Low-Complexity Iterative Per-Antenna MAP Equalizer for MIMO Frequency Selective Fading Channels

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Abstract— **We consider the equalization of a (N_t, N_r) MIMO, $L+1$-tap fading channel. We first evaluate the performance of a full complexity, vector MAP equalizer which runs the forward/backward algorithm on a trellis which has $M^{N_t \times L}$ states with M^{N_t} transitions from each state when an $M-$ary constellation is used. This MAP equalizer achieves the $N_r \times (L+1)$ diversity benefit while realizing $N_t \times \log_2(M)$ bits/sec/Hz. We then propose a novel iterative per-antenna MAP(PAMAP) approach which can be used (1) to reduce the complexity and (2) to achieve the full diversity order $N_t \times N_r \times (L+1)$ at the rate of $\log_2(M)$ bits/sec/Hz. The proposed equalizer consists of a probabilistic "signal separator" and a bank of N_t PAMAPs each having M^L states with M transitions from each state. The signal separator and the PAMAPs exchange extrinsic information during iterations. Simulation results indicate that the proposed receiver closely achieves the performance of the full complexity MAP within 2 to 3 iterations. The proposed scheme saves a significant amount of complexity in uncoded systems with a large number of transmit antennas and a high modulation order. In coded systems, the PAMAP scheme becomes more beneficial when iterative equalization and decoding is used.**

I. Introduction

Wireless signals experience time-varying frequency selective fading due to the combined effect of multipath propagations, mobility of transceivers and changing environments. Realizing the maximum diversity benefit, which may be available in space, frequency or time domain, is critical in designing robust transceivers for these harsh conditions. Spatial diversity at the receiver, frequency and time diversity have all been widely studied [1]. Recently, transmit diversity has received widespread attention sparked by the capacity calculations [2], [3] which promise a linear increase in capacity with the number of transmit antennas in a rich scattering channel environment.

The transmit diversity can be used to achieve a gain either in capacity or in SNR. For example, the BLAST architecture [4] uses the transmit diversity to obtain increased capacity and the space-time codes [5] mainly use it for getting an SNR gain. In this paper, we address both directions with a general assumption of time-varying frequency selective fading channels, which is modelled with N_t-transmit, N_r-receive antennas and $L+1$ time-varying taps for each of the sub-channels. We thus have an additional diversity factor of $L+1$ which may be realized with employment of a good equalizer. For the capacity gain approach, we first consider the transmission of uncoded, independent streams of data symbols at each transmit antenna(also called *direct* transmission). In this scheme, the transmission rate is $N_t \times \log_2(M)$ bits/sec/Hz and the expected diversity order is $N_r \times (L+1)$. For the SNR gain approach, we transmit identical information at each transmit antenna. A sequence transmitted at one transmit antenna is simply an random-interleaved version of the others transmitted at different antennas. Thus, the rate is $\log_2(M)$ bits/sec/Hz–not increasing with N_t, and the maximum diversity order $N_t \times N_r \times (L+1)$ is achieved–proportionally increasing with N_t. These two examples show that the rate and diversity orders may be conveniently exchanged for one another. It shall be noted, however, that each of the two transmission schemes must be accompanied with an enabling equalizer at the receiver which indeed materializes all the potential benefits.

Frequency-selective channels introduce inter-symbol interference(ISI) into the transmit signals which may also be viewed as additional diversity available in the frequency domain. In order to exploit this benefit, the use of a good equalizer is essential[1]. The optimal equalizer, the maximum *aposteriori* probability(MAP) [6] rule, is extremely complex and a number of low-complexity equalizers for MIMO ISI channels have been proposed. Choi and Cioffi [7] used space-time block codes to obtain diversity after canceling ISI using a linear equalizer. Choi, Cheong and Cioffi proposed a low-complexity iterative soft-interference canceller [8] to counter ISI in this scenario. Tehrani *et al* [9] formulated a recursive least-squares(RLS) solution for the decision feedback equalizer(DFE). Bjerke and Proakis [10] gave theoretical analysis of MLSE and sub-optimal detectors(linear and decision feedback detectors) in a MIMO fading ISI channel.

We propose a low-complexity iterative equalizer which employs the MAP criterion as the underlying detection principle. Besides good performance, motivation for choosing the MAP rule is at least twofold: (1) the turbo-like message passing can be implemented naturally with the MAP, which works well with the low complexity equalization scheme proposed in this paper; (2) it can be easily extended to allow for iterative demodulation and decoding schemes[2].

The main contributions of the paper are as follows. We propose a novel probabilistic detection framework consisting of a

†This work was carried out while the authors were with HRL Laboratories, L.L.C., Mr. Gulati as a summer intern in year 2001 and Dr. Lee as a research staff. The corresponding author for this work is Heung-no Lee who is currently with the Electrical Engineering Department of the University of Pittsburgh.

