Matched Filter Bounds on q-ary QAM Symbol Error Probability for Diversity Receptions and Multipath Fading ISI Channels

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Abstract— **This paper extends previous works [1][2][3] in matched filter bounds and provides a theoretical calculation of detection probability of q-ary QAM signals transmitted over the diversity reception and multipath fading ISI channels. The matched filter bounds represent the best attainable detection performance of a particular system, which may or may not be realized with a practical system. While an exact analytical expression of detection performance of a transceiver system is difficult to obtain for multipath fading ISI channels, the matched filter bounds is relatively easy to obtain, provides many useful information, and gives a good comparison with the simulation results of realistic systems [5][6].**

Keywords—**Matched filter bound, QAM modulation, equalization and diversity combining.**

I. Introduction

BASED on the matched filter theory (see Chapter 6 of [4]), the detection SNR of any linearly filtered receive-signal is maximized if the matched filter, obtained assuming the receiver has the perfect knowledge of the filter, is applied to the received signal perturbed by the additive noise. In this paper, we use the matched filters to derive the symbol error probability of q-ary QAM signals transmitted over the diversity reception and multipath fading ISI channels; for the description of communication channel, see Fig.1.

A complete system of a transceiver involves many different function blocks, including a channel estimation and tracking, a synchronization, an adaptive diversity combining and equalization, and possibly an adaptive sequence decision [5][6]. In such systems, an exact calculation of detection probability is difficult. Furthermore, the approach may also dependent upon a particular selection of fading rate, detection and estimation conditions. Thus, most of times it is considered to be better off to perform simulation to obtain the exact detection performance of a system. Meanwhile, as an analytical tool to calculate link-budget requirement or as a tool to support/verify the simulation results, a fundamental capacity calculation which can be computed relatively easily and applied regardless of any particular estimation and detection scheme is extremely useful.

Previous works in calculating the matched filter bounds on fading channels include Mazo [1], Clark et. al [2], Ling [3], Proakis [7], and Jakes [9]. In Jakes and Proakis, the matched filter bounds are derived for the maximal ratio combining receiver, where each of the receive diversity-antennas in Fig.1 is assumed to be a single tap Rayleigh fading. Mazo, Clark and Ling extended the results for multipath fading frequency-selective channels with BPSK, 4-PSK or QPSK signaling. Summarizing their approaches, we note that there are three major steps. The first is to obtain the bit error or symbol error probability (SEP) expression of a single and static channel. The second is to taking the expectation of the SEP over ensemble of the channel. This results in a well-known integral form which involves an integral of error probability function over a gamma distribution. The gamma distribution represents the probability density function, denoting the probability of matched filter SNR taking a particular value. The third part is to generalize the second to diversity reception and fading ISI channels. This part involves the use of eigenvalue decomposition to decorrelate the matched filter SNR. The matched filter SNR is the quadratic combination of correlated random variables, resulting from common transmit-shape filter, the matched filter operation and diversity combining, which add up all the available SNR at each of the multipath components and diversity antennas. Eigenvalue decomposition provides the tools to obtain decorrelated SNR random variables. Then, the rest is repeated application of the second part to each of the decorrelated SNR random variables.

The derivation of matched filter bounds in this paper follows the same general framework of above three steps. The major contribution of the paper includes that the matched filter bounds are obtained for q-ary QAM signalling, where $q = 4, 16, 64$. In addition, the matched filter bounds are derived for fractionally-spaced channels with the diversity reception channels. Thus, with a specific example of fractionally-spaced multipath-power delay profiles (MPDPs), one can readily obtain the matched filter bounds and compare with their computer-simulation results.

Organization of the paper is as follows: Section II. provides system description. Section III. describes the detailed derivation of matched filter bound. Section IV. discusses three cases of interests. The three areas are 1) a single ISI channel case, 2) maximal ratio combining case, and 3) receive diversity-channels, each of the channel being ISI. Section V. provides concluding remarks.

II. System Description

Fig.1 describes the underlying channel and matched fil-

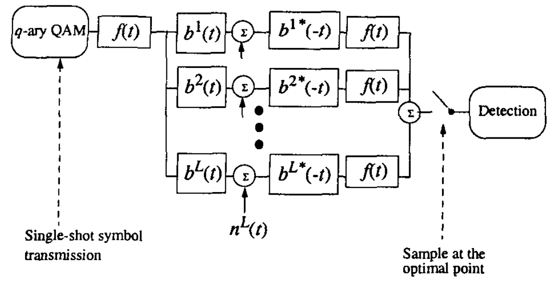

Fig. 1. Single-stot, diversity receive channel model, optimal matched filtering and combining, and detection at the ideal sampling point

ter system. q-ary QAM symbols are generated and pulse-shaped by the transmit filter $f(t)$. The transmitted signal propagates through the wireless channel and arrive to the space-diversity antennas at the receiver. The independent wireless channel is modeled as filters $\{b^l(t), l = 1, ..., L\}$. Upon receiving the signal, the optimal receiver performs matched filtering at each branch, samples at the optimum sampling time, and combines the samples from all the branches. The detection performance will be evaluated on the combined sample of the received signal. Before starting with the derivations, we shall describe some of the important assumptions we make in the derivation:

