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Performance Analysis of Binary DPSK Modulation Schemes over Hoyt Fading Channels

Nazih Hajri (Ecole Supérieure de Communications de Tunis, Sup'Com, Tunisia); Neji Youssef (Ecole superieure des communications de Tunis, Tunisia); Matthias Pätzold (University of Agder, Norway)

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Schnorr-Euchner Sphere Decoder with Statistical Pruning for MIMO Systems

Junil Ahn (GIST, Korea); Heung-No Lee (University of Pittsburgh, USA); Kiseon Kim (GIST - Gwangju, Republic of Korea, Korea)

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Schnorr-Euchner Sphere Decoder with Statistical Pruning for MIMO Systems

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***Abstract*—A near-maximum-likelihood (ML) detection algorithm for spatially multiplexed multiple-input multiple-output (MIMO) systems has been considered. The sphere decoder (SD) is one of the promising techniques to solve the ML problem. However the SD has a loose necessary condition for pruning branches, and it becomes impractical in large dimensional systems. We propose a Schnorr-Euchner SD with statistical pruning (SP-SESD) in order to further reduce complexity with small performance degradation. Squared *statistical constraint radius* (SCR) and expected partial path metric from unvisited levels are defined from statistics of noises, and two pruning conditions are jointly applied to search tree for detection efficiency. A flexible trade-off between bit error rate (BER) and complexity can be supported by selecting two pruning probabilities in the proposed scheme, and hence one can design various MIMO detectors according to system demands. Simulation results show the proposed SP-SESD requires lower computational complexity than any statistical pruning approaches, although performance degradation is negligible. The proposed algorithm is effective for MIMO systems with any number of antennas.**

I. Introduction

The maximum likelihood (ML) detector provides optimal bit error rate (BER) performance for spatially multiplexed MIMO (SM-MIMO) systems and solves the integer least-squares problem by exhaustive search over multiple dimensions. This is well-known to be NP-hard. To reduce computational burden with the ML performance, the sphere decoder (SD) has been introduced in [1]–[5]. The expected complexity of SD behaves polynomially, about cubic, for moderate numbers of transmit antennas and constellation orders [5]. The main idea of SD is to search the closest lattice point inside a hypersphere centered at the received signal vector. One SD algorithm known as the Fincke-Pohst sphere decoder (FPSD) is based on enumeration strategy proposed in [1] for finding the shortest lattice vector. In [2], FPSD was applied to complex-valued MIMO channels for lattice code decoding. The Schnorr-Euchner SD (SESD) was suggested as a refined version of FPSD [3], [4]; SESD tries to firstly search candidate symbol with shorter branch metric at each level, thus the shrinkage of search space is facilitated. Indeed SESD shows better computational efficiency than FPSD dose.

Although FPSD and SESD can provide the ML performance for SM-MIMO systems, they still have substantial complexity burdens than suboptimal methods such as linear detectors and Vertical Bell Laboratories Layered Space Time (V-BLAST) detector [6]. Therefore, there have been various researches on the algorithmic modification of SD aiming to alleviate computational complexity. One promising technique is a statistical pruning strategy. In [7], one statistical pruning approach called the increasing radii algorithm (IRA) was presented. It adopted a schedule of radii to prune branches which seem to constitute incorrect paths. IRA was developed on the basis with enumeration method of FPSD. Another statistical pruning approach referred to as the radius-adaptive SD (RA-SD) was applied to the SESD in [8]. RA-SD regarded the probabilistic distribution of noises as expected metric from unvisited levels. From this metric, a tighter necessary condition was obtained to shrink search space at each level.

From the results of papers [7], [8], we note that detection complexity still remains to be a significant problem for especially large dimensional systems. We aim to address this issue and propose a powerful statistical pruning rule. This rule is developed under the framework of SESD, and we call it SP-SESD in this paper. It is an unified statistical pruning approach and can subsume previous statistical pruning rules by carefully choosing design parameters. The proposed SP-SESD is obtained by devising a pruning algorithm which takes the merits of both IRA and RA-SD algorithms and can achieve fully pruning gain in the search process. It makes use of *statistical constraint radius* (SCR) and expected partial path metric from unvisited levels with which we can control the early elimination of unlikely paths. As a result, SP-SESD algorithm provides superior performance to any existing algorithms. To the best of author's knowledge, the proposed detection method is novel.

The rest of this paper is organized as follows: In Section II, MIMO system model is described, the SD algorithm and enumeration methods of FPSD and SESD are briefly reviewed. Section III presents the proposed SP-SESD in detail. In Section IV, simulation results show advantages of our scheme. Finally, conclusion is offered in Section V.

