Message Passing Aided Least Square Recovery for Compressive Sensing

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I. Introduction and Motivation

Compressive sensing (CS) have got attention as a promising signal processing technique to reduce information rate of sparse signals [1]. One line of CS related researches are to devise low complexity recovery algorithms since the conventional L1-norm based recovery algorithms still have high computational complexity for practical applications. Recently, a few researchers have made an attempt to apply probabilistic message passing (PMP) ideas to CS recovery [2], [3] since PMP has provided a successful solution for low complexity decoding while showing suboptimal performance in channel coding problems, such as low-density parity check codes [4].

Motivated by such previous works, in this paper, we propose a new least square estimation (LSE) based CS recovery algorithm by applying PMP, called PMP-LSE. It is well known that CS recovery is basically an underdetermined system and it can be reformed as an overdetermined system with the support set information (SI). Therefore, in the proposed algorithm, PMP undertakes to find the SI of the signal to reform the recovery to an overdetermined case, and then LSE completes the recovery using the SI. Mainly, PMP-LSE has two strong benefits. First, PMP-LSE shows outstanding performance with noisy measurements by removing the noise effect from elements belonging to the non-support set. Second, PMP-LSE prevents the recovery from diverging. Under certain conditions, PMP based algorithms fails in the recovery due to divergence caused by a large number of iterations. In the algorithm, however, the possibility of the divergence highly decreases since PMP is only used to search the SI with a few iterations.

II. Problem setup

We consider a sparse signal $\mathbf{x} \in \mathbf{R}^N$ whose sparsity is characterized by q, named sparsity rate. With the sparsity rate q, each element of $\mathbf{x}$ belongs to the support set denoted by $\mathbf{S}$. Hence, $|\mathbf{S}|$ corresponds to Binomial random variable with $\mathbf{B}(N,q)$. Let $\mathbf{x_S} \in \mathbf{R}^{|\mathbf{S}|}$ denote a vector consisting of nonzero elements belonging to $\mathbf{S}$, and assume that each element of $\mathbf{x_S}$ follows Gaussian distribution with $\mathbf{N}(0,\sigma_x^2)$. We also assume that the sensing matrix is a well-designed binary matrix, *i.e.*, $\mathbf{\Phi} \in \{0,1\}^{M\times N}$, according to [5] such that the measurements $\mathbf{y} \in \mathbf{R}^M$ are generated by $\mathbf{y} = \mathbf{\Phi}\mathbf{x}$. Then, noisy measurements $\mathbf{z} \in \mathbf{R}^M$ at the decoder are described as $\mathbf{z} = \mathbf{y} + \mathbf{n}$, where each element of $\mathbf{n} \in \mathbf{R}^M$ is Gaussian noise with $\mathbf{N}(0,\sigma_n^2)$.

III. Algorithm

The algorithm is divided into two parts: PMP and LSE.

i) PMP: PMP consists of two kinds of probability calculations based on Bayesian rule: Variable to check message (VCM, $\mathbf{v}_{i\to j}$) and check to variable message (CVM, $\mathbf{c}_{j\to i}$) calculation where i and j indicate the index of elements of $\mathbf{x}$ and $\mathbf{z}$, respectively.

$$\text{VCM}: \mathbf{v}_{i\to j}^{l} := p\{x_i|\mathbf{z}\} = C^l p\{x_i\} \times \prod_{k:\phi_{ki}=1, k\neq j} \mathbf{c}_{k\to i}^{l-1} \quad (1)$$

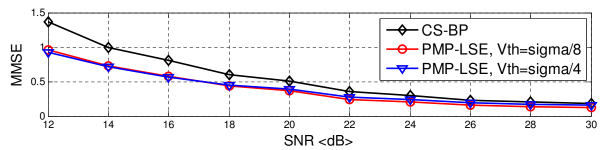

Fig. 1. MMSE performance of PMP-LSE (N=100,M=80,q=0.1,$Niter$=3)

$$\begin{aligned}\text{CVM}: \mathbf{c}_{j\to i}^{l} := \quad & P\{z_j|x_i\} = P\{z_j - \sum_{k:\phi_{jk}=1, k\neq i} x_k + x_i | x_i\} \\ = \quad & p\{z_j | any\, x_k : \phi_{jk}=1\} * \mathbf{v}_{k_1\to j}^{l} * \cdots * \mathbf{v}_{k_{L_j-1}\to j}^{l} \quad (2)\end{aligned}$$

Here, l is the number of iteration, ϕ_{ji} is the (j,i) th element of $\mathbf{\Phi}$, L_j is the number of ones in jth row of $\mathbf{\Phi}$, and C^l is the normalization constant for lth VCM. And, $*$ indicates the convolution operation. At each iteration, PMP updates VCM and CVM by exchanging the probabilistic messages among the elements of $\mathbf{x}$ and $\mathbf{z}$. After a few iterations, PMP distinguish the elements of the support set with a certain threshold denoted by V_{th}.

ii) LSE: Once the SI is given, $\mathbf{x_S}$ is easily estimated only using the corresponding columns of $\mathbf{\Phi}$, denoted by $\mathbf{\Phi_S}$, *i.e.*, $\mathbf{x_S} = (\mathbf{\Phi_S}^T\mathbf{\Phi_S})^{-1}\mathbf{\Phi_S}^T\mathbf{z}$. By combining the SI and $\mathbf{x_S}$, PMP-LSE completes to find the recovered signal $\hat{\mathbf{x}}$.

IV. Numerical results

To demonstrate the performance, we simulated PMP-LSE with CS-BP [2]. Figure 1 plots the MMSE per elements as a function of SNR for variety of thresholds with three PMP iterations. Figure 1 shows that PMP-LSE outperforms CS-BP notably in low SNR region. The reason is that PMP-LSE prevents the corruption of zero elements from the noise effect by pre-detecting the support set using PMP.

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