1. Matched filter bound is based on a *single-shot* symbol transmission and detection, such that it avoids the difficulty of dealing with intersymbol interference.
2. The matched filter theory applies also for the colored noise; however in this paper we assume that the noise is complex-valued white Gaussian.
3. The channel is assumed to be time-invariant over the duration of the overall pulse, which includes the channel and the transmit shaping filter (and the anti-aliasing receive filter).
4. The transmit shaping filter is assumed to employ an excess bandwidth of less than 100%, and thus the channel can be modeled as a half-symbol spaced finite impulse response filter without loss of any information.
5. The half-spaced fading components of the channel are mutually uncorrelated, (i.e., the *wide-sense stationary uncorrelated scattering assumption*[7]).

A. Single-shot system equation

Based on the assumptions made, we first define the basic equation of the receive signal for a single channel case. Cases can be generalized to multiple antennas and the results will be discussed in IV.B and IV.C. The received signal for a single-shot transmission of a pulse modulated by the information symbol I_0 can be written as

$$x_s(t) = \overbrace{\sum_{p=0}^{N_R-1} b_p f(t - pT_B)}^{h(t)} I_0 + n(t) = h(t)I_0 + n(t), \quad (1)$$

where

- I_0 denotes the q-ary QAM symbol,
- b_p denotes the p-th component of the half symbol-spaced finite impulse response (FIR) filter of the channel at a fixed instant of time, (we use $N_R = 3$ for illustration in this paper).
- From the assumption-5, we note that

$$E\{b_p^* b_{p+q}\} = \alpha_p^2 \delta(q) =: \phi_c(p). \quad (2)$$

Thus, we may define the results as the half-symbol spaced multi-path power delay profile (MPDP) $\phi_c(p)$ and note that α_p^2 denotes the average power of the p-th component.

- $f(t)$ is a square root raised cosine filter, and $F(\omega)$ denotes the Fourier transform of $f(t)$,

$$F(\omega) = \int_{-\infty}^{\infty} f(t) exp(-j\omega t) dt \quad (3)$$

- $n(t)$ denotes the zero-mean, complex-valued additive white Gaussian noise with two-side power spectral density N_0.
- $h(t)$ denotes the compound channel response for $-\infty < t < \infty$

Now consider the Fourier transform of $h(t)$, which is denoted as,

$$H(\omega) := \mathrm{F}\{h(t)\} = F(\omega) \sum_{p=0}^{N_R-1} b_p exp(-j\omega p T_B/2), \quad (4)$$

where T_B denotes the symbol-period. Then, the complex-conjugate $H^*(\omega)$ can be written as

$$H^*(\omega) = F^*(\omega) \sum_{p=0}^{N_R-1} b_p^* exp(j\omega p T_B/2). \quad (5)$$

Based on the matched filter theory [4], $H^*(\omega)$ is the optimal filter that maximizes the detection SNR. Now applying the matched filter response $H^*(\omega)$ to the received signal $x(t)$, we have the matched filtered signal which can be written in the Fourier transform domain as

$$H^*(\omega)X(\omega) = H^*(\omega)H(\omega)I_0 + H^*(\omega)N_0. \quad (6)$$

The inverse Fouier transform of (6) provides the time-domain response of the matched filtered signal. We now note that the power spectral density $H^*(\omega)H(\omega)$ is the channel and the inverse Fouier transform, $\frac{1}{2\pi}\int_{-\infty}^{\infty} H^*(\omega)H(\omega)exp(j\omega\tau)d\omega$, the autocorrelation function is the channel impulse response, being Hermitian symmetric around $\tau = 0$. Thus, by sampling the matched filtered output response at $\tau = 0$, we achieve the optimal matched filtering.

B. Sampled, matched filter output

Now let's denote z_s the receive signal sample at the optimal sampling time $\tau = 0$. Then, for the detection of I_0 the following equation shall provide the sufficient statistic,

$$z_s = A_s I_0 + v_s, \quad (7)$$

where

• A_s denotes the value of zeroth lag of the autocorrelation channel. It is a random variable representing the instantaneous energy of the cascade response $h(t)$ and can be written as,

$$\begin{aligned} A_s &= \frac{1}{2\pi}\int_{-\infty}^{\infty} H^*(\omega)H(\omega)exp(j\omega\tau)d\omega\Big|_{\tau=0} \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} H^*(\omega)H(\omega)d\omega \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} F^*(\omega)F(\omega)\sum_{p=0}^{N_R-1} b_p^* exp(j\omega pT_B/2) \\ &\times \sum_{q=0}^{N_R-1} b_q exp(-j\omega qT_B/2)d\omega. \end{aligned} \quad (8)$$