Notation: Matrices and column vectors are denoted using upper and lower bold face letters, respectively; $(\cdot)^T$ and $(\cdot)^\dagger$ denote the matrix transposition and the pseudoinverse operation; $\|(\cdot)\|$ denotes the Euclidean norm of $(\cdot)$; $\mathcal{C}^{N\times M}$ and $\mathcal{C}^M$ represent the set of all complex $N \times M$ matrices and the set of all complex $M \times 1$ vectors, respectively; $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ denote real and imaginary part of $(\cdot)$, respectively; $\chi_k^2(\sigma^2)$ stands for the Chi-square distribution with k degrees of freedom having probability density function (pdf) $f(x) = \frac{1}{2^{k/2}\Gamma(k/2)\sigma^k} x^{(k/2)-1} e^{-x/(2\sigma^2)}$ where $\Gamma(k)$ denotes the Gamma function, and especially χ_k^2 represents the standard Chi-square distribution ($\sigma^2 = 1$) with k degrees of freedom; $\sim$ means "follow distribution as".

II. System Model And Sphere Decoder

The baseband model for an uncoded SM-MIMO system with n_R receive antennas and n_T transmit antennas ($n_R \geq n_T$) is represented by

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{x}} + \tilde{\mathbf{z}} \tag{1}$$

where $\tilde{\mathbf{x}} = [\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_{n_T}]^T$ is the vector of transmitted symbols in which each symbol is selected independently from a complex signal constellation and satisfies the component-wise power constraint $\mathrm{E}[\|x_i\|^2] = E_s/n_T$, and the total transmit power is E_s in each channel use. $\tilde{\mathbf{y}} \in \mathcal{C}^{n_R}$ is the received signal vector, $\tilde{\mathbf{z}} \in \mathcal{C}^{n_R}$ is the independent and identically distributed (i.i.d.) complex additive white Gaussian noise (AWGN) vector of which components have zero mean and a variance of $\sigma^2 = \frac{N_0}{2}$ per dimension, $\tilde{\mathbf{H}} \in \mathcal{C}^{n_R \times n_T}$ is uncorrelated slow Rayleigh fading baseband MIMO channel model and has a full column rank, and elements $\tilde{h}_{i,j}$ of $\tilde{H}$ are assumed to be i.i.d complex Gaussian variable with zero mean and a variance of 0.5 per dimension. We assume perfect channel knowledge is given at the receiver.

One can rewrite the complex model (1) as a real-valued model, i.e.,

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z} \tag{2}$$

where $\mathbf{y} = [\mathrm{Re}(\tilde{\mathbf{y}})^T \mathrm{Im}(\tilde{\mathbf{y}})^T]^T$, $\mathbf{x} = [\mathrm{Re}(\tilde{\mathbf{x}})^T \mathrm{Im}(\tilde{\mathbf{x}})^T]^T$, $\mathbf{z} = [\mathrm{Re}(\tilde{\mathbf{z}})^T \mathrm{Im}(\tilde{\mathbf{z}})^T]^T$, and

$$\mathbf{H} = \begin{bmatrix} \mathrm{Re}(\tilde{\mathbf{H}}) & -\mathrm{Im}(\tilde{\mathbf{H}}) \\ \mathrm{Im}(\tilde{\mathbf{H}}) & \mathrm{Re}(\tilde{\mathbf{H}}) \end{bmatrix}. \tag{3}$$

Here, $\mathbf{H}$ is a $N \times M$ real-valued MIMO channel matrix where $N = 2n_R$, $M = 2n_T$. The ML detector tries to find the closest lattice point $\hat{\mathbf{x}}_{\mathrm{ML}}$ for a lattice $\Lambda(\mathbf{H})$, i.e.,

$$\hat{\mathbf{x}}_{\mathrm{ML}} = \arg\min_{\mathbf{x} \in \mathcal{D}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 \tag{4}$$

where we assume $\mathcal{D} = \mathcal{A}^M$ and $\mathcal{A}$ is a real-valued signal constellation set; $\mathcal{A} = \{-3, -1, 1, 3\}$ for 16-QAM.

A. Fincke-Pohst Sphere Decoding

Basically, FPSD and SESD can be formulated in a similar manner. They simply employ different strategies to enumerate candidate symbols. Hence, the original SD algorithm is constructed from the notion of Finke and Pohst algorithm [1].

