A New Framework for Sensor Networks via Compressive Sensing (Study)

Jun-Ho Lee, HyunU Song, Heung-No Lee*
INFONET Lab.(Internship), Gwangju Institute of Science and Technology, Korea
Email : {ljh8097, olivaceous85}@gmail.com, heungno@gist.ac.kr,
The asterisk * indicates corr. author <technical report>

Abstract— Compressive sensing(CS) is a new signal acquisition framework for sparse and compressible signals. In sensor networks, sensor measurements are often correlated with each other. This paper proposes a CS framework that aims to exploit both intra-signal and inter-signal correlation and minimize the number of required samples/bits. A joint sparsity decoding is performed at a center unit. We show the total number of required bits is a far fewer than a conventional sensor networks scheme.

Keywords : Compressive sensing, sparsity, joint reconstruction, correlation, sensor networks

I. Introduction

We consider a sensor network of *S* sensors and study the amount of information required to represent the sensed signal. In sensor network, measurements are typically highly correlated with each other. Two kinds of sensor measurement correlations are frequent which include sensor-to-sensor(inter sensor) correlation and within a sensor (intra-sensor) correlation. And in a sensor network, sensors are battery operated and thus energy sensitive.

In a conventional sensor system, each sensor node performs analog-to-digital conversion (ADC) at the Nyquist rate which should be set at least twice the maximum frequency of the measured signal. The uniform spaced samples are obtained, say *N*, during a fixed time period. These raw samples are redundant and they can be compressed before being transmitted. So transmission of the redundant raw data is inefficient. To save the transmittal power, compression can be done so that the number of bits transmitted can be reduced. However, for this conventional approach, two things are required, 1) all the sensed samples need to be gathered and stored at a collaboration location, and thus inter sensor correlation cannot be performed unless all the sensed samples are at a single location 2) compression operations can be performed at the central location. But the compression operations require their own computation which burns the battery power.

In this paper, we propose a compressive sensing framework which requires a far fewer number of samples than the conventional approach does. The inter and the intra sensor correlations are exploited without collaboration and a joint decoding of multiple signals can be performed at the center unit. The framework allows distributed compression, while requires joint decompression.

A. Compressive sensing (CS)

Compressive sensing is a sampling method that converts high dimension input signal into signal that lie in a space of smaller dimension, $M << N$. Let $\underline{x} \in \mathbb{R}^N$ be a k -sparse vector that has k nonzero coefficient in Ψ , $\underline{x} = \Psi \underline{s}$. CS computes inner product with randomized incoherent basis, i.e., $\underline{y} = \Phi \underline{x}$. where $\Phi \in \mathbb{R}^{M \times N}$ is measurement matrix. Then, the signal is recovered by convex optimization. If $\underline{x} = \Psi \underline{s}$ such that $\| \underline{s} \|_0 = k$ and Φ is an $M \times N$ measurement matrix with independent identically distributed (i.i.d.) Gaussian entries, only $k+1$ projections of the signal onto the incoherent is required to reconstruct with high probability. Unfortunately, since reconstruction problem via l_0 optimization is NP-complete, tractable recovery procedures based on linear programming that use ck projections to reconstruct the signal are proposed. And iterative greedy algorithms allow far faster reconstruction though slightly more measurements are expensed.[1],[2]

B. Distributed compressive sensing (DCS)

In [3], the potential of DCS for sensor network is demonstrated. In a typical DCS scenario, a number of sensors measure signals that represent in some basis and has correlation among sensors. Each sensor performs CS individually and then transmit a few resulting measurements to the data sink. Since the measurement rates relate directly to the signals' conditional sparsities, joint decoding each sensed signal precisely in the data sink requires fewer the number of measurements. And simulations in [4] show the number of measurements per signal required to exactly recovery each signal approximate to $k+1$ as the number of sensors increases.

In this paper, jointly decoding is performed for the original signal not each sensed signals at a destination. And we show how efficiently sensor network limited for battery power operate in jointly decoding original signal.

This paper is organized as follows. In section II, the system model that individually encoding and joint reconstruction for original signal are performed is described. Then, the simulation results are given in section III. In step 1 and 2, we assume sensor network using two sensors measures sparse signal and compressible signal respectively; In step 3, as increasing the number of sensors. And, finally, we conclude this paper in section IV.

