



Throughput behavior of link adaptive 802.11 DCF with MUD capable access node[☆]

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ABSTRACT

Conventional IEEE 802.11 medium access control (MAC) protocol discourages simultaneous transmission to avoid collisions. With fast advances in physical layer technologies, multi-user detection (MUD) capable receivers which can detect multiple frames from different users simultaneously become available. If we are to utilize them in today's wireless LAN, however, it is not entirely clear how we should change the MAC and how much benefit is available and can be obtained by doing so. The primary objective of this paper is to investigate such questions. We approach this objective by developing a new throughput expression for 802.11 distributed coordination function (DCF). The derived expression has been verified in simulation. We show that significant throughput gain can be garnered with slight modification in 802.11 DCF.

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1. Introduction

The single packet reception constraint at the access node in conventional wireless multiple access networks can be relaxed using multi-user detection (MUD) techniques which can decode information transmitted from multiple stations simultaneously. In the past, a MUD receiver [1] was viewed as an exclusive technique, perhaps deemed suitable only for high performance base stations in cellular networks, as high complexity operations were usually required for it. Recent advances in the graph code theory such as low-density parity-check codes and interleaved convolutional codes have made MUD less computationally intensive via the use of turbo-iterative algorithms, see [2–5]. Users can be differentiated by channel codes. This user separation via channel code alone has received attention in the past and proven to be more spectrum efficient than what the conventional approach of separated spreading and channel coding can provide [6].

The downside of such MUD receivers is that the receiver complexity still increases, at least linearly to the number of users.

Therefore, for an access node in a typical Wireless Local Area Network (WLAN) application, the number of users, say m , that a MUD receiver can detect simultaneously will likely be limited up to several signals at maximum. We will call such an access network m -MUD enabled.

We are interested in examining the throughput behavior of a WLAN when the access node is m -MUD enabled. We aim to achieve this goal by deriving a new throughput expression.

With employment of an m -MUD enabled access node, lesser collision and thus increased effective information transmission is expected as more than one stations can transmit successfully to the access node simultaneously [7]. With the employment of a centrally controlled MAC protocol, such as the point coordination function in 802.11, polling of stations can be used and exactly m stations can be scheduled to access the channel simultaneously. This case is not of interest in this paper.

We are interested in a distributive MAC and thus aim to analyze the distributed coordination function (DCF) [8] in 802.11. The basic access mechanism used in 802.11 MAC protocol is carrier sense multiple access with collision avoidance (CSMA/CA). Stations are not allowed to send frames whenever they sense signaling activity in the channel. On top of CSMA/CA, stations follow the distributed coordination function (DCF) with which they share the medium while avoiding collision via randomized transmissions. In the standard 802.11 MAC, simultaneous transmissions from multiple stations to the access node is considered as collision and hence discouraged. The control is done by

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providing each station with a prescribed contention window (CW) size. For m -MUD enabled 802.11 DCF, we need to encourage simultaneous transmissions up to a certain level. One can achieve this objective by decreasing the size of CW. But what sizes of CWs we should allow the stations to use vary for different m and for different groups where users in different group uses different transmission rate via link adaptation (see next paragraph for more explanation). Our throughput expression can be used to evaluate this problem and provide a solution.

The throughput expression in this paper, therefore, is derived under the assumption that each station in the network employs link adaptation [9,10]. With link adaptation, stations choose a data rate depending on underlying channel condition. As the signal to noise ratio (SNR) available to a station increases, it uses higher data rates to transmit its frame, hence reaching closer to channel capacity. Link adaptation in WLAN changes the throughput behavior of WLAN significantly [11,12]. Thus, it is meaningful to include link adaptation in our analysis and see its impact on the throughput behavior.

The contributions of this paper are therefore as follows: first, we derive the throughput expression for the multi-rate 802.11 MAC protocol with the support of MUD capable access node. To the best of our knowledge, this is novel (see our comparative literature analysis in the next section). Second, we show how this throughput expression can be used in the optimal control of 802.11 DCF.

The rest of the paper is organized as follows. In Section 2, we provide comparison of this work to prior works. In Section 3, we describe the system model of our m -MUD proposed WLAN. The backoff process of 802.11 is then modeled as a Markov chain. Analytical framework to investigate network throughput is presented in Section 4. Section 5 contains numerical results and their comparison with simulations. Performance enhancement mechanisms and MAC protocol that may be employed to get the maximum advantage of MUD capability are discussed in Section 6, following which we conclude.

2. Comparison to prior work

Many works exist in the literature, which analyzed the network throughput of IEEE 802.11. Bianchi [13,8] modeled 802.11 DCF using Markov chains and evaluated the performance of WLANs using single transmission rate. Improvements were then presented by several researchers which include the details of 802.11 DCF in the Markov chain model for performance analysis. It should be pointed out that while substantial research is conducted on throughput analysis assuming single data rate, not comparable attention is given to the scenario where stations are allowed to use different data transmission rates, a typical case in modern 802.11 WLANs.

More recently conducted researches reported in [11,14–16] accommodate multi-rate transmission. Yang et al. [11], assume exponential backoff procedure, as outlined in the IEEE 802.11 DCF. The model, an improved version of the one presented in [8], accommodates multi-rate transmission. With employment of multi-rate transmissions, “performance anomaly” problem was observed: low rate users hold the channel longer in time and thus the overall network throughput suffers. The authors have tried to address this problem by controlling access parameters of stations such as the initial contention window size, the frame size, and the maximum backoff stage. All these parameters are well defined in the IEEE 802.11 standard. This problem of performance anomaly was also observed in [16]. In [15], the authors analyze multi-rate 802.11 WLANs, where they assume two Markov chain models for

stations and channels. However, the remedial solution for the performance anomaly was not addressed.

The authors in [17] attempted to apply a MUD capable access node in IEEE 802.11 WLANs and proposed a modified MAC. They assumed a simple scenario in which all nodes have the same SNR and use geometrically distributed random backoff interval as compared to the exponential backoff in DCF. In [18,7], the authors attempted to implement multiple packet reception (MPR) in 802.11 WLANs using DCF. CSMA was modified for MPR scenario in [12] and a modified cross layer CSMA for MPR, named XL-CSMA was proposed. Although decentralized, it did not assume DCF mechanism for backoff. We point out that these prior works with 802.11 and MUD mentioned here assume single transmission rate for throughput analysis.

In [19], the authors have addressed the throughput unfairness problem for a network of spatially distributed nodes. Distant nodes due to poor channel conditions suffer from low throughput. This is in fact the same phenomenon observed in [17]. There they aim to increase the overall throughput. Here the point is fairness. To enhance the throughput of distant nodes, the authors of [19] proposed a simple modification in a MAC with which a node selects a transmission probability from two possible values – high and low. A station with an unsuccessful transmission history is allowed to select the high transmission probability.

The works compared so far in this section start with the assumption of the classic CSMA/CA framework where simultaneous transmissions are discouraged. Among the earlier works on multi-packet reception (MPR) in random access networks, the authors of the paper [20] studied slotted ALOHA (SA) systems under infinite number of users and single buffer assumption. Stations under SA transmit at the start of each frame whenever they have a frame to send. Thus, simultaneous transmission is easier than under CSMA/CA to be encouraged via manipulating the transmission probabilities. However, there are downsides as well which make SA less attractive than 802.11. The throughput of SA is poor. For example, the throughput is only compared to the offered load (for single user detection receivers); the throughput vanishes as the offered load increases. SA is known to have instability problem as the offered load increases requiring a separate remedial treatment [20]. Furthermore, most of the existing MPR MAC protocols [20,21], assume the existence of a central coordinator that schedules transmissions from stations. Hence, these prior works are not applicable to our 802.11-based distributed approach here.