In order to find the ML solution, the SD searches over only lattice points that lie inside a hypersphere of a radius C_0 centered at received signal vector $\mathbf{y}$ [2]. This concept can be written as

$$\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 \leq C_0^2. \tag{5}$$

If the radius C_0 is large enough to include the ML solution, the SD can guarantee to provide the optimal ML solution with reduced complexity, contrary to the exhaustive ML detector. The QR decomposition is applied to the real-valued channel matrix $\mathbf{H}$ in (5) and then, the SD constraint of (5) can be rewritten as

$$\begin{aligned} &\left\| \mathbf{y} - \mathbf{Q} \begin{bmatrix} \mathbf{R} \\ \mathbf{0}_{(N-M)\times M} \end{bmatrix} \mathbf{x} \right\|^2 \\ &= \left\|\mathbf{Q}_1^T\mathbf{y} - \mathbf{R}\mathbf{x}\right\|^2 + \left\|\mathbf{Q}_2^T\mathbf{y}\right\|^2 \leq C_0^2 \end{aligned} \tag{6}$$

where $\mathbf{Q} = [\mathbf{Q}_1 \mathbf{Q}_2]$ is an $N \times N$ orthogonal matrix which is partitioned into an $N \times M$ matrix of $\mathbf{Q}_1$ and an $N \times (N-M)$ matrix of $\mathbf{Q}_2$ respectively, and $\mathbf{R}$ is an $M \times M$ upper triangular matrix with non-negative diagonal entries. Here, we let the modified radius squares as $C^2 \triangleq C_0^2 - \left\|\mathbf{Q}_2^T\mathbf{y}\right\|^2$, and (6) is equivalent with a new inequality

$$\|\mathbf{R}(\hat{\mathbf{x}} - \mathbf{x})\|^2 \leq C^2 \tag{7}$$

where $\hat{\mathbf{x}} = \mathbf{R}^{-1}\mathbf{Q}_1^T\mathbf{y} = \mathbf{H}^\dagger\mathbf{y}$ is the unconstrained least-squares (LS) solution. Owing to the upper triangular property of $\mathbf{R}$, (7) can be iteratively evaluated from the M-th to the 1-st dimension for $\mathbf{x}$, i.e., searching backward.

Accordingly, we can expand (7) as

$$\begin{aligned} &\|\mathbf{R}(\hat{\mathbf{x}} - \mathbf{x})\|^2 \\ &= r_{M,M}^2(\hat{x}_M - x_M)^2 \\ &+ r_{M-1,M-1}^2\left((\hat{x}_{M-1} - x_{M-1}) + \tfrac{r_{M-1,M}}{r_{M-1,M-1}}(\hat{x}_M - x_M)\right)^2 \\ &+ ... + B(1) \leq C^2 \end{aligned} \tag{8}$$

From (8), we are able to construct a tree with depth M which is indexed from level $k = M$ to 1. In that tree, branches at level k (or, at depth $M - k + 1$) correspond to candidate symbols x_k, and paths can be regarded as candidate symbol vectors $\mathbf{x}$. Each term in right-hand side of (8) represents branch metric $B(k)$ at level k which is calculated as

$$\begin{aligned} B(k) = r_{k,k}^2 &\left[(\hat{x}_k - x_k) + \sum_{j=k+1}^{M} \tfrac{r_{k,j}}{r_{k,k}}(\hat{x}_j - x_j)\right]^2 \\ &\text{for } k = M, M-1, ..., 1. \end{aligned} \tag{9}$$

A partial path metric from root to level k is given by

$$P(k) = \sum_{j=k}^{M} B(j) \text{ for } k = M, M-1, ..., 1. \tag{10}$$

From (8), x_k belongs to the corresponding range of

$$x_k^{LB} \leq x_k \leq x_k^{UB} \tag{11}$$

where

$$x_k^{LB} = \left\lceil \hat{x}_k + \sum_{j=k+1}^{M} \frac{r_{k,j}}{r_{k,k}}(\hat{x}_j - x_j) - \frac{\sqrt{C^2 - P(k+1)}}{r_{k,k}} \right\rceil, \tag{12}$$

$$x_k^{UB} = \left\lfloor \hat{x}_k + \sum_{j=k+1}^{M} \frac{r_{k,j}}{r_{k,k}}(\hat{x}_j - x_j) + \frac{\sqrt{C^2 - P(k+1)}}{r_{k,k}} \right\rfloor. \tag{13}$$

During the search process of the SD, whenever a symbol vector $\mathbf{x}$ satisfying (7) is found at bottom level $k = 1$, the radius square C^2 is reduced to the path metric of searched symbol vector, and then the SD algorithm is restarted with new C^2. This search process continues until only one symbol vector $\mathbf{x}$, which coincides with the ML solution remains within the radius C.