Due to the uncorrelated scattering assumption (Assumption 5), (8) can be written as

$$\begin{aligned} A_s &= \sum_{p=0}^{N_R-1}\sum_{q=0}^{N_R-1} b_p^* b_q \\ &\times \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 exp(-j\omega(q-p)T_B/2)d\omega \\ &= \sum_{p=0}^{N_R-1}\sum_{q=0}^{N_R-1} b_p^* b_q f_{rc}(t=(q-p)T_B/2), \end{aligned} \quad (9)$$

where $f_{rc}(t)$ is the raised cosine filter response,

• I_0 denotes the transmitted q-ary QAM symbol, $E\{I_0\} = 0.0$ and

$$Var(I_0) = 2(q-1)/3 \quad (10)$$

• v_s denotes the matched filtered noise output sample at $t = 0$,

$$v_s = \int_{-\infty}^{\infty} n(\tau)h(t-\tau)d\tau\Big|_{t=0}. \quad (11)$$

Thus, v_s has zero mean and $Var(v_s) = N_o A_s$.

III. MATCHED FILTER BOUND CALCULATION

A. Square-QAM symbol error probability

In this section, the symbol error probability will be evaluated for square QAM constellations, i.e. for even q. Referring to Fig.2, we start with a summary of following relationships which shall be found useful for the discussion of materials in the sequel:

• The average energy of the square-QAM constellation can be computed as, using (10) and the definition given in Fig.2

$$E_s = E[I_0^2]\left(\frac{A_s}{\sqrt{2}}\right)^2 = 2(q-1)A_s^2/3 \quad (12)$$

• The minimum Euclidean distance of the square-QAM constellation is

$$d_{min} = \sqrt{2}A_s \quad (13)$$

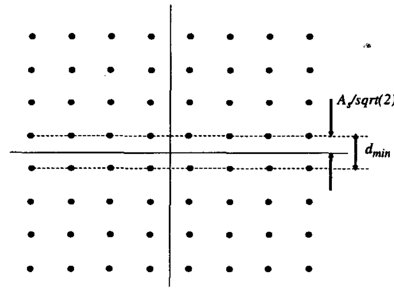

Fig. 2. Receive-constellation for 64-QAM, where A_s denotes the instantaneous channel gain, combining all the channel branches.

• The instantaneous signal to noise ratio is

$$\frac{SignalPower}{NoisePower} = \gamma = k\gamma_b = \frac{E_s}{A_s N_o} = \frac{(q-1)A_s}{3N_o}, \quad (14)$$

where γ is the instantaneous SNR, $k = \log_2(\sqrt{q})$ the number of bits per symbol, γ_b is the instantaneous SNR/bit.

Then, the q-ary square-QAM symbol error probability at a particular channel gain $A_s = a$, can be computed as

$$\begin{aligned} P_q(A_s = a) &= 2\left(1-\frac{1}{\sqrt{q}}\right) erfc\left(\frac{d_{min}(a)}{2\sqrt{Var(v_s)}}\right) \\ &\times \left\{1 - \frac{1}{2}(1-\frac{1}{\sqrt{q}})erfc(\frac{d_{min}(a)}{2\sqrt{Var(v_s)}})\right\}. \end{aligned} \quad (15)$$

(15) can be tightly upper-bounded by the first term, which is (16). We note that (16) is *asymptotically efficient* and very tight approximation of (15). Fig.3 shows how good

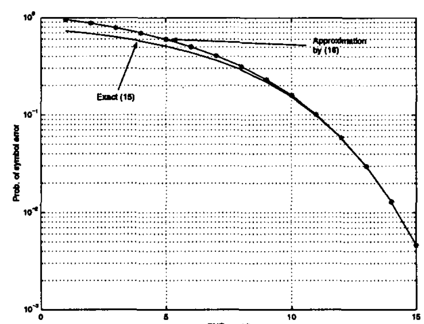

Fig. 3. Illustration of tight upper bound of the square QAM symbol error rate, an example with M = 64

the approximation is. In the sequel, we shall use the approximation for $P_q(a)$, i.e.

$$P_q(a) \approx 2\left(1-\frac{1}{\sqrt{q}}\right) erfc\left(\frac{d_{min}(a)}{2\sqrt{Var(v_s)}}\right). \quad (16)$$

Now, solving for a particular value a of A_s from (12), i.e., $a^2 = 3E_s/(q-1)$ we have

$$d_{min}(a) = \sqrt{2}a = \sqrt{2\cdot 3E_s/(q-1)}, \quad (17)$$

but using (14), (17) is

$$d_{min}(a) = \sqrt{2}a = \sqrt{\frac{2\cdot 3}{q-1}ak\gamma_b N_o}, \tag{18}$$

Then, the approximation of the symbol error probability (16) can be written as

$$P_q(a) = 2\left(1-\frac{1}{\sqrt{q}}\right) erfc\left(\sqrt{\frac{3k\gamma}{2(q-1)}}\right), \tag{19}$$

or simply

$$P_q(a) = 2\left(1-\frac{1}{\sqrt{q}}\right) erfc\left(\sqrt{\frac{a}{2N_o}}\right). \tag{20}$$

We shall use this approximation (20) to derive the averaged symbol error probability in the sequel.