To the best of our knowledge, this paper provides the first work which attempts to analyze multi-user detection (MUD) in a multi-rate 802.11 WLANs in which stations use DCF to enter into exponential backoff before transmitting their frames.

3. System model for multi-rate 802.11 m-MUD

3.1. System description

Fig. 1 depicts a basic service set (BSS) system in consideration. All the stations in the BSS are divided into N groups; those belonging to each group transmit their frames at the data rate of r_i for $i = 1, 2, \dots, N$. It is further assumed that there are a total of M stations in the network, each group having M_i number of stations having their frames ready to be transmitted, i.e. $M = \sum_{i=1}^N M_i$. There are n_i stations from the i th group which start their transmission simultaneously at the start of the same timeslot. Hence, overall, there are n number of stations starting transmission simultaneously, where $n = \sum_{i=1}^N n_i$.

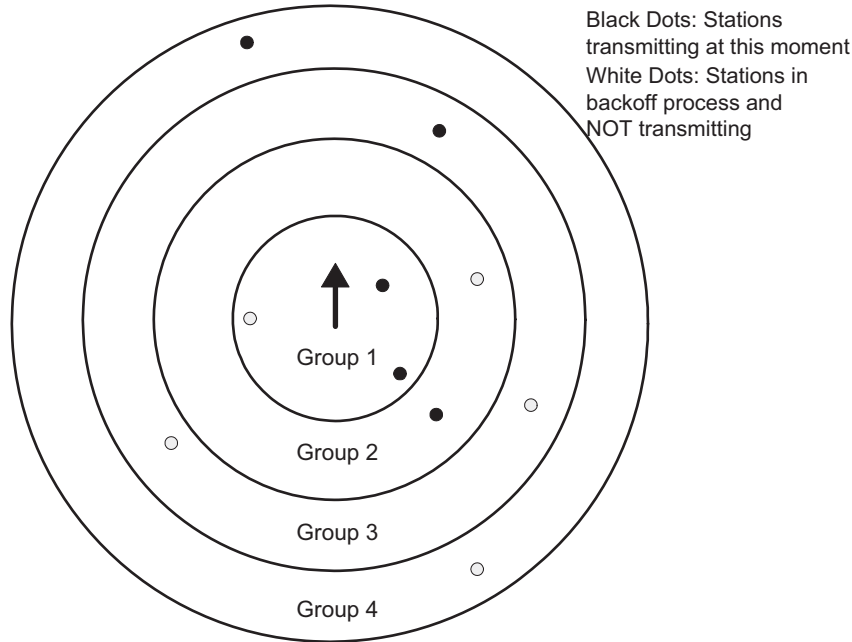


Fig. 1. The basic service set system: there are four groups $N = 4$. There are total of 10 stations, $M = 10$. The number of stations in each group is $M_1 = 3, M_2 = 2, M_3 = 3,$ and $M_4 = 2$. There are total five stations transmitting at the moment, $n = 5$. Among them two are in Group 1, $n_1 = 2$, one in Group 2, $n_2 = 1$, three in Group 3 $n_3 = 3$, and one in Group 4, $n_4 = 1$.

We consider the *saturation* condition in which stations always have another frame ready for transmission after a successful transmission, as done in [11,17,8,14,15].

3.2. Modeling the backoff mechanism in DCF

In this subsection, we use a discrete-time Markov chain model to describe the exponential backoff process defined in the distributed coordination function (DCF) mode of operation of IEEE 802.11. Yang et al. [11], an extended work of [8] with the inclusion of link rate adaptation in the analysis, also used a discrete-time Markov chain. We, however, allocate a space here to remodel the Markov chain because with our m -MUD system, the collision event used in the DCF needs to be carefully redefined. Our m -MUD system with $m = 1$ should render Yang et al.'s result.

Now let us consider the Markov chain (in Fig. 2) as depicted in [11]. The definition of modified collision event along with other details will follow.

Under the DCF mode, a station senses channel before transmitting. If sensed idle, the station just starts transmission (after waiting an inter frame space (IFS) and finding the channel idle again). If it is sensed busy, the station defers its transmission for a time equal to DCF inter-frame space (DIFS). After the DIFS, the station chooses a random backoff counter and takes an additional deferral period before transmitting.

The value of the backoff counter is chosen randomly from a uniform distribution over the interval $[0, CW_{i,0} - 1]$, where $CW_{i,k}$ is defined as the size of the contention window (CW) of a station in the i th group in the k th stage. The stage index starts at zero and is incremented by 1 each time collision occurs. Hence $CW_{i,0}$ is the size of initial contention window. Whenever a timeslot is sensed idle, the backoff counter is decremented. If the slot is sensed busy at anytime, the process is suspended, and starts again when an idle slot is encountered. When the counter reaches zero, the frame is transmitted. When collision occurs, those stations involved in the collision enter into their own backoff process. For an example of the 1st collision, the stage index is incremented by one, i.e.,

$k = 1$. In addition, the size of the contention window is increased by

$$CW_{i,k} = \begin{cases} 2^k CW_{i,0}, & 0 < k \leq k_{\max,i} - 1 \\ 2^{k_{\max,i}} CW_{i,0}, & k_{\max,i} \leq k \leq k_{\text{retry},i} \end{cases} \quad (1)$$

With each collision, the size of CW is increased until it reaches up to its maximum, i.e., $CW_{\max,i} = 2^{k_{\max,i}} CW_{i,0}$ after which it remains the same. The frame is dropped after $k_{\text{retry},i}$ attempts.

In the conventional CSMA/CA, the collision occurs if more than one stations attempt to transmit their frames simultaneously. In the context of m -MUD capable physical layer, the collision should be redefined and said to have occurred when more than m stations attempt to transmit their frame simultaneously, where m is the maximum number of users that can be detected simultaneously by the MUD capable access node. The definition of busy medium may remain the same as it refers to absence of idle period, or when one or more stations are transmitting simultaneously.

Now, we define $X(i, t) = \{K(i, t), L(i, t)\}$ as a discrete-time Markov Chain. We use the same assumptions made in [1] that the conditional busy probability $p_{b,i}$, that the channel is sensed busy by an i th group station, and the conditional collision probability $p_{c,i}$, that the transmitted frame of i th group station collides with any other station, are assumed independent of the backoff mechanism. Here, $K(i, t)$ and $L(i, t)$ are random processes representing the backoff stage of a station in the i th group and the size of the backoff counter of a station in the i th group, respectively. We define (i, k, l) as a state in the Markov chain which represents the i th group, k th stage and the value of random backoff counter l . Therefore, the ranges of these indexes are given by $1 \leq i \leq N, 0 \leq k \leq k_{\text{retry},i}$ and $0 \leq l \leq CW_{i,k} - 1$.