B. Enumeration methods in FPSD and SESD

In the FPSD algorithm, the enumeration of candidate symbols x_k is based on natural spanning called the Pohst enumeration [1]; x_k is evaluated as the ordered set of

$$x_k \in \left\{x_k^{LB}, x_k^{LB} + \Delta, x_k^{LB} + 2\Delta, ..., x_k^{UB}\right\} \tag{14}$$

where Δ is a distance between two closest lattice points.

The SESD algorithm leads to much faster shrinkage of search space using the Schnorr-Euchner enumeration [3], [4], since it uses efficient ordering of candidate symbols. The Schnorr-Euchner enumeration sorts candidate symbols in a zig-zag manner, and the start point is

$$x_k^{SE} = \left\lfloor \hat{x}_k + \sum_{j=k+1}^{M} \frac{r_{k,j}}{r_{k,k}} (\hat{x}_j - x_j) \right\rceil \tag{15}$$

where $\lfloor \cdot \rceil$ rounds to the nearest lattice point of its argument in the real-valued constellation set $\mathcal{A}$. Thus, when $\hat{x}_k + \sum_{j=k+1}^{M} \frac{r_{k,j}}{r_{k,k}} (\hat{x}_j - x_j) \geq x_k^{SE}$, x_k is examined as the ordered set of

$$x_k \in \left\{x_k^{SE}, x_k^{SE} + \Delta, x_k^{SE} - \Delta, x_k^{SE} + 2\Delta, x_k^{SE} - 2\Delta, ...\right\} \cap \mathcal{A}. \tag{16}$$

The first lattice point explored by SESD with the radius $C = \infty$ is always the Babai point [4] which is referred to as the nulling and canceling (NC) point [9]. If search radius is sufficiently large in SESD, the initial point found can be guaranteed to be constant if search radius is sufficiently large. Hence, SESD has an advantage over FPSD in the choosing of initial radius. In the viewpoint of tree search, branches are examined according to the ascending order of branch metric in SESD. If one candidate symbol violates sphere constraint in the Schnorr-Euchner enumeration, SESD can remove next candidates which do not comply with constraint condition. Hence, SESD has much less computational complexity than FPSD.

III. Proposed SESD With Statistical Pruning

In this section, we propose a new powerful statistical pruning rule to fully use pruning gain for SESD.

A. Statistical Punning

The amount of pruned branches is usually not significant around early search phase of SD. This is a big problem, especially for large dimensional MIMO systems. Without solving this problem, the SD may become impractical.

Recently, IRA was proposed for FPSD [7]. The IRA employs a schedule of radii, and the radii are selected as an increasing sequence. If there are no feasible candidate symbols within reduced search space by IRA, decoding will be restarted with a set of larger radii. In SP-SESD, these radii are referred to as *statistical constraint radii*, *viz.*, SCRs, because SCRs are obtained directly from a statistical function, i.e., the inverse cumulative distribution function (cdf) of $\chi_k^2(\sigma^2)$. In [7], they were chosen as a linear schedule, i.e., $r_i^2 = (\delta \log M + i)\sigma^2$ for $i = 1, ..., M$ (see [7] for more detail). In addition, the proposed SP-SESD employs the Schnorr-Euchner enumeration for smaller complexity.

We also consider expected metrics for unvisited levels. This is done to further expedite search speed. When partial path metric $P(k)$ is compared with the squared radius C^2, expected path metric from unvisited levels is also regarded as cost metric with $P(k)$. Hence, SP-SESD can achieve both pruning conditions by SCR and expected partial path metric respectively. They allow us to use flexible pruning conditions according to pruning probabilities.