B. Average symbol error probability for square-QAM

Now,the symbol error probability, averaged over the ensemble of the channel or equivalently that of A_s, can be computed from

$$\bar{P}_q(\bar{\gamma}_b) = \int_0^{\infty} P_q(a)Pr(A_s = a)da, \tag{21}$$

where $\bar{P}_q(\bar{\gamma}_b)$ denotes the averaged symbol error probability of q-ary QAM system for the average input SNR which is defined as

$$\bar{\gamma}_b = E\{\gamma_b\} = \frac{q-1}{3kN_o}E\{A_s\}. \tag{22}$$

Thus, we need to know the distribution of the random variable A_s. From (9), we may note that the random variable can be written as follows. We illustrate the case with $N_R = 3$ without loss of generality:

$$A_s = \begin{pmatrix} b_1^* & b_2^* & b_3^* \end{pmatrix} \times \underbrace{\begin{pmatrix} f_{rc}(0) & f_{rc}T_B/2) & f_{rc}(T_B) \\ f_{rc}(-T_B/2) & f_{rc}(0) & f_{rc}(T_B/2) \\ f_{rc}(T_B) & f_{rc}(-T_B/2) & f_{rc}(0) \end{pmatrix}}_{\mathbf{F}_{rc}} \overbrace{\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}}^{\mathbf{b}}. \tag{23}$$

Denote the matrix in the middle as $\mathbf{F}_{rc}$, where $f_{rc}(t)$ denote the raised cosine function. Now, we represent each of the fading channel tap as $b_i = \alpha_i \rho_i$, multiplication of an attenuation factor and the unit-variance, complex-valued Gaussian random variable ρ_i. We now can write the channel vector b as

$$b := \alpha\rho = \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix}, \tag{24}$$

where $E\{\rho\rho^H\}$ is a 3 x 3 identity matrix because $\rho_i, i = 1,2,3$, are assumed mutually uncorrelated. Using (24), (23) can now be rewritten,

$$A_s = b^H \mathbf{F}_{rc} b = \rho^H \alpha^H \mathbf{F}_{rc} \alpha\rho = \rho^H \mathbf{G}\rho, \tag{25}$$

where we have defined $\mathbf{G} := \alpha^H \mathbf{F}_{rc}\alpha$ for the last equality. It is important to note that for a fixed MPDP, $\mathbf{G}$ is fixed. Also note that $\mathbf{G}$ is Hermitian symmetric. In addition, since A_s is the energy of the cascade filter (8) it is non-negative definite. For any non-negative definite Hermitian symmetric matrix , there exist an orthonormal matrix such that

$$\mathbf{G} = \mathbf{Q}^H \Lambda \mathbf{Q}, \tag{26}$$

where Λ is a diagonal matrix with the diagonal elements, $\lambda \geq 0, p = 1,2,3$, being the eigenvalues of the matrix $\mathbf{G}$. Now rewriting (25) using (26) we have

$$A_s = \rho^H \mathbf{G}\rho = \rho^H \mathbf{Q}^H \Lambda \mathbf{Q}\rho = \dot{\rho}^H \Lambda \dot{\rho} = \sum_{p=0}^{N_R-1} \lambda_p |\dot{\rho}_p|^2, \tag{27}$$

where we define $\dot{\rho} = \mathbf{Q}\rho$. Note that $\dot{\rho}_p, p = 1,2,3$, are again mutually independent, complex-valued Gaussian random variables with zero-mean and unit-variance, and thus $\lambda_p|\dot{\rho}_p|^2, p = 1,2,3$, are the χ^2-distributed random variables with the characteristic function $\frac{1}{1-jv\lambda_p}$. Thus, the characteristic function of A_s has the product form

$$E\{\exp(jvA_s)\} = \prod_{p=1}^{N_R} \frac{1}{1-jv\lambda_p}. \tag{28}$$

IV. Three cases of interests

We now want to evaluate the average probability of symbol errors. For easy of illustration, we divide the tasks based on three cases of interests. The first is the case with a single, finite ISI channel, and A_s is to be represented with N_R distinctive eigenvalues. The second is the case with L-diversity antenna channels, having flat-fading channel at each antenna, and A_s is to be represented with a single eigenvalue repeating L-times. The third is the case with a combination of the first two, and A_s is to be represented with L-repeated set of N_R-distinctive eigenvalues. With an input of set of parameters for MPDPs, the shaping filter and averaged input SNR, the derivation in this section shall allow us to obtain the averaged, square-QAM symbol error probability as a function of averaged input SNR for any L.