3.3. Transition probabilities

For the description of the transition probabilities of the Markov chain, it is useful to see Fig. 2 again. The state transition

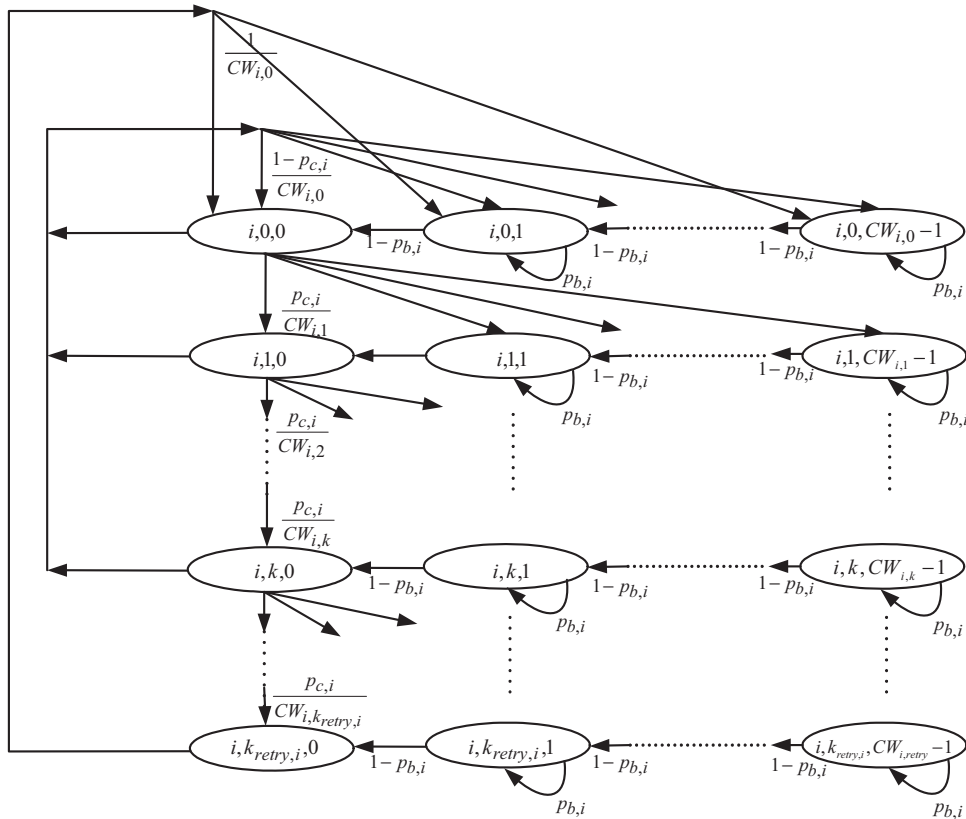


Fig. 2. Markov chain (of an i th group station) is constructed according to the IEEE distributed coordination function for the multi-rate 802.11 MUD MAC protocol.

probability of the Markov chain $X(i, t)$, defined by

$$P\{i, k_1, l_1 | i, k_0, l_0\} = P\{K(i, t) = k_1, L(i, t) = l_1 | K(i, t) = k_0, L(i, t) = l_0\} \tag{2}$$

can be obtained from the *busy and collision probabilities* in the following way:

$$P\{i, k, l - 1 | i, k, l\} = 1 - p_{b,i} \quad \text{for } 1 \leq l \leq CW_{i,k} - 1 \tag{3}$$

$$P\{i, k, l | i, k, l\} = p_{b,i} \quad \text{for } l = 0 \tag{4}$$

$$P\{i, k + 1, l | i, k, 0\} = \frac{p_{c,i}}{CW_{i,k+1}} \quad \text{for } 0 \leq k \leq k_{retry,i}, 0 \leq l \leq CW_{i,k+1} - 1 \tag{5}$$

$$P\{i, 0, l | i, k, 0\} = \frac{1 - p_{c,i}}{CW_{i,0}} \quad \text{for } 0 \leq k \leq k_{retry,i}, 0 \leq l \leq CW_{i,0} - 1 \tag{6}$$

$$P\{i, 0, l | i, k_{retry,i}, 0\} = \frac{1}{CW_{i,0}} \quad \text{for } 0 \leq l \leq CW_{i,0} - 1 \tag{7}$$

Eq. (3) shows that the backoff counter is reduced by 1 each time the channel is sensed idle, whose probability is $1 - p_{b,i}$. Eq. (4) shows that the chain halts reducing the backoff counter when the channel is sensed busy (happens with the busy probability $p_{b,i}$). Eq. (5) shows that the stage index k of the chain is incremented by 1 when collision occurs. The size of contention window determines the range of the counter value. Since uniformly distributed, each counter value l in the range $0 \leq l \leq CW_{i,k+1} - 1$ is equally probable. Eq. (6) shows that the chain jumps downward from k th stage to 0th stage with a successful transmission. Again a uniform

randomly selected counter value from the range $0 \leq l \leq CW_{i,0}$ will determine the next counter state l . Eq. (7) suggests that the station will restart the backoff process with a new frame to transmit. These equations are constructed to model the backoff process of 802.11.

3.4. Steady state distribution and transmission probabilities

Let us now define a steady state distribution, $s_{i,k,l}$, of the Markov chain where it is defined as $s_{i,k,l} = \lim_{t \rightarrow \infty} P\{K(i, t) = k, L(i, t) = l\}$. Using the definition of probability, we have

$$\sum_{k=0}^{k_{retry,i}} \sum_{l=0}^{CW_{i,k}-1} s_{i,k,l} = 1 \tag{8}$$

Using the balance equations on the Markov chain, we obtain

$$s_{i,k,0} = p_{c,i}^k s_{i,0,0} \tag{9}$$

and

$$s_{i,k,l} = \frac{CW_{i,k} - l}{CW_{i,k}(1 - p_{b,i})} s_{i,k,0} \quad \text{for } 0 \leq k \leq k_{retry,i}, 0 \leq l \leq CW_{i,k} - 1 \tag{10}$$

Since a station transmits its frame when its backoff counter reaches zero, the probability p_i that a station in the i th group transmits its frame can then be calculated as the sum of the probabilities of such events, i.e.,

$$p_i = \sum_{k=0}^{k_{retry,i}} s_{i,k,0} = \sum_{k=0}^{k_{retry,i}} p_{c,i}^k s_{i,0,0} = \frac{1 - p_{c,i}^{k_{retry,i}}}{1 - p_{c,i}} s_{i,0,0} \tag{11}$$

Using (10), (9), and (8) in the respective order, we can find $s_{i,0,0}$

$$s_{i,0,0} = \left[\sum_{k=0}^{k_{\text{retry},i}} \sum_{l=0}^{CW_{i,k}-1} \frac{CW_{i,k}-l}{CW_{i,k}} \frac{1}{1-p_{b,i}} p_{c,i}^k \right]^{-1} = \left[\sum_{k=0}^{k_{\text{retry},i}} \frac{p_{c,i}^k}{(1-p_{b,i})} \left(1 + \frac{CW_{i,k}-1}{2} \right) \right]^{-1} \quad (12)$$

Using (11) and (12), p_i for $i = 1, 2, \dots, N$ can be obtained.

$$p_i = \frac{1 - p_{c,i}^{k_{\text{retry},i}}}{1 - p_{c,i}} \left[\sum_{k=0}^{k_{\text{retry},i}} \frac{p_{c,i}^k}{(1-p_{b,i})} \left(1 + \frac{CW_{i,k}-1}{2} \right) \right]^{-1} \quad (13)$$

The transmission probabilities, p_i , are proven to be the key statistics for further analysis. Since they dictate how much access any station from the i th group will have on the network resources, they directly influence the system throughput.