[1]There are other modulation technologies such as CDMA or OFDM which do not need an explicit equalization. However, they are not within the scope of this research.

[2]We simulated an iterative decoding and demodulation scheme with an LDPC code [11] as the channel code. The results show that the proposed equalizer is within 0.5dB of the MAP equalizer [12]

"signal separator" and a bank of "per-antenna" MAP(PAMAP) equalizers. The signal separator singles out the contribution of a particular transmit-receive antenna pair at a given time instant. The PAMAPs, working on the output of the signal-separator, make a sequence-based soft decision on the transmit symbols. The two signal processing blocks exchange extrinsic information in a turbo-like fashion as iterations proceed. We then show the following simulation results. First, the full complexity vector-MAP achieves the diversity order of $N_r \times (L+1)$ at the rate of $N_t \times \log_2(M)$ bits/sec/Hz. Second, the sub-optimal PAMAP approach achieves the full diversity order, indicating only a few dB SNR difference from the performance of the full MAP. Third, for random interleaved sequence transmission scheme at a rate $\log_2(M)$ bits/sec/Hz the PAMAP again achieves the full diversity order of $N_t \times N_r \times (L+1)$. For this transmission scheme, it should be noted, the full MAP is unavailable due to the use of the random interleaver. The proposed technique is able to exploit all the diversity available in both the spatial and frequency domains with a much reduced complexity.

The rest of the paper is organized as follows. Section II describes the system model. Section III describes the proposed solution. Simulation results are presented in section IV. We conclude with some remarks in section V.

II. System Model

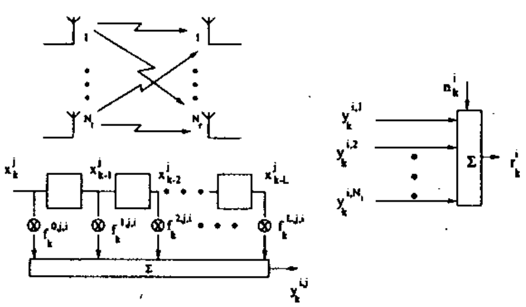

Fig. 1. The MIMO-ISI channel model.

We consider the transmission of data through a time-varying, frequency-selective fading channel using N_t transmit antennas, in blocks of size N. These $N \times N_t$ symbols may either be raw data or coded symbols. The channel is modeled as a symbol-spaced $L+1$ tap Rayleigh fading channel. Let N_r be the number of receive antennas(Figure 1). The received signal on antenna i at time instant k is given by the super-position of the signals transmitted on each transmit antenna and the inter-symbol interference experienced by these symbols:

$$
\begin{aligned}
r_k^i &= \sum_{j=0}^{N_t} \left[\sum_{l=0}^{L} f_k^{l,j,i} x_{k-l}^j \right] + n_k^i \\
&= \sum_{j}^{N_t} y_k^{i,j} + n_k^i \\
&= y_k^i + n_k^i
\end{aligned}
\tag{1}
$$

Thus, there are $M^{(L+1)N_t}$ possible values of y_k^i. It would be convenient to collect all the transmit and receive signals into column vectors:

$$
\mathbf{x}_k = \begin{bmatrix} x_k^1 \\ x_k^2 \\ \vdots \\ x_k^{N_t} \end{bmatrix} \quad \text{and} \quad \mathbf{r}_k = \begin{bmatrix} r_k^1 \\ r_k^2 \\ \vdots \\ r_k^{N_r} \end{bmatrix}
$$

Also, define the $N_r \times N_t$ matrix of "clean" signals as below:

$$
\begin{aligned}
\mathbf{Y}_k &= \begin{bmatrix} y_k^{1,1} & y_k^{1,2} & \cdots & y_l^{1,N_t} \\ y_k^{2,1} & y_k^{2,2} & \cdots & y_l^{2,N_t} \\ \vdots & \vdots & \ddots & \vdots \\ y_k^{N_r,1} & y_k^{N_r,2} & \cdots & y_l^{N_r,N_t} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{y}_k^1 \\ \mathbf{y}_k^2 \\ \vdots \\ \mathbf{y}_k^{N_r} \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{y}}_k^1 & \bar{\mathbf{y}}_k^2 & \cdots & \bar{\mathbf{y}}_k^{N_t} \end{bmatrix}
\end{aligned}
$$