B. Proposed SP-SESD

We assume an actual transmitted symbol vector $\mathbf{x}$, then

$$\|\mathbf{R}(\hat{\mathbf{x}} - \mathbf{x})\|^2 = \left\|\mathbf{Q}_1^T \mathbf{z}\right\|^2 = \|\mathbf{v}\|^2 \tag{17}$$

can be derived where $\mathbf{v}$ is an i.i.d Gaussian noise vector with statistical properties equaling to $\mathbf{z}$; $\mathbf{Q}$ is an unitary matrix. Hence, the partial path metric of actual transmitted symbol vector $\mathbf{x}$ from root to level k is

$$P(k) = \sum_{j=k}^{M} B(j) = \sum_{j=k}^{M} |v_j|^2 \quad \text{for } k = M, M-1, ..., 1. \tag{18}$$

where v_j is the j-th entry of $\mathbf{v}$, this partial path metric $P(k)$ is the Chi-square random variable (RV) with $M - k + 1$ degrees of freedom, i.e., $P(k) \sim \chi_{M-k+1}^2(\sigma^2)$. Here, if we assume successful detection over unvisited levels at current level k, from (17) expected partial path metric from unvisited levels, i.e., from level $k-1$ to level 1 (a leaf node), is described as

$$V(k-1) = \sum_{j=1}^{k-1} |v_j|^2 \tag{19}$$

where $V(k-1) \sim \chi_{k-1}^2(\sigma^2)$.

In SP-SESD, the squared SCR D_k^2 can be achieved at level k such that

$$Pr[P(k) \leq D_k^2] \geq 1 - \alpha \quad \text{for } k = M, M-1, ...,1 \tag{20}$$

where $0 \leq \alpha \leq 1$, pruning probability α would be selected by user's design. Since $P(k) \sim \chi_{M-k+1}^2(\sigma^2)$, D_k^2 is obtained from

$$D_k^2 = F^{-1}(1 - \alpha; M - k + 1)\sigma^2 \tag{21}$$

where $F^{-1}(x; k)$ is the inverse cdf of χ_k^2, $F(x;k) = \frac{\gamma(k/2, x/2)}{\Gamma(k/2)}$ is the cdf of χ_k^2, and $\gamma(k, x)$ is the incomplete Gamma function.

Partial path metric $P(k)$ of (10) is assessed with D_k^2 in order to prune branches which are probably incorrect solutions. Thus, a necessary condition from the squared SCR D_k^2 can be obtained as

$$P(k) \leq D_k^2. \tag{22}$$

Now we consider expected partial path metric from unvisited levels $V(k-1)$ at current level k. SP-SESD tries to prune branches satisfying

$$\begin{aligned} &Pr[V(k-1) \leq C^2 - P(k)] < \beta \\ &\text{for } k = M, M-1, ..., 2 \end{aligned} \tag{23}$$

where $0 \leq \beta \leq 1$, pruning probability β would also be selected according to system demands. From (23), $F(\frac{C^2-P(k)}{\sigma^2}; k-1) < \beta$, and $V(k-1)$ is determined for level k as

$$V(k-1) = F^{-1}(\beta; k-1)\sigma^2. \tag{24}$$

We add $V(k-1)$ to $P(k)$ in order to make a much tighten necessary condition than that of (8), and hence, from (10) and (19), an another necessary condition at level k for satisfying tighter sphere constraint can be given as

$$P(k) + V(k-1) \leq C^2. \tag{25}$$

As a result, SP-SESD explores more reduced search space by two necessary conditions of (22) and (25), we can expect to achieve benefits in the aspect of detection complexity and pruning gain. Also, SP-SESD obtains adaptable system performance by adjusting two pruning probabilities of α, β. We formalize the proposed algorithm as follows.

Proposed SP-SESD Algorithm:
Input: $\mathbf{R}$, $\hat{\mathbf{x}}$, C^2,
$D_{k,\alpha}^2 = F^{-1}(1-\alpha; M-k+1)\sigma^2$
for $1 \leq k \leq M$ and $\alpha \in \{\alpha_1, \alpha_2, ..., \alpha_L\}$,
$V_\beta(k-1) = F^{-1}(\beta; k-1)\sigma^2$
for $2 \leq k \leq M$ and $\beta \in \{\beta_1, \beta_2, ..., \beta_L\}$.
Output: s