3.5. Busy and collision probabilities

Now we define P_n the probability that n number of stations start transmitting their frames simultaneously. Recall that $n = \sum_{i=1}^N n_i$ where n_i is the random variable denoting the number of stations from the i th group start to making simultaneous transmissions. Note that all n_i are mutually independent with each other.

To obtain P_n , we first determine the probability mass function (pmf) of the random variable n_i . We note that with p_i known, the pmf under discussion can be obtained as the binomial distribution $P\{n_i = k_i\} = \binom{M_i}{k_i} p_i^{k_i} (1 - p_i)^{M_i - k_i}$. Since all n_i are independent of each other, the joint pmf can be written as

$$P\{n_1 = k_1, \dots, n_N = k_N\} = \prod_{i=1}^N \binom{M_i}{k_i} p_i^{k_i} (1 - p_i)^{M_i - k_i} \quad (14)$$

Using (14) and defining a set $S_1 \triangleq \{(x_1, \dots, x_N) : \sum_{i=1}^N x_i = n, 0 \leq x_i \leq M_i\}$, P_n can be written as

$$P_n = \sum_{S_1} P\{n_1 = x_1, \dots, n_N = x_N\} \quad (15)$$

A conditional joint pmf which is defined under the condition that at least one station transmits will be useful later on. Using the busy channel probability defined as $p_b \triangleq P\{n \geq 1\}$, it can be written by

$$P(\mathbf{n}|n \geq 1) \triangleq \frac{P\{n_1 = k_1, \dots, n_N = k_N | n \geq 1\}}{P_b} = \frac{P\{n_1 = k_1, \dots, n_N = k_N, \sum_{i=1}^N k_i \geq 1\}}{P_b} \quad (16)$$

Given P_n , we can obtain the conditional busy and collision probabilities $p_{b,i}$ and $p_{c,i}$. We note that an i th group station will sense the channel whether it is busy or not. Reserving one station from the i th group as the station sensing the channel, therefore, the conditional busy probability $p_{b,i}$ is calculated. In a similar fashion, conditioning upon the event that an i th group station transmits its frame, the conditional collision probability $p_{c,i}$ is calculated. Namely, assuming an m -MUD, a collision occurs if m or more other stations are also transmitting their frames in a particular time slot. For Eqs. (17) and (18) where we define $p_{b,i}$ and $p_{c,i}$, therefore, we will use $P_n(M_i - 1)$ to imply the probability P_n in (16) obtained from excluding exactly one station from i th group; and then we have

$$p_{b,i} = \sum_{n=1}^{M-1} P_n(M_i - 1) \quad \text{for } i = 1, 2, \dots, N \quad (17)$$

and

$$p_{c,i} = \sum_{n=m}^{M-1} P_n(M_i - 1) \quad \text{for } i = 1, 2, \dots, N \quad (18)$$

Similarly, we can calculate the busy probability p_b and the idle probability p_{idle} :

$$p_b = \sum_{n=1}^M P_n \quad (19)$$

$$p_{\text{idle}} = 1 - p_b \quad (20)$$

Given the set of network parameters, such as contention windows $\{CW_{i,k}\}$, number of stations in each group $\{M_i\}$, and the backoff stage limits $\{k_{\text{max},i}, k_{\text{retry},i}\}$, we notice that the transmission probabilities $\{p_i\}$, the busy probabilities and the collision probabilities are fixed. Thus, they are obtained from numerical evaluation of Eqs. (13), (15), (17), and (18).

4. Throughput derivation

Given the transmission probabilities obtained in the previous section, we may carry our analysis with the renewal theorem. The Markov chain renews itself every time a successful transmission is made. Then, the average throughput can be obtained by analyzing a single renewal period as shown in Fig. 3.

As described earlier, each station enters the exponential backoff process upon sensing the channel busy or experiencing a collision. There may be more than one stations who finish their backoff process earlier than others and hence start transmitting their frames simultaneously. Therefore the shared communication medium is idle at the start of each renewal period until the time a backoff process of any station(s) comes to an end. Those stations whose backoff counter reaches zero then start transmitting their frames simultaneously. Since CSMA/CA protocol requires any station to sense the medium before transmitting its frame, no station can start communication as long as the transmission of the maximum time consuming frame finishes. The data transmitted in one busy period is taken to be the sum of effective data transmitted by all the stations. We further explain each of them, idle period, busy period, and equivalent successful data transmission in the subsequent pages. We can now define the network throughput as follows:

$$\text{Throughput} = \frac{\bar{U}}{\bar{I} + \bar{B}} \quad (21)$$

where \bar{U} denotes the average data size of successful data transmission, \bar{I} the average idle period, and \bar{B} the average busy period.

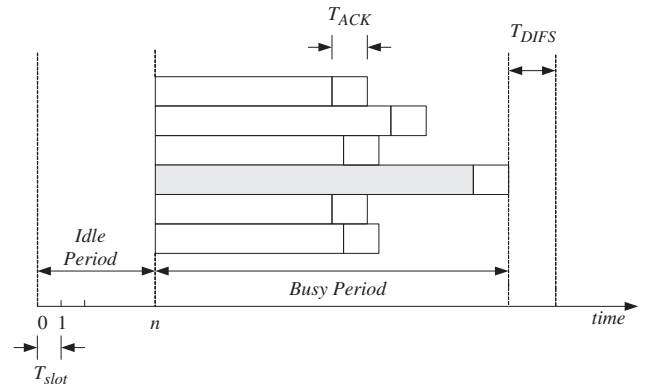


Fig. 3. The renewal period considered for the throughput analysis of multi-rate 802.11 MUD MAC protocol.

The average idle period is defined as the time before any of the stations start their frame transmission. We obtained the probability when no station is transmitting its frame in a slot, p_{idle} (20) in the last section. The average idle period (number of slots in fact) is calculated as the average of the geometric distribution, i.e.,

$$\bar{I} = \sum_{n=0}^{\infty} np_{idle}^n(1-p_{idle}) = \frac{p_{idle}}{1-p_{idle}} \text{ [slots]} \quad (22)$$

4.1. Busy period \bar{B}

The busy period (in the unit of slots) is defined as the duration of the maximum time-consuming frame, after the idle period has finished. Note that the frame may be generated from any station in any group. The frame length of any stations regardless of membership to a group is identically distributed, and it is geometrically distributed with parameter q . The probability that the packet length is l is given by $P\{L=l\} = q(1-q)^{l-1}$ for $l=1,2,\dots$. The distribution function is then given by

$$P\{L \leq l\} = 1 - (1-q)^l \text{ for } l=1,2,\dots \quad (23)$$

Note that average frame length is $1/q$ [bits].

We now move on to model the maximum length of a frame in the i th group, which is transmitted at the rate r_i bit-per-second [bps]. Note that with multi-rate link adaptation, the rate is different for each group $i=1,2,\dots,N$. After that, the maximum time consuming frame is modeled. Note that the maximum time consuming frame may be transmitted by any station from any group. Let us define the unit for frame lengths as the number of bits.

We define $L_{max,i}$ as the maximum size of a frame generated by a station in i th group, where $L_{j,i}$ is defined as the frame length by j th station in the i th group. Since it is assumed that there are n_i stations transmitting their frames simultaneously from the i th group, the maximum length given n_i can be written as $L_{max,i}(n_i) := \max\{L_{1,i}, L_{2,i}, L_{3,i}, \dots, L_{n_i,i}\}$. Then, we can find the distribution function for this random variable $L_{max,i}$, given that there are n_i number of stations from the i th group which are transmitting their data. It is given by

$$P\{L_{max,i}(n_i) \leq l\} = P\{L_{1,i} \leq l, L_{2,i} \leq l, \dots, L_{n_i,i} \leq l\} \\ = \prod_{j=1}^{n_i} P\{L_{j,i} \leq l\} = [1 - (1-q)^l]^{n_i} \quad (24)$$

where we use the fact that the frame lengths of all the stations are mutually independent with each other and (23).