Further, (1) may be re-written as:

$$
\begin{aligned}
\mathbf{r}_k &:= \begin{bmatrix} r_k^1 \\ r_k^2 \\ \vdots \\ r_k^{N_r} \end{bmatrix} = \begin{bmatrix} y_k^1 + n_k^1 \\ y_k^2 + n_k^2 \\ \vdots \\ y_k^{N_r} + y_k^{N_r} \end{bmatrix} \\
&= \begin{bmatrix} y_k^{1,1} + y_k^{1,2} + \cdots + y_k^{1,N_t} \\ y_k^{2,1} + y_k^{2,2} + \cdots + y_k^{2,N_t} \\ \vdots \\ y_k^{N_r,1} + y_k^{N_r,2} + \cdots + y_k^{N_r,N_t} \end{bmatrix} + \begin{bmatrix} n_k^1 \\ n_k^2 \\ \vdots \\ n_k^{N_r} \end{bmatrix} \\
&= \begin{bmatrix} y_k^{1,1} \\ y_k^{2,1} \\ \vdots \\ y_k^{N_r,1} \end{bmatrix} + \begin{bmatrix} y_k^{1,2} \\ y_k^{2,2} \\ \vdots \\ y_k^{N_r,2} \end{bmatrix} + \cdots + \begin{bmatrix} y_k^{1,N_t} \\ y_k^{2,N_t} \\ \vdots \\ y_k^{N_r,N_t} \end{bmatrix} + \mathbf{n}_k \\
&= \bar{\mathbf{y}}_k^1 + \bar{\mathbf{y}}_k^2 + \cdots + \bar{\mathbf{y}}_k^{N_t} + \mathbf{n}_k
\end{aligned}
\tag{2}
$$

The fade coefficient $f_k^{l,j,i}$, representing the $l-$th tap between transmit antenna j and receive antenna i at time instant k, is a sample of a Rayleigh fading process. All the taps on all antennas are assumed to be fading independent of one another. The auto-correlation of all fading processes is assumed to be identical and depends only on the normalized Doppler rate. The power-delay profile is normalized:

$$
\begin{aligned}
\sum_{l=0}^{L} \mathrm{E}\left[\left| f_k^{l,j,i} \right|^2 \right] &= 1 \quad \forall i,j \\
& \quad i = 1, \ldots, N_r \text{ and } j = 1, \ldots, N_t
\end{aligned}
\tag{3}
$$

where $E[x]$ denotes the expectation of x.

The transmit symbols x_k^j belong to an $M-$ary constellation. Let E_s be the energy of each transmit symbol. The noise samples n_k^i are zero mean, complex Gaussian random variables with variance $N_0/2$ in each dimension. Signal-to-noise ratio(SNR) is defined in terms of the average SNR per bit. That is, SNR $= 1/N_r \sum_{i=1}^{N_r} \text{SNR}_i$. For the uncoded system, the overall rate is $N_t \times \log_2(M)$ bits/s/Hz and the SNR is given by $N_t E_s/N_0$. In the sequel we also assume that all the fades are known(perfect CSI). In practice, the fast fading channel taps may be estimated by employing, for example, snap-shot channel estimation using pilot symbols and interpolation [13].

III. Iterative Per-antenna MAP Equalizer

In this section we first describe the full complexity, vector MAP equalizer, which is a straightforward extension of the well known sequence-based MAP symbol estimation criterion [6], [14] to the MIMO ISI channel. Next, we describe the proposed low complexity iterative equalizer. Lastly, we make a complexity comparison.

A. Vector MAP Equalizer

The MAP equalizer makes a sequence-based decision on the transmit symbols using the maximum *aposteriori* criterion [6]. For our MIMO setting, this can be written as

$$\hat{\mathbf{x}}_k = \arg\max_{\mathbf{x}\in\Omega^{N_t}} \Pr\{\mathbf{x}_k|\mathbf{r}_j, j=1,2,\ldots,N\} \qquad (4)$$

where Ω denotes the $M-$ary signal constellation set. The probabilities involved in the maximization above are the soft information generated by the equalizer. Since the ISI channel is equivalent to a Markov process, the MAP rule may be described in terms of operations on a trellis. Each trellis transition, originating from a state S_{s-1} and terminating in a state S_s, is marked with an input(or transmit) vector $\mathbf{x}^s$ and an output(or "clean") vector $\mathbf{y}^s$. The state at any time k is defined completely by the $N_t \times L$ past signals. Let $\chi_x := \{(x_{k-1}, x_{k-2}, \ldots, x_{k-L}) : x_j \in \Omega\}$. Thus, the possible states belong to the set $\chi_x^{N_t}$, which has a cardinality M^{LN_t}. The output vectors may similarly be identified using the elements of the set $\chi_y^{N_t}$ where $\chi_y := \{(x_k, x_{k-1}, x_{k-2}, \ldots, x_{k-L}) : x_j \in \Omega\}$. The probability term in (4) is, thus, the sum of probabilities of making all the transitions that have $\mathbf{x}$ as the input vector at time epoch k:

