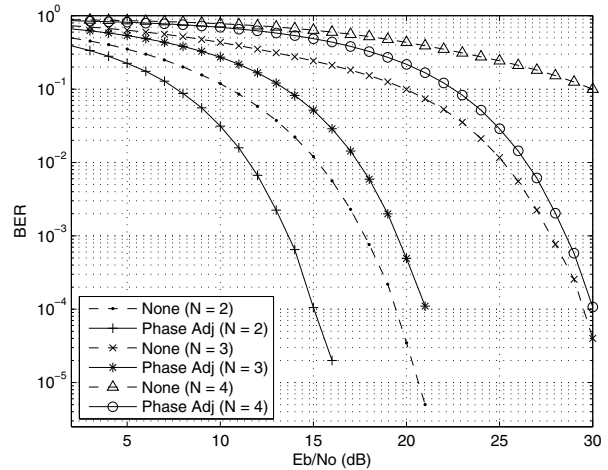
### B. Results for QPSK modulation

Fig. 6 shows the BERs of the proposed carrier phase adjustment scheme and no carrier phase adjustment case for QPSK modulation with respect to the SNR and the number of transmitters (i.e., the number of distinct signals that are compounded into the received signal at the receiver side). Fig. 6(a) depicts the BER when the random carrier phase error distribution follows three-mean Gaussian distribution with a variance of 1,  $\theta_e \sim G(3, 1)$ . In the entire SNR range, the proposed scheme gives lower BER values than the unmodified QPSK. When there are two transmitters, the proposed scheme shows a gain of about 2.5 dB at a BER of  $10^{-4}$ ; this performance gain then becomes larger as the SNR increases. In the case of three transmitters, the BER gain becomes almost twice as large as that of the two-transmitter case.

As the random carrier phase error becomes larger, the channel condition becomes more unstable. When the carrier phase errors are doubled, the proposed scheme much outperforms the unmodified QPSK as shown in Fig. 6(b). When the number of signals is two, the BER gain is comparable to that in three-mean Gaussian random carrier phase error case. As the number of transmitters increases, the BER gain of the proposed scheme becomes quite considerable. For example, with four transmitters, the proposed scheme gives approximately 8 dB better performance than the unmodified QPSK. This improvement is almost twice of the lower carrier phase error scenario at a BER of  $10^{-4}$  with the same number of transmitters. This result implies that the proposed scheme effectively adjusts the carrier phase errors so that the signals from multiple transmitters are well separated over a wide range of carrier phase error variations, and that the MPR capability is fully utilized.



(a)  $\theta_e = 3$



(b)  $\theta_e = 6$

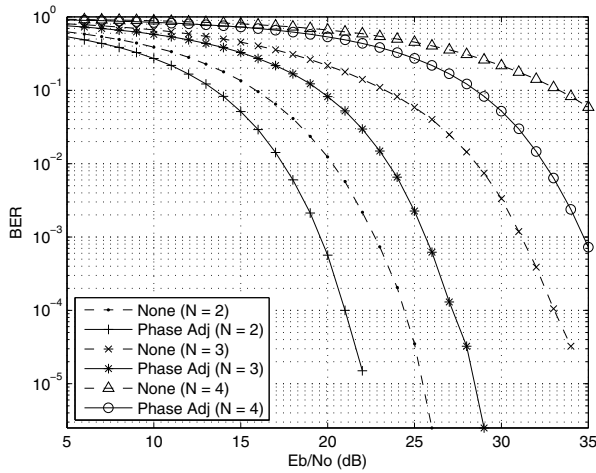
Fig. 6. BER performance for QPSK modulation for carrier phase errors of (a)  $\theta_e = 3$  and (b)  $\theta_e = 6$ .

### C. Results for 8PSK modulation

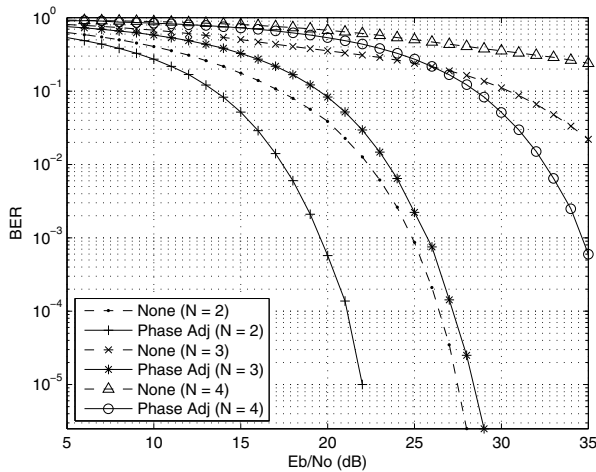
Fig. 7 shows the BER performance of the proposed scheme and the unmodified 8PSK modulation. As in the previous results for the QPSK modulation, for all cases the proposed scheme gives much lower BER values than the unmodified 8PSK. When the carrier phase error distribution is  $\theta_e \sim G(2, 1)$ , an SNR gain of almost 5 dB was obtained with four transmitters at a BER of  $10^{-4}$ ; when the carrier phase error was doubled (i.e.,  $\theta_e \sim G(4, 1)$ ) an even higher BER performance gain was achieved. However, it should be noted that with a carrier phase error of  $G(4, 1)$  the unmodified 8PSK has a very long error floor range, and thus does not ensure reliable transmission.

## V. CONCLUSION

In this paper, we proposed a feedback-based carrier phase adjustment scheme in order to fully take advantage of the



(a)  $\theta_e = 2$



(b)  $\theta_e = 4$

Fig. 7. BER performance for 8PSK modulation for carrier phase errors of (a)  $\theta_e = 2$  and (b)  $\theta_e = 4$ .

MUD technique in multi-packet reception (MPR) capable wireless networks by coordinating the carrier phase of each transmitter. To determine the optimal constellation placement of the compounded signal at the receiver, we formulated an optimization problem and then numerically obtained the optimal carrier phase offsets for 2–4 user cases for M-PSK modulation. Under the proposed carrier phase adjustment scheme, the compounded signal from multiple transmitters became more separable on its constellation at the receiver; as a result, the BER performance significantly improved in comparison with the no carrier phase adjustment cases.

As future work, we will further extend this carrier phase adjustment scheme in consideration of unbalanced power cases for realistic scenarios, as in this work we assumed that the power received for all transmitters was the same as at the receiver. Also, for a more empirical performance evaluation we plan to implement MPR capable wireless communications

based on this carrier phase adjustment scheme on software-defined-radio (SDR).

#### ACKNOWLEDGMENT

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