Let us recall that the unit of our time is the number of slots. Without loss of generality, take the slot duration is equal to 1 s. We use (24) to get the distribution function for $t_{max,i}$, the maximum time consumed by a frame transmitted by an i th group station, where $L_{max,i} = r_i t_{max,i}$ [bits]:

$$P\{t_{max,i}(n_i) \leq \tau\} = P\{L_{max,i}(n_i) \leq r_i \tau\} = [1 - (1-q)^{r_i \tau}]^{n_i} \quad (25)$$

Similarly, since the lengths of frames of all the stations are independent with each other, $t_{max,i}$ are also independent of each other for all groups. Therefore, the maximum time taken by a frame transmitted by a station belonging to any group, t_{max} , can be defined as, given that there are a total of n_i stations belonging to the i th group transmitting their frames,

$$t_{max}(\mathbf{n}) = \max\{t_{max,1}(n_1), \dots, t_{max,N}(n_N)\} \quad (26)$$

where $\mathbf{n} := (n_1 \ n_2 \ \dots \ n_N)$. Now the distribution of t_{max} can be found, i.e.,

$$P\{t_{max}(\mathbf{n}) \leq \tau\} = P\{t_{max,1}(n_1) \leq \tau, \dots, t_{max,N}(n_N) \leq \tau\} = \prod_{i=1}^N [1 - (1-q)^{r_i \tau}]^{n_i} \quad (27)$$

With the above distribution function, we can calculate the expected value of t_{max} , i.e.,

$$E_{\tau}(t_{max}(\mathbf{n})) = \int_0^{\infty} P\{t_{max}(\mathbf{n}) > \tau\} d\tau \\ = \int_0^{\infty} \left(1 - \prod_{i=1}^N [1 - (1-q)^{r_i \tau}]^{n_i}\right) d\tau \\ = \int_0^{\infty} \left(-\prod_{i=1}^N \sum_{x_i=0}^{n_i} \binom{n_i}{x_i} (-1)^{r_i \tau x_i}\right) d\tau \\ = \int_{\tau=0}^{\infty} \left[-\sum_{x_1=1}^{n_1} \dots \sum_{x_N=1}^{n_N} \binom{n_1}{x_1} \dots \binom{n_N}{x_N}\right] \\ \cdot (1-q)^{(r_1 x_1 + \dots + r_N x_N) \tau} d\tau \quad (28)$$

where the third line is due to the use of the binomial expansion and excluding the case when $x_1 = \dots = x_N = 0$, which is the first term of the binomial expansion and equals to 1. We note that with $\alpha \triangleq \sum_{i=1}^N x_i$, (28) can be written as

$$E_{\tau}(t_{max}(\mathbf{n})) = \sum_{x_1=1}^{n_1} \dots \sum_{x_N=1}^{n_N} \frac{(-1)^{\alpha} \prod_{i=1}^N \binom{n_i}{x_i}}{\left(\sum_{i=1}^N x_i r_i\right) \ln(1-q)} \quad (29)$$

Averaging the expected value (29) over all possible vectors \mathbf{n} will give us the average busy period:

$$\bar{B} = \sum_{S_B} P\{n_1 = k_1, \dots, n_N = k_N | n \geq 1\} \cdot E_{\tau}\{t_{max}(k_1, k_2, \dots, k_N)\} \text{ [slots]} \quad (30)$$

where $S_B \triangleq \{(k_1, \dots, k_N) : 0 \leq k_i \leq M_i, \sum_{i=1}^N k_i \geq 1\}$. The protocol overheads such as T_{ACK} and T_{DIFS} in terms of number of slots can be added to the busy period.

4.2. Successful effective data transmitted \bar{U}

The effective data transmitted in one renewal period is defined as the averaged data being successfully served by the access node within a single renewal period.

For m -MUD, the access node is capable of detecting up to m frames simultaneously; a collision occurs if more than m stations simultaneously attempt to transmit their frames. Hence the effective data transmitted is zero if $n > m$. We define a random variable U to represent the good data successfully served. Recall that n_i is the total number of stations transmitting their frames simultaneously from the i th group. The random variable is defined as a function of the vector $\mathbf{n} := (n_1, n_2 \ \dots \ n_N)$, i.e.,

$$U \triangleq \begin{cases} \sum_{i=1}^N \sum_{x=1}^{n_i} L_{x,i} & \text{for } n \leq m \\ 0 & \text{for } n > m \end{cases} \quad (31)$$

Then the average effective data being transmitted, $barU$, can be found by averaging over vector \mathbf{n} where $n \neq 0$ as was done for the case for the average busy period, as well as over the frame length, which is given by

$$\bar{U} = \sum_{S_U} P\{n_1 = k_1, \dots, n_N = k_N | n \geq 1\} \sum_{i=1}^N n_i \bar{L} \text{ [bits]} \quad (32)$$

where $\bar{L} = 1/q$ is the average frame length [bits] and the set S_U is defined as $S_U \triangleq \{(k_1, \dots, k_N) : 1 \leq \sum_{i=1}^N k_i \leq m, 0 \leq k_i \leq M_i\}$. Finally, we can substitute (32), (30), and (22) into the (33) to get throughput. The protocol overheads are included.

$$\text{Throughput} = \frac{\bar{U}}{\bar{I} + \bar{B} + \sum_{S_U} P(\mathbf{n}|n \geq 1)T_{ACK} + T_{DIFS}} \quad (33)$$

The unit of the throughput is bits/slot. The protocol overhead such as T_{DIFS} and T_{ACK} should be given as the number of slots. We have not substituted expressions in place of \bar{I} , \bar{B} and \bar{U} into (33) to avoid repetition.

5. Numerical results

In the previous section, we have obtained the throughput expression for the multi-rate 802.11 m -MUD. Let us refer to it as 802.11- m from now on. In this section, we use the throughput expression to evaluate and compare the performance of 802.11- m ($m \geq 2$) with the conventional one, i.e., 802.11-1 ($m = 1$). Note that choosing $m = 1$, our analysis for 802.11- m reduces itself to the conventional system. Throughput optimization is then discussed and it is shown that the performance of 802.11- m network increases significantly with increasing m .

In order to validate our model, we have developed a network simulation program in MATLAB using the same conditions and assumptions under which the analysis was conducted. The

simulation starts with stations having a frame ready to transmit, whose lengths are uniformly distributed. The DCF algorithm as outlined in the paper is then used in the simulation to coordinate multiple transmissions using backoff counters. When the idle period ends (backoff process of one or more stations finishes), relevant information, e.g. the total successful data transmitted and the total idle time, is used to calculate network throughput as per (21). This is done by comparing the number of stations whose backoff process ends at the same time (hence they start transmission), and the number of stations m whose transmissions can be successfully decoded at the receiver. With this marking the end of one renewal period, the simulation is run for 10,000 renewal periods to generate the averaged results which we report in this paper along closely predict the simulation results.

We assume that the stations are divided into four groups; each group uses a different data rate. Group 1 uses the highest available data rate while Group 4 employs the lowest. They are 11, 5.5, 3 and 1 Mbps, respectively. The other parameters used for the generation of numerical results are listed in Table 1.