A. Distinct eigenvalues (no eigenvalues in multiplicity)

When all the eigenvalues are distinct, (28) can be expressed as

$$E\{\exp(jvA_s)\} = \sum_{p=1}^{N_R} \frac{\pi_p}{1-jv\lambda_p} \tag{29}$$

where we have defined the weight of an individual random variable to be

$$\pi_p = \prod_{\substack{q=1 \\ p\neq q}}^{N_R} \frac{1}{1-\lambda_q/\lambda_q}. \tag{30}$$

We now write the probability density function for N_s which is the weighted sum of N_R χ^2-distributed random variables. That is,

$$Pr\{A_s = a\} = \sum_{p=1}^{N_R} \pi_p \frac{\exp(-a/\lambda_p)}{\lambda_p}. \quad (31)$$

Now substituting (31) and (20) into (21) we have

$$\begin{aligned}\bar{P}_q(\bar{\gamma}_b) &= \int_0^\infty P_q(a) Pr(A_s = a) da, \\ &= 2\left(1 - \tfrac{1}{\sqrt{q}}\right) \sum_{p=1}^{N_R} \pi_p \int_0^\infty erfc\left(\sqrt{\tfrac{a}{2N_o}}\right) \tfrac{\exp(-a/\lambda_p)}{\lambda_p} da.\end{aligned} \quad (32)$$

Now define

$$Y := \frac{A_s}{2N_o}, \quad (33)$$

then by the change of variable (32) can be rewritten

$$\begin{aligned}\bar{P}_q(\bar{\gamma}_b) =& 4\left(1 - \tfrac{1}{\sqrt{q}}\right) \Bigg\{ \tfrac{1}{2} \sum_{p=1}^{N_R} \pi_p \\ & \times \int_0^\infty erfc(\sqrt{y}) \tfrac{\exp(-y/\dot{\lambda}_p)}{\dot{\lambda}_p} dy \Bigg\},\end{aligned} \quad (34)$$

where we defined $\dot{\lambda}_p := \frac{\lambda_p}{2N_o}, p = 1, \ldots, N_R$. Note that the weight terms π_p stays the same. Then, (34) becomes

$$\bar{P}_q(\bar{\gamma}_b) = 4\left(1 - \frac{1}{\sqrt{q}}\right) \left\{ \frac{1}{2} \sum_{p=1}^{N_R} \pi_p \left[1 - \sqrt{\frac{\dot{\lambda}_p}{1 + \dot{\lambda}_p}} \right] \right\}. \quad (35)$$

We are now at the last step of computing the matched filter bound. For this, it shall be advantageous to straighten out the relationship between the average SNR/bits and the eigenvalues:

$$\bar{\gamma}_b = \frac{2q-1}{3k} \frac{E\{A_s\}}{2N_o} = \frac{2q-1}{3k} E\{Y\} = \frac{2q-1}{3k} \frac{1}{2N_o} \sum_{p=1}^{N_R} \lambda_p. \quad (36)$$

Now, the following steps describe the procedure of how to compute the matched filter symbol error probability when the input parameters are the average SNR/bits $\bar{\gamma}_b$, the constellation size q, the multipath power delay profile and $L = 1$.

- Evaluate the eigenvalues $\{\lambda_i, i = 1, \ldots, N_R\}$ using the MPDP and the transmit shaping filter, as described in (23) to (27).
- Now determine the value of $\frac{1}{2N_o}$ for the given value of $\bar{\gamma}_b$ and q by

$$\frac{1}{2N_o} = \frac{3\bar{\gamma}_b \log_2 q}{2(q-1)\sum_{i=0}^{N_R} \lambda_i} \quad (37)$$

- Calculate $\{\dot{\lambda}_i, i = 1, \ldots, N_R\}$ by evaluating

$$\dot{\lambda}_i = \frac{\lambda_p}{2N_o} \quad (38)$$

- Finally, substitute (38) into (35) to calculate the average symbol error probability.

