Performance Limits of the Measurements on Compressive Sensing for Multiple Sensor System

Sangjun Park, Hwanchol Jang and Heung-No Lee*
Gwangju Institute of Science and Technology, South Korea. Email: {sjpark1,hcjang,heungno*}@gist.ac.kr

***Abstract*—A performance analysis of Multiple-Sensor-System(MSS) on a compressive sensing(CS)[1] w.r.t. the per-sensor-measurements(PSM) is studied. In the proposed MSS, sensors make measurements using CS and the decoder jointly recover signals from them. We obtain the upper bound on the recovery failure probability for given K-sparse signals, derive the relationship between PSM and the number of sensors(S) for the recovery. We examine the effect of SNR and S for the recovery. We use the concept of joint typicality proposed by Shannon[6]. We shows that PSM converges to the sparsity(K) as S increases for given K-sparse signals. Theoretical result is consistent with [3][4][5].**

***Index Terms*—Compressive Sensing, Multiple Sensor System. Joint Recovery.**

I. Introduction and Motivation

Multiple-Sensor-System (MSS) deploys many sensors to a limited region and uses them to measure the signal from a common information source in different locations. In MSS, high resolution signal can be obtained as many sensors are used to measure a common phenomenon from many places. However, the coverage areas of sensors may significantly overlap with each other as they are distributed in a limited region. This causes redundancy in the measurement signal. The transmission of the redundant signal to the fusion center is a significant communication costs. There is tradeoff between the resolution and the redundancy on the number of sensors. To work on this tradeoff relationship, we use the idea of the compressive sensing [1]. CS reduces the number of measurements while it recovers the signal perfectly. Using this technique, it is possible to reduce the redundancy and obtain high resolution simultaneously by reducing the per-sensor-measurements (PSM).

To investigate our problem, we propose to use an information theoretic tool, the concept of Jointly Typicality [6]. It was also used by Akcakaya and Tarokh [2] for the single sensor case. Using this tool, we can derive the upper bound on the failure probability as a function of PSM, the number of sensors, the sparsity and the noise variance.

Clearly, the MSS problem is different from a single sensor system in many aspects. For an appropriate modification of the tool for MSS problem, we should consider these differences. One big difference is the signal correlation among the sensors. For a successful extension, we use the inter-signal correlation in the system model and the decoder also takes advantage of this signal correlation for a signal recovery. To make the correlation model, we assume that each sensor has the same sparsity and shares the same support set which is the set of indices for the non-zero elements. Obviously, in the recovery, the decoder using this prior information gains benefits.

II. Theorems

Theorem 1: Let the rank of $\mathbf{F}_{s,J}$ be K for each s and J be any candidate set, $M > K$, $\sigma^2 = min(\sum_{i \in I \setminus J} x_s(i)^2)$ over s, and $\delta > 0$. Then, P$\{\texttt{Fail}|\mathbf{x}\}$ converges to zero as the number of sensors increases.

Theorem 2: Let the rank of $\mathbf{F}_{s,J}$ be K for each s and J be any candidate set, $M > K$, $\sigma^2 = min(\sum_{i \in I \setminus J} x_s(i)^2)$ over s, $\delta > 0$, S_i be the number of sensors of the i^{th} MSS, σ_i^2 be the noise variance of the i^{th} MSS and P$_1\{\texttt{Fail}|\mathbf{x}\} \le \gamma$. If the noise variance increases, i.e.,$\sigma_1^2 < \sigma_2^2$, then, the sufficient condition for P$_2\{\texttt{Fail}|\mathbf{x}\} \le \gamma$ is

$$S_2 \ge S_1 \max\left(\frac{f\left(\frac{\delta}{\sigma_1^2}\frac{M}{M-K}\right)}{f\left(\frac{\delta}{\sigma_2^2}\frac{M}{M-K}\right)}, \frac{g\left(\frac{\sigma_1^2}{\sigma_{\min,1}^2} + \frac{\delta}{\sigma_{\min,1}^2}\frac{M}{M-K}\right)}{g\left(\frac{\sigma_2^2}{\sigma_{\min,2}^2} + \frac{\delta}{\sigma_{\min,2}^2}\frac{M}{M-K}\right)} \right). \tag{1}$$

We note that $f(x) = log(1+x) - x$, $g(x) = log(x) - x + 1$, $\sigma_{min,i}^2 \equiv min(\sum_{j \in I \setminus J} x_s(j)^2) + \sigma_i^2$ over s and J, J denotes any subset with size K expect for I and I denotes the set whose entries are corresponding to indices of the nonzero elements in signal. All theorems will be explained in the next section.

III. Contributions and Conclusions

We use the described correlation model with noisy observation. First, we have found how many per-sensor-measurements (PSM) are needed for successful recovery in the MSS problem. As the number of sensors increases, how does PSM change? There is a limit we have found. We will show this behavior and will show how PSM depends on the sparsity. We have Theorem 1 that the infimum of PSM is the sparsity obtained as the number of sensors increases. Different from the results in [3], [4], [5], the work of ours gives analytical results. Our analysis works for a small number of sensors as well. Second, we have shown that the decoder which uses the prior information obtains benefit in terms of the Signal to Noise Ratio (SNR). Specifically, Theorem 2 tells us how the required SNR decreases as the number of sensors changes.

IV. Acknowledgment

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (Do-Yak Research Program, N0. 2010-0017944).

References

[1] Dvaid L. Donoho, “Compressive sensing,” *IEEE Trans. On Information Theory,* vol. 52, pp. 1289-1306, 2006.
[2] Mehmet Akcakaya and Vahid Tarokh, ”Shannon-Theoretic Limits on Noisy Compressive Sampling”, *IEEE Trans. On Information Theory,* vol. 56, 2010.
[3] S. Sarvotham et. al, ”Analysis of the DCS one-stage greedy algorithm for common sparse supports, Technical Report TREE-0503: Rice University, Department of Electrical and Computer Engineering,” Oct. 2005.
[4] D.Baron et. al ”Distributed compressive sensing,” 2009.
[5] Pablo Vinuelas-Peris and Antonio Artes-Rodriguez, ”Bayesian Joint Recovery of Correlated Signals in Distributed Compressed Sensing,” 2010 2^{nd} International Workshop on Cognitive Information Processing
[6] Thomas M. Cover and Joy A. Thomas, ”Elements of Information Theory second Edition,” 2006