5.1. Influence of initial contention window size

In this subsection, we study the effect of initial contention window on network throughput of our 802.11- m system. The effect of maximum backoff stage, is negligible, a fact that is verified by our numerical computations. This view is also shared by Yang et al. [1] who have reported minimal impact of $k_{max,i}$ on network throughput for conventional 802.11.

Fig. 4 aims to show how throughput varies as the sizes of initial contention windows of stations belonging to Group 3 and Group 4 are increased. The number of stations in each group we have used are $M_1 = 3$, $M_2 = 2$, $M_3 = 3$ and $M_4 = 2$, respectively. These values are also used for all other graphs except when they are varied in Figs. 5 and 6. Initial contention window sizes of the faster rate groups are fixed at 32. Since these stations are using the lowest data rates, as the size of their initial CW increases, their chance to transmit frames – p_i the transmission probability of a station belong to Group i – decreases. We note

Table 1
System parameters used in numerical analysis.

Parameter	Value
Slot duration	20 μ s
DIFS duration	50 μ s
SIFS duration	10 μ s
ACK	14 bytes
Average frame length	5000 bits
Header size	192 bits
Data rate for header and ACK transmission	1 Mbps

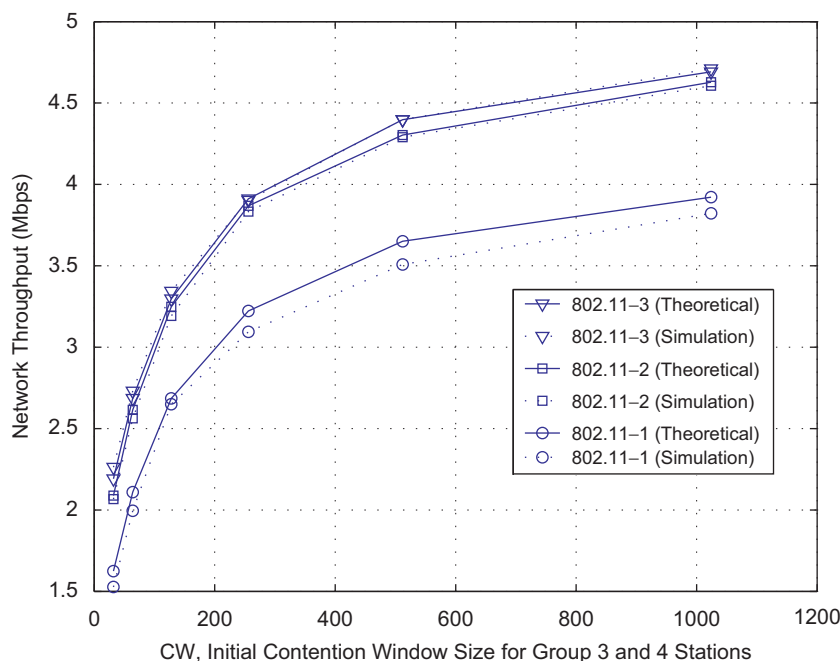


Fig. 4. Network throughput vs. $CW_{3,0}$ and $CW_{4,0}$ plotted for 802.11-1, 802.11-2 and 802.11-3 MAC.

that the throughput – the sum of all throughputs of individual groups is largely dominated by the fastest rate group. Similar behavior can be observed in simulation results presented in Fig. 4. Note that our simulation results are consistent with the analytical results, as verified in each comparison in the sequel. Simulation results are dotted lines; analytical results are solid.

A considerable increase in throughput, about 32%, is shown for 802.11-2 in Fig. 4, as compared to the conventional system; while that for 802.11-3 is only about 3.0%. Note that the throughput increase for $m = 3$ is very small in this setting. We will talk about the issue of throughput maximization as m changes in the next subsection.

Next we analyze the cases when the number of stations of the lowest rate group M_i increases, from 1 to 10. As shown in Fig. 5, the throughput now decreases for a fixed initial contention window size, set at 32 for all groups. This can be explained as

follows: with increase in M_i , the number of stations contending for network resources belonging to Group 4 increases. Hence more and more stations using the lowest data rate get a chance to grab the channel, and they obviously tend to hold the channel longer. This will obviously reduce the throughput. Another factor which reduces the throughput is that as the total number of stations increase, the chance of collisions increases as well. Hence the effective amount of data successfully transferred will reduce.

However, by increasing the initial contention window size of Group 4 stations, their probability of transmission p_4 can be made smaller. Hence the effect of increasing M_4 can be mitigated. Fig. 6 indicates this situation. As the contention window size is increased from 32 to 1024, we notice that, the throughput can hold out well.

The frame size is another important parameter which influences the throughput heavily. In the earlier section, we have

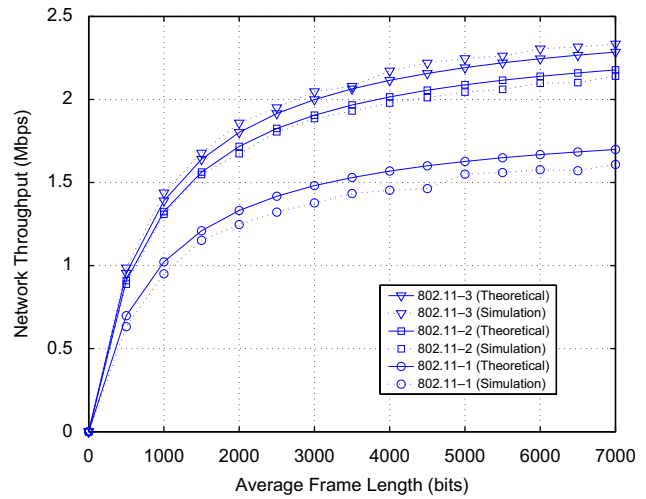
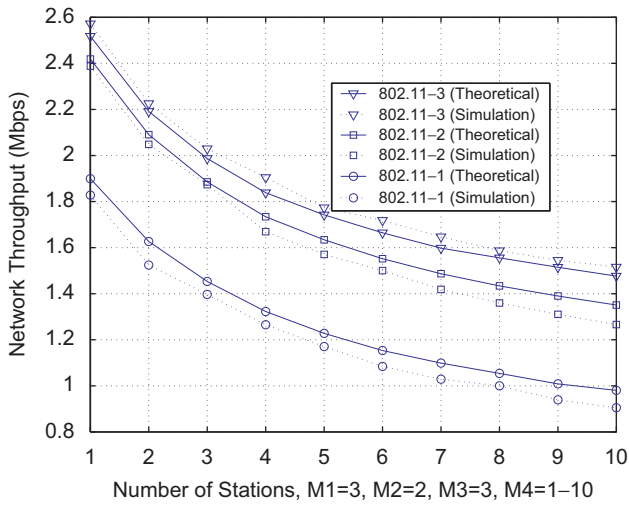


Fig. 5. Network throughput vs. M , the total number of stations. Only M_4 is increased here. Graphs are plotted for 802.11-1, 802.11-2 and 802.11-3 MAC.

Fig. 7. Network throughput vs. average length of frame. Graphs are plotted for 802.11-1, 802.11-2 and 802.11-3 MAC.