Step 1: Set $i = 1$.
Step 2: Set $k = M$, $\hat{x}_{k|k+1} = \hat{x}_k$, and $P(k+1) = 0$.
Step 3: Set $\alpha = \alpha_i$, $\beta = \beta_i$;
$D_k^2 = D_{k,\alpha}^2$, $V(k-1) = V_\beta(k-1)$ for all possible k.
Step 4: Set $x_k = \lfloor \hat{x}_{k|k+1} \rceil$, $\Delta_k = sign\left(\hat{x}_{k|k+1} - x_k\right)\Delta$.
Step 5: Let $B(k) = r_{k,k}^2\left(\hat{x}_{k|k+1} - x_k\right)^2$,
$P(k) = P(k+1) + B(k)$;
If $D_k^2 \geq P(k)$, go to **Step 6**; Else, go to **Step 8**.
Step 6: If $C^2 - V(k-1) \geq P(k)$ and $x_k \in \mathcal{A}$, go to **Step 7**; Else if $C^2 - V(k-1) \geq P(k)$ and $x_k \notin \mathcal{A}$, go to **Step 10**; Else, go to **Step 8**.
Step 7: If $k = 1$, let $C^2 = P(k)$, $\mathbf{s} = \mathbf{x}$, $k = k+1$ and go to **Step 10**; Else if k $\neq 1$, let $k = k-1$,
$\hat{x}_{k|k+1} = \hat{x}_k + \sum_{j=k+1}^{M} \frac{r_{k,j}}{r_{k,k}}(\hat{x}_j - x_j)$,
and go to **Step 4**.
Step 8: Let $k = k+1$; If $k = M+1$, go to **Step 9**; Else, go to **Step 10**.
Step 9: If s is empty, let $i = i+1$, go to **Step 2**; Else, terminate algorithm.
Step 10: Set $x_k = x_k + \Delta_k$, $\Delta_k = -\Delta_k - sign(\Delta_k)\Delta$ and go to **Step 5**.

IV. Simulation Results

To demonstrate the performance of proposed SP-SESD, we present simulation results. SP-SESD is compared with SESD, IRA, and RA-SD in terms of BER and average floating point operations (FLOPs). The FLOPs is counted by MATLAB to measure computational complexity. (Note that IRA was also simulated with SE enumeration and employed squared SCRs same as SP-SESD instead of a linear schedule of [7] in order to gain fairness for performance comparison here.) Since we assume quasi-static Rayleigh fading channel, MIMO channel remains constant for one packet transmission. Thus, the complexity of preprocessing stage can be ignored.

We consider uncoded SM-MIMO system with $M = N = 16$, and 16-QAM constellation is adopted for each transmit antenna. The number of channel generation is at least 10000, and MIMO channel matrix $\mathbf{H}$ remains fixed for 100 symbol times. Signal-to-noise ratio (SNR) is defined as bit energy-to-noise power ratio (E_b/N_0). Here $E_b/N_0 = (NE_s)/(MN_0S)$, E_b denotes average power per one bit, and S is number of bits per a transmitted symbol.

For statistical pruning approaches, we select pruning probabilities of α, β in logarithmic fashion. The set of pruning probability α is chosen with $\{0.1, 0.01, 0.001, 0\}$ for IRA, RA-SD uses pruning probability $\beta = 0.1$, and SP-SESD employs two sets of α and β such that $\{0.1, 0.01, 0.001, 0\}$ respectively. For IRA and SP-SESD, sets of pruning probabilities α, β contain 0 so as to ensure finding solution; IRA and SP-SESD can avoid fail in decoding. From sets of pruning probabilities α and β, input parameters such as D_k^2 and $V(k-1)$ can be pre-computed numerically by mathematical tools, e.g., MATLAB. These are utilized as a form of look-up table during the search process. And, we let an initial radius such as $C = \infty$ for all detectors.

Fig. 1 shows BER curves of several detectors as a function of E_b/N_0. SESD presents the optimal ML performance and accomplishes spatial diversity order of 8. IRA, RA-SD and SP-SESD also achieve same spatial diversity order. As shown in Fig. 1, our SP-SESD achieves BER performance very similar to those of IRA and RA-SD over all SNR regime. Hence, the performance loss of SP-SESD with respect to SESD can be negligible, therefore we can say that SP-SESD is a near-ML algorithm.

Fig. 2 illustrates the average FLOPs of considered detectors in order to detect symbols at each symbol time. We can see that IRA, RA-SD, and SP-SESD have lower computational complexity than SESD. The average FLOPs of our SP-SESD is smaller than that of RA-SD over all SNRs. It is also smaller than or similar to that of IRA over almost SNRs. The complexity reduction of SP-SESD is remarkable with