$$\begin{aligned}
&\Pr\{\mathbf{x}_k|\mathbf{r}_j, j=1,2,\ldots,N\}\\
&= \sum_{\mathbf{m}\in\chi_x^{N_t}} \Pr(\mathbf{x}^s = \mathbf{x}_k, S_k = \mathbf{m}|\mathbf{r}_j, j=1,2,\ldots,N)\\
&= \frac{1}{\Pr(\mathbf{r}_j, j=1,2,\ldots,N)} \times \sum_{\mathbf{m}'\in\chi_x^{N_t}} \sum_{\mathbf{m}\in\chi_x^{N_t}}\\
&\qquad \Pr\{\mathbf{x}^s = \mathbf{x}_k, S_{k-1} = \mathbf{m}', S_k = \mathbf{m}, \mathbf{r}_j, j=1,2,\ldots,N\}\\
&= K \sum_{\mathbf{m}'}\sum_{\mathbf{m}} \Pr(\mathbf{r}_j, j=k+1,\ldots,N|S_k=\mathbf{m})\\
&\qquad \times \Pr(S_{k-1}=\mathbf{m}'|\mathbf{r}_j, j=1,\ldots,k)\\
&\qquad \times \Pr(\mathbf{x}^s = \mathbf{x}_k, S_k=\mathbf{m}, \mathbf{r}_k|S_{k-1}=\mathbf{m}') \qquad (5)
\end{aligned}$$

The factor K is just a constant normalization factor and the last decomposition follows from the Bayes rule and the Markovian property. Next, as in [15] we define the following probability functions:

$$\alpha_k(\mathbf{m}) = \Pr(S_k = \mathbf{m}|\mathbf{r}_j, j=1,2,\ldots,k) \qquad (6)$$

$$\beta_k(\mathbf{m}) = \frac{\Pr(\mathbf{r}_j, j=k+1,\ldots,N|S_k=\mathbf{m})}{\Pr(\mathbf{r}_j, j=k+1,\ldots,N|\mathbf{r}_j, j=1,\ldots,k)} \qquad (7)$$

$$\gamma(\mathbf{x}^s, \mathbf{r}_k, \mathbf{m}', \mathbf{m}) = \Pr(\mathbf{x}_k = \mathbf{x}^s, S_k = \mathbf{m}, \mathbf{r}_k|S_{k-1} = \mathbf{m}') \qquad (8)$$

Thus, (5) may be rewritten as:

$$\begin{aligned}
&\Pr\{\mathbf{x}_k|\mathbf{r}_j, j=1,2,\ldots,N\} =\\
&\qquad K\sum_{\mathbf{m}'}\sum_{\mathbf{m}} \alpha_{k-1}(\mathbf{m}')\gamma(\mathbf{x}^s, \mathbf{r}_k, \mathbf{m}', \mathbf{m})\beta_k(\mathbf{m}) \qquad (9)
\end{aligned}$$

The quantities $\alpha_k(\mathbf{m})$ and $\beta_k(\mathbf{m})$ satisfy the following recursions [15], [14]

$$\alpha_k(\mathbf{m}) = \frac{\sum_{\mathbf{m}'}\sum_{\mathbf{x}^s} \gamma(\mathbf{x}^s,\mathbf{r}_k,\mathbf{m}',\mathbf{m})\alpha_{k-1}(\mathbf{m}')}{\sum_{\mathbf{m}}\sum_{\mathbf{m}'}\sum_{\mathbf{x}^s} \gamma(\mathbf{x}^s,\mathbf{r}_k,\mathbf{m}',\mathbf{m})\alpha_{k-1}(\mathbf{m}')} \qquad (10)$$

$$\beta_k(\mathbf{m}) = \frac{\sum_{\mathbf{m}'}\sum_{\mathbf{x}^s} \gamma(\mathbf{x}^s,\mathbf{r}_k,\mathbf{m},\mathbf{m}')\beta_{k+1}(\mathbf{m}')}{\sum_{\mathbf{m}}\sum_{\mathbf{m}'}\sum_{\mathbf{x}^s} \gamma(\mathbf{x}^s,\mathbf{r}_k,\mathbf{m},\mathbf{m}')\beta_{k+1}(\mathbf{m}')} \qquad (11)$$