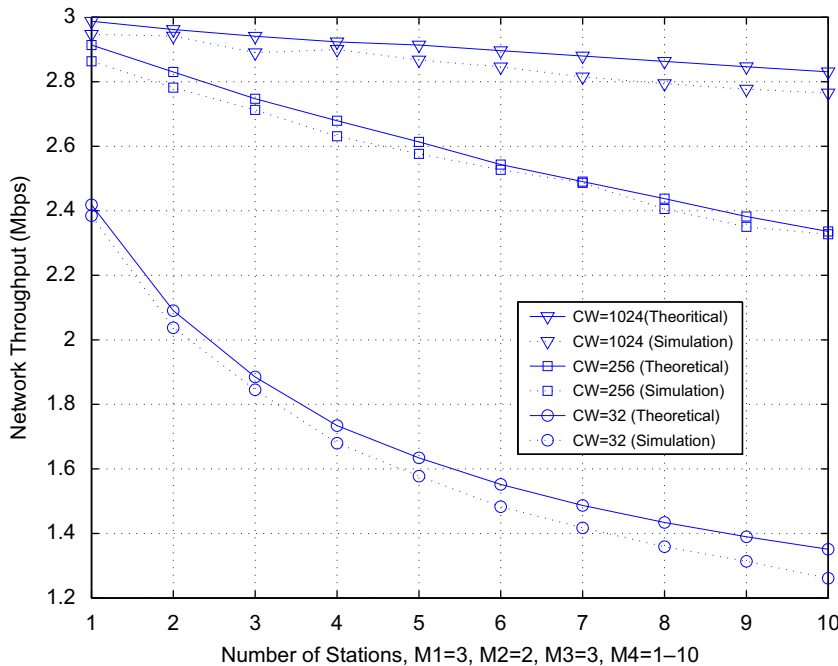


Fig. 6. Network throughput vs. M , total number of stations. Only M_4 is increased here. Graphs are plotted for three values of initial contention window of the fourth group, which are 32, 256, and 1024.

modeled the frame size of each station as a geometric distribution with parameter q . We now show in Fig. 7 how throughput varies as the average frame length, or $1/q$, is increased. The protocol overheads, shown in Table 1, will play a bigger role when smaller average frame lengths are used. The throughput will thus increase as the average frame length increases. Average frame length also needs to be carefully chosen as the trend shows saturation behavior after 6000 bits. Using a larger average frame length will cause a larger busy period and hence stations need to wait longer for their turn to transmit, so the quality of service degrades.

5.2. Throughput optimization vs. fairness issue

Now we study how throughput can be optimized for the multi-rate 802.11- m system. As we learned from earlier examples in this paper, the initial contention window size determines the transmission probability p_i and hence the throughput. One trivial optimization solution with maximum throughput can be obtained when casting all of the network resources to the fastest rate group stations. However, then the other lower rate group stations will not be able to transmit any frames. This can be observed in Fig. 9. We consider a BSS consisting of only two groups and throughput is plotted as function of the transmission probabilities, p_1 being the probability for the faster rate group. Note that the optimum point occurs when the transmission probability of the slower rate group set to zero, i.e., $p_2 = 0$.

Therefore, one may consider different criteria other than the aggregate throughput maximization. One is fairness among different users. While different criteria can be used for fairness, in this paper we consider *proportional fairness* and see if a non-trivial optimal point is available for each m . Under the proportional fairness criteria, the transmission probabilities of different groups are varied together in proportion. As we have done so far, let us divide the stations into four groups. Now in this paper, we investigate a special case by making p_i equal to each other, i.e., $p_1 = p_2 = p_3 = p_4$. Note that with our throughput expression, one can choose other non-proportional fairness as well. With this selection, every station in the network has an equal chance of

transmitting frames. We observe from Fig. 8 that there exists an optimal p_1 for each m and substantial throughput enhancement can be maintained as m increases. Another point of interest that can be studied from Fig. 8 is that as m increases, the optimal transmission probabilities also increase, hence giving the stations more chance to transmit their frames. This makes sense. As m increases, simultaneous transmissions are encouraged by increasing the transmission probabilities (or by decreasing the initial contention window size).

6. Proposed mechanism for performance enhancement

Having established that the m -MUD enabled 802.11 system as discussed in this paper do provide significant throughput enhancement even in the distributed MAC, we now would like to discuss enabling system components by which this enhancement can be realized.

6.1. Physical layer

At the physical layer, we want to use non-spreading based techniques to perform MUD to improve on spectral efficiency. It has been shown that low density parity check (LDPC) codes can be used to identify and decode information from several sources transmitted simultaneously [20,21] using message passing iterative decoders. As for the multi-rate scenario, all the stations transmit their physical layer header using 1 Mbps. The signal field of header contains information about the rate at which the station is going to transmit [1]. After extracting the data rate information, the access node can start decoding at that data rate.

6.2. MAC layer

At the MAC layer, all stations will use DCF with CSMA/CA to acquire network resources. It is possible that more than one stations finish their backoff process simultaneously. They will hence start transmission. A collision will occur in multi-rate

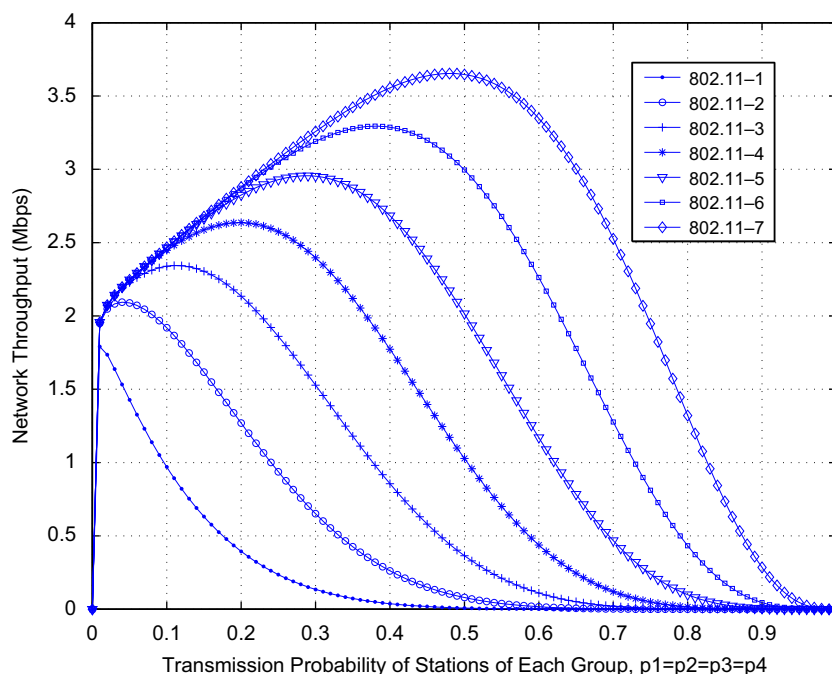


Fig. 8. Network throughput vs. p_1 — the probability that a Group 1 station transmits. Graphs are plotted for 802.11-1, 802.11-2, ..., and 802.11-7 MAC.