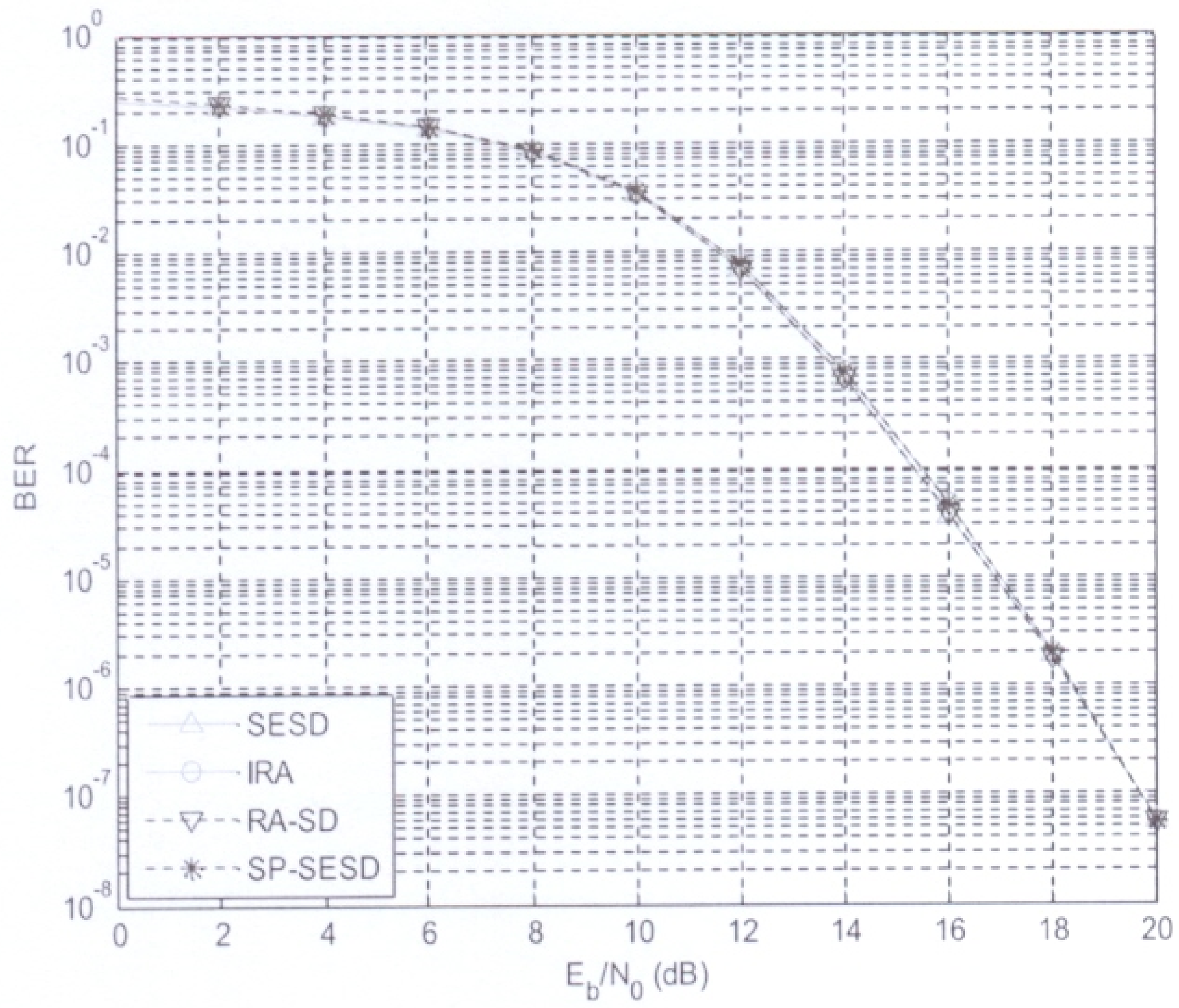

Fig. 1. BER curves versus E_b/N_0 for an uncoded SM-MIMO system with $M = N = 16$, 16-QAM modulation.