The probability $\gamma(\mathbf{x}^s, \mathbf{r}_k, \mathbf{m}', \mathbf{m})$ may be written as a product of three terms:

$$\begin{aligned}
\gamma(\mathbf{x}^s, \mathbf{r}_k, \mathbf{m}', \mathbf{m}) &= p(\mathbf{r}_k|\mathbf{x}_k = \mathbf{x}^s, S_k = \mathbf{m}, S_{k-1} = \mathbf{m}')\\
&\quad \times q(\mathbf{x}_k = \mathbf{x}^s|S_k = \mathbf{m}, S_{k-1} = \mathbf{m}')\\
&\quad \times \pi(S_k = \mathbf{m}|S_{k-1} = \mathbf{m}') \qquad (12)
\end{aligned}$$

where $p(.|.)$ is the transition probability of the MIMO channel, $q(\mathbf{x}_k = \mathbf{x}^s|S_k = \mathbf{m}, S_{k-1} = \mathbf{m}')$ is a zero or a one depending on whether the transition from state $\mathbf{m}'$ to state $\mathbf{m}$ is marked with $\mathbf{x}_k$ or not and $\pi(S_k = \mathbf{m}|S_{k-1} = \mathbf{m}')$ is the *apriori* probability of this transition. The *apriori* probability is useful in a coded system where this information is obtained from the channel decoder. In the uncoded case, this probability is the same for all transitions and may be thought of as a normalization term.

There are a total of $M^{N_t \times L}$ states with M^{N_t} transitions from each state. Thus, the complexity of this algorithm is of the order of $(M^{(L+1)N_t})$. This excludes the complexity involved in computing the transition probabilities $p(\mathbf{r}_k|\mathbf{x}_k = \mathbf{x}^s, S_k = \mathbf{m}, S_{k-1} = \mathbf{m}')$ from the received vectors $\mathbf{r}_k$.

B. Proposed equalizer

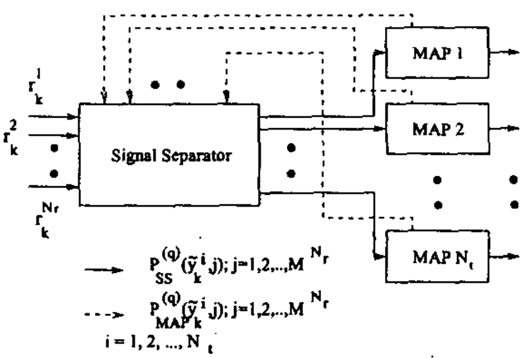

Fig. 2. The "per-antenna" MAP Equalizer.

The high complexity of the full MAP equalizer results from the fact that it searches the full state space $\mathbf{y}^i = y^{i,1} + y^{i,2} + \cdots + y^{i,N_t}, i = 1,2,\ldots,N_r$. Each of the $y^{i,j}$ is the output of the channel which has L memory elements. In this section, we develop a novel receiver structure that probabilistically singles out the contribution of a single transmit antenna, say $y^{i,j}, i = 1, 2, \ldots, N_r$, from the received signal $\mathbf{r}_k$ and then performs an MAP search on the reduced state space of $\tilde{\mathbf{y}}^j = [y^{1,j}\ y^{2,j}\ \ldots\ y^{N_r,j}]^T$.

The proposed equalizer consists of two parts – the signal separator and the per-antenna MAP bank. The signal separator generates the probabilities of the "clean signal" vectors $\tilde{\mathbf{y}}_k^i, i = 1, 2, \ldots, N_t$ from the received signal $\mathbf{r}_k$ as follows:

$$
\begin{aligned}
\Pr(\tilde{\mathbf{y}}_k^1 = \tilde{\mathbf{y}}_1, \tilde{\mathbf{y}}_k^2 = \tilde{\mathbf{y}}_2, \ldots, \tilde{\mathbf{y}}_k^{N_t} = \tilde{\mathbf{y}}_{N_t} | \mathbf{r}_k) = \\
\frac{\Pr(\mathbf{r}_k | \tilde{\mathbf{y}}_k^1 = \tilde{\mathbf{y}}_1, \tilde{\mathbf{y}}_k^2 = \tilde{\mathbf{y}}_2, \ldots, \tilde{\mathbf{y}}_k^{N_t} = \tilde{\mathbf{y}}_{N_t})}{\Pr(\mathbf{r}_k)} \\
\times \prod_{i=1}^{N_t} \Pr(\tilde{\mathbf{y}}_k^i = \tilde{\mathbf{y}}_i) \qquad (13)
\end{aligned}
$$

The term in the numerator is the transition probability of a MIMO channel. The term in the denominator is a constant and may be treated as a normalizing factor. Each of the terms in the product denotes the *apriori* probabilities. During the first iteration, all the symbols are assumed to be equally likely. During the later iterations, the extrinsic information obtained from the PAMAPs is treated as *apriori* information by the signal separator. Specifically, during the q−th iteration:

$$
\Pr(\tilde{\mathbf{y}}_k^i = \tilde{\mathbf{y}}_i) \quad = \quad P_{\text{MAP}}^{(q-1)}(\tilde{\mathbf{y}}_k^i, i) \qquad (14)
$$