B. Eigenvalues occurring in multiplicity

We now consider the case with an eigenvalue repeating L times, i.e.

$$E\{\exp(jvA_s)\} = (\frac{1}{1 - jv\lambda_1})^L. \quad (39)$$

This case arises when we have equal gain, L independent diversity antennas. Then, (27) takes the expression

$$A_s = \sum_{p=1}^{L} \lambda_1 |\dot{\rho}_p|^2, \quad (40)$$

where $|\dot{\rho}_p|^2$ again are iid χ^2- distribution with zeo mean and unit variance. The distribution function for this case is $Pr\{A_s = a\} = \frac{1}{(L-1)!\lambda_1^L} a^{L-1}$. Then, the average symbol error probability is

$$\begin{aligned}\bar{P}_q(\bar{\gamma}_b) &= \int_0^\infty P_q(a) Pr(A_s = a) da \\ &= 4\left(1 - \frac{1}{\sqrt{q}}\right) \int_0^\infty \frac{1}{2} erfc\left(\sqrt{\frac{a}{2N_o}}\right) \\ &\quad \times \frac{1}{(L-1)!\lambda_1^L} a^{L-1} \exp(-a/\lambda_1) da.\end{aligned} \quad (41)$$

Then, by defining $Y = \frac{A_s}{2N_o}$, we have

$$\begin{aligned}\bar{P}_q(\bar{\gamma}_b) &= \int_0^\infty P_q(a) Pr(A_s = a) da \\ &= 4\left(1 - \frac{1}{\sqrt{q}}\right) \int_0^\infty \frac{1}{2} erfc(\sqrt{y}) \\ &\quad \times \frac{1}{(L-1)!\dot{\lambda}_1^L} y^{L-1} \exp(-y/\dot{\lambda}_1) dy,\end{aligned} \quad (42)$$

where we again defined $\dot{\lambda}_p := \frac{\lambda_p}{2N_o}$. Then, we obtain[1]

$$\bar{P}_q(\bar{\gamma}_b) = 4\left(1 - \tfrac{1}{\sqrt{q}}\right) \left(\tfrac{1-\Omega}{2}\right)^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left(\tfrac{1+\Omega}{2}\right)^k, \quad (43)$$

where we defined $\Omega := \sqrt{\dot{\lambda}_1/(1 + \dot{\lambda}_1)}$.

Finally, we have following procedure to compute the averaged symbol error probability with the input parameters of the average SNR/bits $\bar{\gamma}_b$, the constellation size q and the number of equal gain antenna diversity L.

- Determine the value of for the given value of $\bar{\gamma}_b$ and q by

$$\frac{1}{2N_o} = \frac{3\bar{\gamma}_b \log_2 q}{2(q-1)L\lambda_1} \quad (44)$$

- Compute $\frac{\lambda_1}{2N_o}$ and $\Omega = \sqrt{\dot{\lambda}_1/(1 + \dot{\lambda}_1)}$
- Finally, substitute Ω into (43) to calculate the average symbol error probability

The situation considered in this subsection is for when the channel at each diversity branch is modeled as a

[1] Using $\int_0^\infty \frac{1}{2} erfc(\sqrt{y}) \frac{1}{(L-1)!\dot{\lambda}_1^L} y^{L-1} \exp(-y/\dot{\lambda}_1) dy = \left(\frac{1-\Omega}{2}\right)^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left(\frac{1+\Omega}{2}\right)^k$ [7]

single-tap Rayleigh fading and mutually independent. Then, the matched filter combiner becomes the well known *maximal ratio combiner*. Fig.4, Fig.5 and Fig.6 are the matched filter bounds for q-QAM with L-diversity antennas, for $q = 4, 16, 64$ and $L = 1, 2, 4$, repectively.

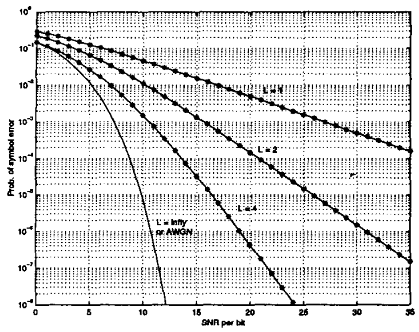

Fig. 4. Matched filter bound SEP for 4-QAM transmission over L-flat fading channels

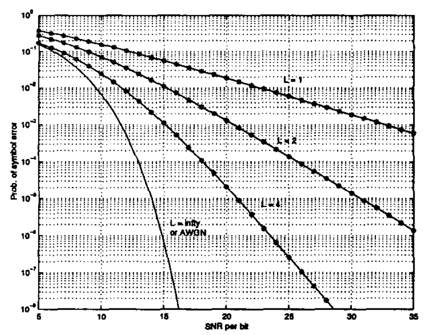

Fig. 5. Matched filter bound SEP for 16-QAM transmission over L-flat fading channels

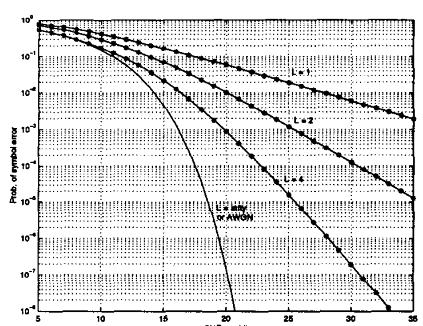

Fig. 6. Matched filter bound SEP for 64-QAM transmission over L-flat fading channels

As the order of diversity increases, we note that the matched filter bounds of fading channel approach the SER performance of the AWGN channels.