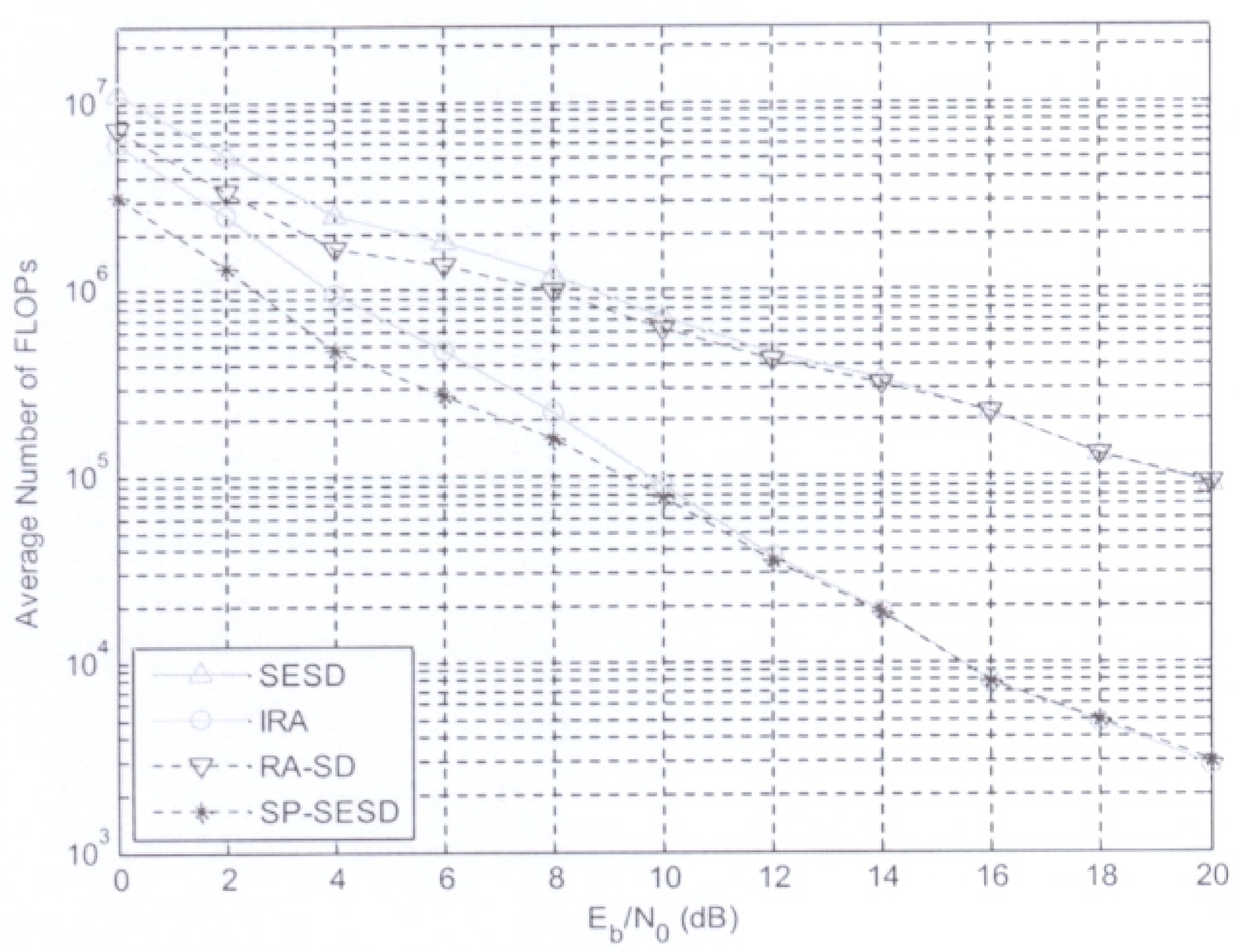

Fig. 2. Average FLOPs curves versus E_b/N_0 for an uncoded SM-MIMO system with $M = N = 16$, 16-QAM modulation.

comparison to other statistical pruning methods over almost SNR regime, although the degradation of BER performance can be neglected. For instance, SP-SESD requires about 48% and 27% of computational complexity compared to IRA and RA-SD respectively at $E_b/N_0 = 4$ dB. If the dimension of MIMO channel becomes much larger or, constellation order becomes increased, then SP-SESD can provide much more complexity reduction than other methods. We can notice that the pruning gain of SP-SESD becomes weak around high SNR regime, both IRA and RA-SD also show same fashion, since the SD algorithms tend to search the ML path immediately when SNR becomes high [5].

V. Conclusion

In this paper, we have presented a new powerful statistical pruning strategy providing the near-ML performance, i.e., SP-SESD, for MIMO systems. Our works are inspired by the fact that the conventional SD uses a loose necessary condition for pruning branches and other statistical pruning strategies still have redundant complexity. Hence, we devise an unified statistical pruning rule to enhance detection efficiency, and SP-SESD can efficiently remove candidate symbols which don't seems to correct branches. Two pruning probabilities of α and β are determined by user's design or system demands, and then D_k^2 and $V(k-1)$ can be computed in offline.

Simulation results demonstrate that SP-SESD requires lower computational complexity than IRA and RA-SD, although the near-ML performance is still achieved. Furthermore, SP-SESD can present a flexible trade-off between BER performance and computational complexity by controlling α and β. For further works, the ordering preprocessing will be combined with SP-SESD to further improve search speed, and it is also interesting work that SP-SESD is applied for soft-decision problems with the channel coding.

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