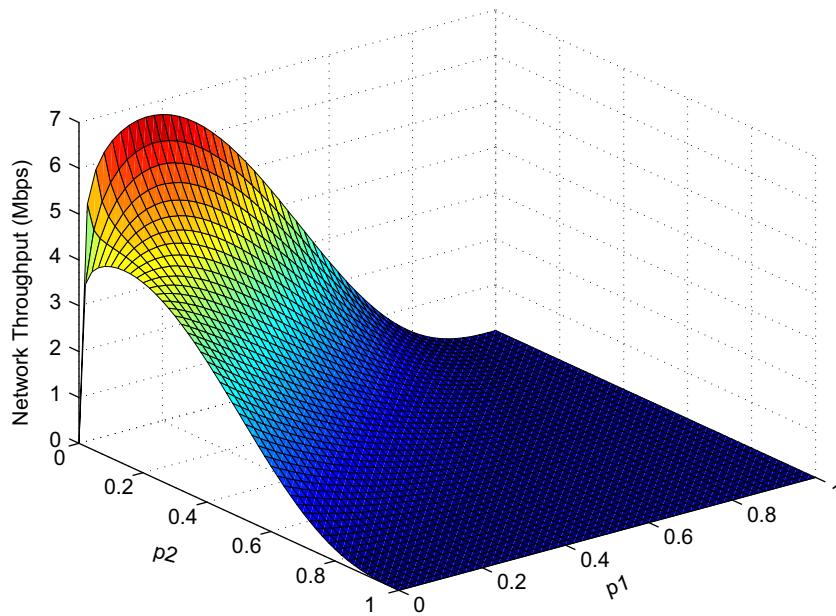


Fig. 9. Throughput vs. p_1 and p_2 . The plot is for 802.11-2 with only two groups.

802.11- m system when more than m stations finish their backoff processes at the same time and start transmission. We are assuming no hidden terminals in this paper and all the stations initiate frame transmission at the start of a time slot. Thus, we are using basic access DCF without request-to-send and clear-to-send (RTS/CTS) procedure. The use of RTS/CTS will be useful for solving the hidden terminal problem. We plan to extend the current work to this situation in our future contribution.

Our work presented here shows that the throughput can be optimized for a given network setting. For example, if each station can get the estimate of the number of stations contending for network resources, it can tune its initial contention window size to achieve the optimal throughput point as in Fig. 8. After the optimal probabilities are obtained, the initial contention window sizes can be back calculated using (13). These optimum probabilities and initial contention window sizes can be pre-calculated for different network loads M and a look-up table can be used. For example, considering 802.11-2 system, the optimal probabilities from Fig. 8 can be observed to be 0.07. This will result in initial contention window sizes of 13 if we set maximum backoff stage of all groups equal to 5. Similar exercise can be performed for other values of m to populate a look up table.

The results so far in this paper indicate that the DCF in 802.11 can be easily modified to exploit the m -MUD capability of the access node by controlling CWs in the multi-rate and multi-user detection wireless multiple access networks. In addition, it is shown that the throughput scales reasonably well as we have seen from Fig. 8.

7. Conclusion

In this work, we have analyzed the proposed multi-rate 802.11- m MAC system where up to m stations can transmit their frames simultaneously using different data rates. We have obtained new analytic expression for the network throughput. By comparing the analysis results with extensive simulation results, the validity of the derived throughput expression has been verified. The analysis result is general enough to subsume previous throughput expressions for 802.11 DCF. While the use of a MUD capable receiver at the physical layer has the potential to

improve performance significantly, it has been shown that appropriate control of the MAC parameters is very important to be able to capitalize on the available benefit. Sometimes, the aim of a network control and resource allocation should not only be the aggregate throughput maximization under which low rate group stations are completely ignored. We have shown that the proposed framework can tell us how the throughput can be maximized under the constraint of proportional fairness among different groups (Fig. 9).

References

- [1] Verdú S. Multiuser detection. New York: Cambridge University Press; 1998.
- [2] Wu J, Lee H-N. Performance analysis for LDPC space-time coded modulation in MIMO multi-access channels. *IEEE Trans Commun* 2007;55:1417–26.
- [3] Sanderovich A, Peleg M, Shamai S. LDPC coded MIMO multiple access with iterative joint decoding. *IEEE Trans Inf Theory* 2005;51:1437–50.
- [4] Ping Li, Guo Qinghua, Tong Jun, J. The OFDM-IDMA approach to wireless communication systems. *IEEE Wireless Commun Mag* 2007: 18–24.
- [5] Li P, Lihai Liu KW, Leung WK. Interleave division multiple-access. *IEEE Trans Wireless Commun* 2006;5:938–47.
- [6] Viterbi AJ. Very low rate convolutional codes for maximum theoretical performance of spread spectrum multiple-access channels. *IEEE J Select Areas Commun* 1990;8:641–9.
- [7] Zheng PX, Zhang YJ, Liew SC. Multipacket reception in wireless local area networks. In: *Proceedings of the IEEE ICC'06, Istanbul, Turkey, 2006*.
- [8] Bianchi G. Performance analysis of the IEEE 802.11 distributed coordinated function. *IEEE J Select Areas Commun* 2000;18:535–47.
- [9] Goldsmith A. Variable-rate variable-power MQAM for fading channels. In: *Proceedings of the IEEE vehicular technology conference, 1996*. p. 815–9.
- [10] Ci S, Sharif H. An link adaptation scheme for improving throughput in the IEEE 802.11 wireless LAN. In: *27th annual IEEE conference on local computer networks, 2002*. p. 205–8.
- [11] Yang Duck-Yong, Tae-Jin Lee, Jang Kyunghun, Chang Jin-Bong, Sunghyun Choi. Performance enhancement of multi rate IEEE 802.11 WLANs with geographically scattered stations. *IEEE Trans Mobile Comput* 2006;5:906–19.
- [12] M. Heusse, F. Rousseau, G. Berger-Sabbatel, A. Duda. Performance Anomaly of 802.11b. In: *Proceedings of IEEE INFOCOM, 2003*.
- [13] Bianchi G. IEEE 802.11 – saturation throughput analysis. *IEEE Commun Lett* 1998;2:318–20.
- [14] Park H, Kwon Kim C. Performance analysis of multi rate IEEE 802.11 WLANs with channel error. In: *Proceedings of the 9th international conference on advanced communication technology, vol. 3, 2007*. p. 1479–81.
- [15] Abu-Sharkh O, Tewfik AH. Performance analysis of multi-rate 802.11 WLANs under finite load and saturation conditions. In: *IEEE vehicular technology conference, vol. 4, 2005*. p. 2652–7.
- [16] Cali F, Gregori E. Dynamic tuning of the IEEE 802.11 protocol to limit. *IEEE/ACM Trans Networking* 2000;8:785–99.

- [17] Zhang J, Lee H-N. Throughput enhancement with a modified 802.11 mac protocol with multi-user detection support. *Int J Electron Commun* 2008;62:365–73.
- [18] Peng Xuan Zheng, Ying Jun Zhang, Soung Chang Liew., Analysis of exponential backoff with multipacket reception in wireless networks. In: *Proceedings of 31st IEEE conference on local computer networks*, 2006. p. 855–62.
- [19] G.D. Celik, G. Zussman, W.F. Khan, and E. Modiano. Mac for networks with multipacket reception capability and spatially distributed node. In: *IEEE INFOCOM*, 2005. p. 18–24.
- [20] S. Ghez, S. Verdu, S.C. Schwartz. Stability properties of slotted aloha with multipacket reception capability. *IEEE Trans Autom Control* 1988;33:640–8.
- [21] L. Tong, Q. Zhao, G. Mergen. Multipacket reception in random access wireless networks: from signal processing to optimal medium access control. *IEEE Commun Mag* 2001;39:108–12.



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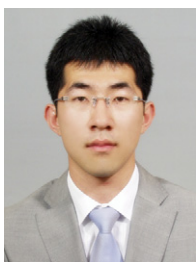
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