C. Combination of distinctive and multiple poles

We now consider the case where each channel is ISI channel. Now, the instantaneous channel gain can be written as

$$A_s = \sum_{p=1}^{L} \sum_{p=0}^{N_R} \lambda_p \left| \dot{\rho}_p^l \right|^2, \tag{45}$$

where $\dot{\rho}_p^l$ are mutually independent, for $p = 1, \ldots, N_R$ and $l = 1, \ldots, L$ are mutually independent, complex-valued Gaussian random number with zero-mean and unit-variance, and thus $\lambda_p \left| \dot{\rho}_p^l \right|^2$ are iid χ^2-distributed random variables. Note that the MPDP stays the same for each of different antennas, and thus the same set of N_R (distinct) eigenvalues should be repeating L times. Thus, the characteristic function becomes

$$E\{\exp(jvA_s)\} = \prod_{p=1}^{N_R} \frac{1}{(1 - jv\lambda_p)^L}. \tag{46}$$

Now, let's proceed with an example of $L = 2$ and $N_R = 3$. Using the method of partial fraction expansion (46) can be decomposed into

$$\prod_{p=1}^{3} \frac{1}{(1 - jv\lambda_p)^2} = \sum_{p=1}^{3} \left(\frac{\Gamma_{2,p}}{(1 - jv\lambda_p)^2} + \frac{\Gamma_{1,p}}{1 - jv\lambda_p} \right), \tag{47}$$

where $\Gamma_{\cdot,\cdot}$ denotes the expansion coefficients from partial fraction operation. Then, the probability density function is

$$Pr\{A_s = a\} = \sum_{p=0}^{2} \left(\Gamma_{1,p} \frac{\exp(-a/\lambda_p)}{\lambda_p} + \Gamma_{2,p} \frac{a^{L-1} \exp(-a/\lambda_p)}{(L-1)! \lambda_p^L} \right). \tag{48}$$

Then, the average symbol error probability is

$$\begin{aligned} \bar{P}_q(\bar{\gamma_b}) &= \int_0^\infty P_q(a) Pr(A_s = a) da \\ &= \int_0^\infty 2\left(1 - \tfrac{1}{\sqrt{q}}\right) erfc\left(\sqrt{\tfrac{a}{2N_o}}\right) \\ &\quad \times \sum_{p=0}^{2} \left(\tfrac{\Gamma_{1,p} \exp(-a/\lambda_p)}{\lambda_p} + \tfrac{\Gamma_{2,p} a^{L-1} \exp(-a/\lambda_p)}{(L-1)! \lambda_p^L} \right) da \\ &= 4\left(1 - \tfrac{1}{\sqrt{q}}\right) \sum_{0}^{2} \left(\Gamma_{1,p} P_1(\dot{\lambda}_p) + \Gamma_{2,p} P_2(\dot{\lambda}_p) \right) \end{aligned} \tag{49}$$

where we have defined

- $P_1(\dot{\lambda}_p) = \frac{1}{2}\left(1 - \sqrt{\dot{\lambda}_p/(1 + \dot{\lambda}_p)}\right)$
- $P_2(\dot{\lambda}_p) = \left[\frac{1-\Omega}{2}\right]^L \sum_{p=0}^{L-1} \binom{L-1+p}{p} \left(\left(\frac{1+\Omega}{2}\right)\right)^p$
- $\dot{\lambda}_p = \frac{\lambda_p}{2N_o}$

Finally, we have following steps to compute the average probability, given the MPDP, the number of diversity channel L, the average SNR/bits $\bar{\gamma_b}$ and the constellation size q:

- Define the average SNR/bits (note, this is not the average SNR/bits/channel),

$$\bar{\gamma_b} = \frac{2q - 1}{3k} \frac{E\{A_s\}}{2N_o} = \frac{2q - 1}{3k} E\{Y\} = L \frac{2q - 1}{3k \cdot 2N_o} \sum_{p=1}^{N_R} \lambda_p \tag{50}$$

• Evaluate the eigenvalues $\{\lambda_p, p = 1, \ldots, N_R\}$ for a given L, the MPDP and the transmit shaping filter (taking the same approach as (23) to (27)

• Compute $\frac{1}{2N_o}$ with

$$\frac{1}{2N_o} = \frac{3\bar{\gamma}_b \log_2 q}{2(q-1)L\sum_{p=1}^{N_R}\lambda_p} \tag{51}$$

• Evaluate $\dot{\lambda}_p$

$$\dot{\lambda}_p = \frac{\lambda_p}{2N_o} \tag{52}$$

• Finally, substitute (52) into (49) to calculate the average symbol error probability

Fig.7, Fig.8 and Fig.9 show the matched filter bounds for q-QAM transmission with $L = 1, 2$, for q = 4, 16, and 64 respectively, over the multipath fading frequency-selective channels. The multipath power delay profiles we used are MPDP-1= (0.7413, 0.2343, 0.0234) and MPDP-2= (0.66520.24470.0900). The normalized (in symbol-period) rms delay spreads of the two MPDPs are $0.2494T_B$ and $0.3257T_B$ respectively. We note from the SEP curves that the detection performance for MPDP-2 is about 1 to 2 dB better than that for MPDP-1. From this, we confirm a well known diversity effect of the wireless channel such that the larger the delay spread is the better the expected detection performance, due to inherent diversity benefit of the delay spread channel.