During the q−th iteration, the signal separator passes extrinsic information about the $\tilde{\mathbf{y}}_k^i$ to the PAMAPs:

$$
\begin{aligned}
P_{\text{SS}}^{(q)}(\tilde{\mathbf{y}}_k^i, i) = \Pr_{\text{ext}}(\tilde{\mathbf{y}}_k^i = \tilde{\mathbf{y}}_i) = \\
\frac{1}{P_{\text{MAP}}^{(q-1)}(\tilde{\mathbf{y}}_k^i, i)} \times \sum_{\tilde{y}_1} \cdots \sum_{\tilde{y}_{i-1}} \sum_{\tilde{y}_{i+1}} \cdots \sum_{\tilde{y}_{N_t}} \{ \\
\Pr(\tilde{\mathbf{y}}_k^1 = \tilde{\mathbf{y}}_1, \tilde{\mathbf{y}}_k^2 = \tilde{\mathbf{y}}_2, \ldots, \tilde{\mathbf{y}}_k^{N_t} = \tilde{\mathbf{y}}_{N_t} | \mathbf{r}_k)\} \quad (15)
\end{aligned}
$$

where each $\tilde{\mathbf{y}}_i \in \chi_y^{N_r}$.

The per-antenna MAPs(PAMAPs) treat the extrinsic information generated by the signal separator as outputs of a channel(marked with solid lines in Fig. 2). The processing is identical to what has been described in the previous section. Each of these MAP filters now work on a trellis of only M^L states and there are only M transitions from each state. The trellises are now marked with symbols transmitted from a particular antenna, x^s, and the received vector $\tilde{\mathbf{y}}^s$. Thus, during the q−th iteration, the j-th PAMAP computes the probabilities:

$$
\Pr\left\{ x_k^j = x^j \in \Omega \,\middle|\, P_{\text{SS}}^{(q)}(\tilde{\mathbf{y}}_t^j, i), t = 1, \ldots, N; i = 1, \ldots, M^{LN_r} \right\}
$$

Once again, the kernel term for the j−th PAMAP may be computed easily:

$$
\begin{aligned}
&\gamma^j\{x^s, \tilde{\mathbf{r}}_k^j, \mathbf{m}', \mathbf{m}\} \\
&= \quad \Pr(x_k^j = x^s, S_k = \mathbf{m}, \tilde{\mathbf{r}}_k^j | S_{s-1} = \mathbf{m}') \\
&= \quad p(\tilde{\mathbf{r}}_k^j | x_k^j = x^s, S_k = \mathbf{m}, S_{k-1} = \mathbf{m}') \\
&\quad \times q(x_k^j = x^s | S_k = \mathbf{m}, S_{k-1} = \mathbf{m}') \\
&\quad \times \pi(S_k = \mathbf{m} | S_{k-1} = \mathbf{m}') \qquad (16)
\end{aligned}
$$

where, with a slight abuse of notation, we may write:

$$
p(\tilde{\mathbf{r}}_k^j | x_k^j = x^s, S_k = \mathbf{m}, S_{k-1} = \mathbf{m}') = P_{\text{SS}}^{(q)}(\tilde{\mathbf{y}}_k^j, j)
$$

Further, the terms $\mathbf{m}$ and $\mathbf{m}'$ now take on values in the set χ_x. The forward and the backward recursions for the PAMAPs are also straight-forward to write. In addition to the *aposteriori* probabilities of the transmit signals, these filters also compute the probability updates for the clean signals:

$$
\begin{aligned}
P_{MAP}^{(q)}(\tilde{\mathbf{y}}_k^i, i) = \Pr_{\text{ext}}(\tilde{\mathbf{y}}_k^i = \tilde{\mathbf{y}}_i) = \\
\sum \sum_{\mathbf{m}, \mathbf{m}' : \tilde{y}_k^i = \tilde{y}_i} \frac{\alpha_{k-1}(\mathbf{m}')\gamma(\mathbf{x}^s, \tilde{\mathbf{r}}_k^i, \mathbf{m}', \mathbf{m})\beta_k(\mathbf{m})}{P_{\text{SS}}^{(q)}(\tilde{\mathbf{y}}_k^i, i)} \quad (17)
\end{aligned}
$$

The extrinsic information thus generated by the PAMAPs is passed back to the signal separator(dashed lines in Fig. 2), which treats this information as *apriori* for the next iteration.