V. Conclusions

In this paper, we have derived analytical expressions for symbol error probability using the matched filter SNR for the square-QAM signals transmitted over the diversity frequency-selective channels. These theoretical bounds may not be attainable in reality due to the impractical assumptions made in deriving the bounds. Nonetheless, they provide invaluable information in designing the complex communication systems and serves as easy-to-compute analytical tools that can readily be compared with the simulation results of practical transceiver schemes. Specifically, we shall observe the exact relationship between the asymptotic slopes of SER curves and influences of different shapes of MPDPs. Future work include the extension of the matched filter bounds for trellis-coded modulation cases, which will be useful to be compared with the simulation results [8].

References

[1] J.E. Mazo, "Exact matched filter bound for two-beam rayleigh fading," *IEEE Trans. on Commun.*, vol. 39, no. 7, pp. 1027–30, 1991.

[2] M.V. Clark et. al., "Matched filter performance bounds for diversity combining receivers in digital mobile radio," *IEEE Trans. Veh. Technol.*, vol. 41, no. 4, pp. 356–62, Nov. 1992.

[3] F. Ling, "Matched filter-bound for time-discrete multipath rayleigh fading channels," *IEEE Trans. on Commun.*, vol. 43, no. 2/3/4, pp. 710–13, Feb./Mar./Apr. 1995.

[4] A.D. Whalen, *Detection of Signals in Noise*, Academic Press, Inc., 1971.

[5] Heung-No Lee and G.J. Pottie, "Fast adaptive equalization/diversity combining for time-varying dispersive channels," *IEEE Trans. on Commun.*, vol. 46, pp. 1146–62, Sept. 1998.

[6] Heung-No Lee and G.J. Pottie, "Adaptive sequence detection using t-algorithm for multipath fading isi channels," *IEEE International Conf. on Commun., Proc. of Commun. Theory Mini Conf.*, pp. 125–9, June 1999.

[7] J.G. Proakis, *Digital Communications*, McGraw-Hill, New York, 3rd edition, 1989.

[8] Heung-No Lee and G.J. Pottie, "Adaptive sequence detection of channel-interleaved trellis-coded modulation signals over multipath fading isi channels," *Proc. of IEEE 49th Vehic. Tech. Conf.*, pp. 1474–9, May 1999.

[9] W.C. Jakes, *Microwave Mobile Communications*, Wiley, New York, 1974.

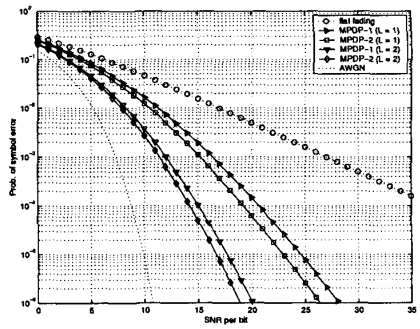

Fig. 7. Matched filter bound SEP for 4-QAM transmission over L-multipath fading ISI channels

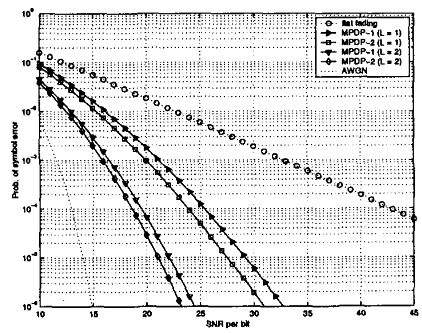

Fig. 8. Matched filter bound SEP for 16-QAM transmission over L-multipath fading ISI channels

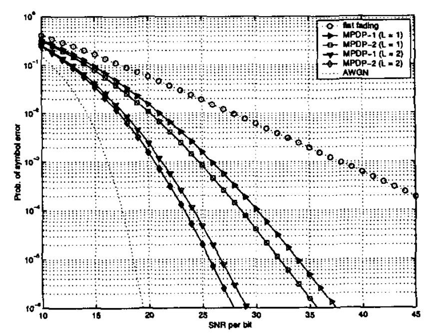

Fig. 9. Matched filter bound SEP for 64-QAM transmission over L-multipath fading ISI channels