C. Complexity Comparison

The MAP equalizer has a complexity of the order of $(M^{(L+1)N_t})$. The computation of probabilities from the received signal in (12) and the signal separator in (15) are of the order of $M^{N_t \times (L+1)}$ and $M^{N_r \times (L+1)}$ respectively. The N_t MAP filters in the proposed scheme require an order of $M^{(L+1)}$ computation. Hence, the overall saving in complexity is given by:

$$
\frac{\alpha M^{(L+1)N_t} + \beta M^{(L+1)N_t}}{N_i \left(\alpha N_t M^{(L+1)} + \beta M^{(L+1)N_r} \right)}
$$

where α and β are constants independent of M, N_t, N_r and L; and N_i is the number of iterations used in the proposed scheme. Since the MAP is much more complex than computing the cross-over probabilities of the MIMO channel, the constant α is much larger than β. Thus, the approximate saving in complexity is about $M^{(L+1)(N_t-1)}/(N_i N_t)$.

Thus, we see that the proposed scheme will give larger savings in complexity when the number of transmit antennas, channel order and modulation order are large. Further, the proposed scheme has an inherently parallelizable structure, making it more suitable for hardware implementation.

IV. Simulation Results

We consider a system with 2 transmit antennas and 1 or 2 receive antennas. The channel is a 3-tap Rayleigh fading channel with a power delay profile $[\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}]$. The modulation is BPSK. The frames consist of 1024 transmissions out of each antenna. All the algorithms are implemented in the log-domain.

Fig. 3 shows the performance of the MAP equalizer and the proposed "per-antenna" MAP equalizer when the normalized Doppler is 0.01. Independent data is transmitted over the two transmit antennas, giving an overall rate of 2 bits/sec/Hz. After 3 iterations, the proposed equalizer is about 3.5dB away from the MAP equalizer when 1 receive antenna is used and about 1.75dB away when 2 receive antennas are used.

The proposed scheme must also be compared for two other parameters other than the uncoded error rates. These are the optimality of the soft-outputs generated and the diversity order achieved. The optimality of the soft-outputs may be determined by employing the equalization scheme in a coded system. Our simulation results with an LDPC code [11] and the proposed

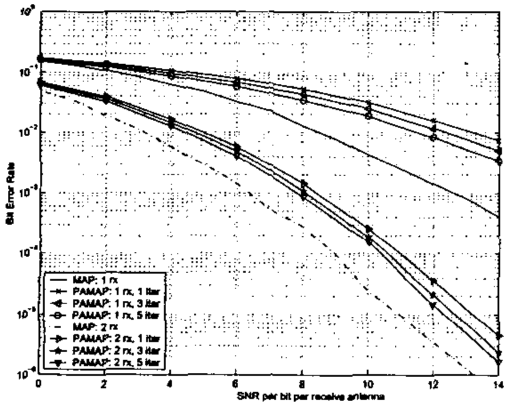

Fig. 3. Uncoded Bit Error Rate Performance, normalized Doppler = 0.01.

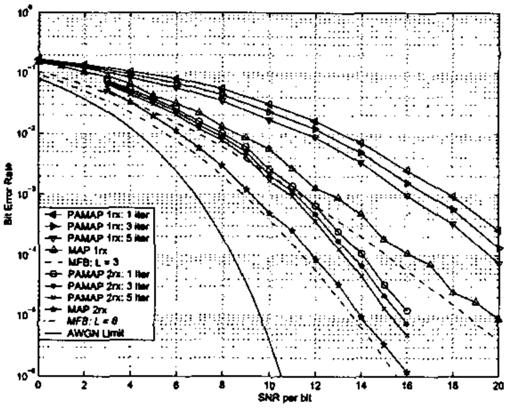

Fig. 4. Comparison with the Matched Filter Bound. Normalized Doppler is 0.001. The slopes of the curves show that the proposed scheme achieves all the space and frequency diversity.

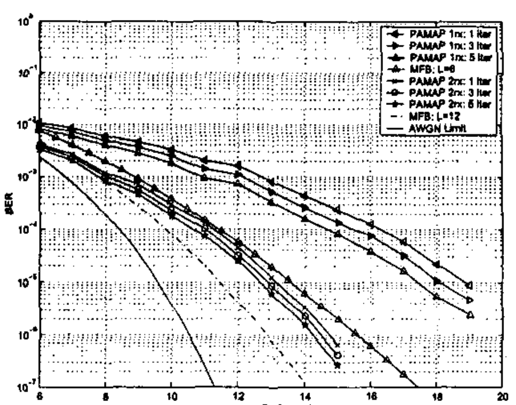

Fig. 5. Achieving all the diversity: comparison with matched filter bounds for maximal ratio combining. Overall rate is 1b/s/Hz. Normalized Doppler is 0.001.

equalizer(results not included here due to space limitation) indicate that the proposed scheme can perform within 0.5dB of the performance obtained using the MAP equalizer.

We compare the diversity order achieved by the proposed scheme with matched-filter bounds in Fig. 4. The bounds have been computed for an equivalent order of receive diversity [16]. When one receive antenna is used, the proposed scheme achieves a diversity order of three. When two receive antennas are used, the achieved diversity order is six. In order to trade rate for diversity, we transmit the data on one transmit antenna and a randomly interleaved version of the same data on the other transmit antenna. The overall rate is now 1 b/s/Hz using BPSK. The achievable diversity orders now become 6 and 12 respectively for 1 and 2 receive antennas(Fig. 5). Thus, the proposed scheme utilizes all the available spatial and frequency diversity.

Fig. 6-8 show the frame error rates for the various cases described above.

V. Conclusions

We have proposed an iterative MAP equalizer for use in MIMO fading ISI channels. The complexity of the proposed scheme increases only linearly with the number of transmit antennas. However, since the underlying rule is still a MAP rule, the complexity of the proposed scheme increases exponentially with the channel order and modulation order. The proposed idea is quite general and has a highly parallelizable structure. Simulation results have shown performance close to the optimal MAP equalizer within a few iterations. The proposed scheme also achieves all the spatial and frequency diversity available.

We have shown that the transmit diversity order can be readily traded with the capacity increase. With the direct transmission scheme, we achieve the rate $N_t \times \log_2(M)$ while achieving the diversity order $N_r \times (L+1)$. With the random interleaving transmission scheme, we achieve the rate $\log_2(M)$ while achieving the maximum diversity $N_t \times N_r \times (L+1)$. It should be noted that for the latter, only the proposed PAMAP equalizer is employable; the full complexity vector MAP is not realizable due to the use of the random interleaver.

The idea of separating equalizers on a per-antenna basis is a general one. Applying this idea to equalizers other than the MAP equalizer(e.g., DFE, Soft-canceller etc) is a topic for our future research.

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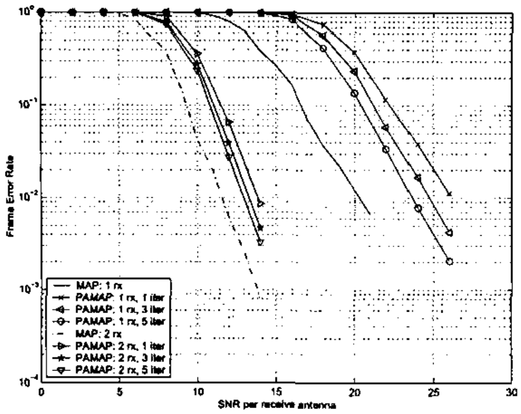

Fig. 6. Uncoded Frame Error Rate Performance, normalized Doppler = 0.01.

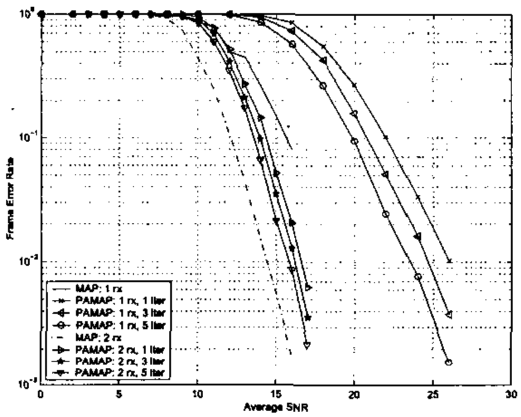

Fig. 7. Uncoded Frame Error Rate Performance, normalized Doppler = 0.001.

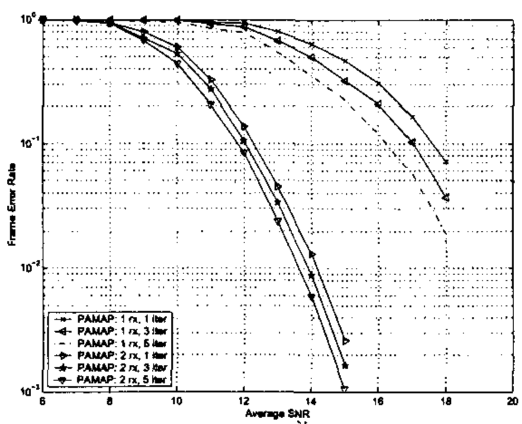

Fig. 8. Frame Error Rate Performance for the 1b/s/Hz system, normalized Doppler = 0.01.

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