Notation : A bold face letter denotes a vector or a matrix; $\mathbf{A}_s$ denotes measurement matrix of s -th sensor node; $\mathbf{A}^{\mathbf{T}}$ denotes transpose of matrix; $\mathbf{C}$ denotes correlation matrix; $\mathbf{D}$ denotes DCT matrix; $\underline{x}_s$ denotes signal vector obtained by s -th sensor node.

II. System Model

In this paper, we consider sparse signal and compressible signal. It is easy to check if the signal is sparse by representing in the sparse basis Ψ. The signals obtained by a number of sensors have both intra- and inter-correlation, so we generate this type of signals. Original signal is $\underline{x}$ and each sensors' signal measured individually is $\underline{x}_1, \underline{x}_2, \cdots, \underline{x}_s$. On the other hand, each signal is represented as $\mathbf{C}_1\underline{x}, \mathbf{C}_2\underline{x}, \cdots, \mathbf{C}_s\underline{x}$.

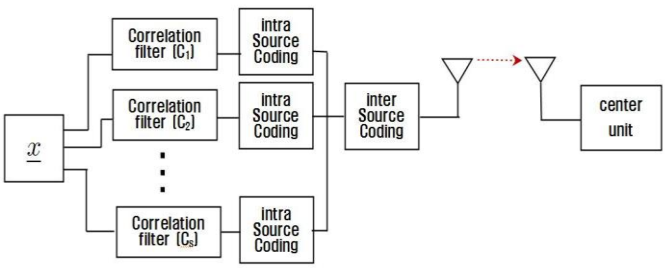

Fig.1-1. Conventional sensor network scheme for compression, SC denotes source coding.

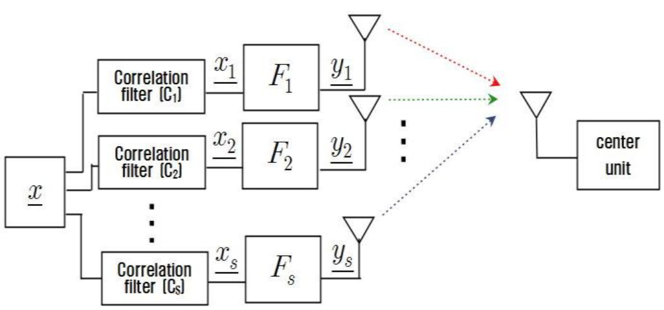

Fig.1-2. Acquisition compressed signal using CS and joint reconstruction scheme.

The sparse signals or the compressible signals $\left[\underline{x}_1 \;\; \underline{x}_2 \;\; \cdots \;\; \underline{x}_s\right]$ are computed with measurement matrixes $\left[\mathbf{F}_1 \;\; \mathbf{F}_2 \;\; \cdots \;\; \mathbf{F}_s\right]$ that are built accordingly through the same procedure independently. Let $\mathbf{F}_s$ be a $M_s \times N$ measurement matrix with i.i.d. Gaussian entries. Though $M_s \times N$ measurement matrix $\mathbf{F}_s$ is $M_s << N$, the measurements $\underline{y}_s = \mathbf{F}_s\underline{x} \in \mathbb{R}^{M_s}$ preserve the essential information about the k -sparse signal in sparse basis Ψ. Therefore, acquisition of measurements $\left[\underline{y}_1 \;\; \underline{y}_2 \;\; \cdots \;\; \underline{y}_s\right]$ is achieved independently by a projection onto the random matrix. By this compressive sensing, the intra-correlation compression is accomplished.

$$\begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \vdots \\ \underline{y}_s \end{bmatrix} = \begin{bmatrix} \mathbf{F}_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{F}_s \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \vdots \\ \underline{x}_s \end{bmatrix} \qquad (1)$$

When the measurements from individual compressive sensing are received by a center unit, joint signal reconstruction algorithm is performed. The number of measurement required to recovery precisely is related to the signals' conditional sparsity and correlation coefficient. The reconstruction technique is based on linear program that seeks the sparsest components. We utilize l_1 minimization with equality constraints as decoding algorism that can be downloaded at [5]. And correlation matrix $\mathbf{C}$ can be obtained from empirical data.

$$\hat{\underline{s}} = \arg\min \|\underline{s}\|_1 \; s.t. \underline{y} = \mathbf{FC\Psi}\underline{s} \;, \; \hat{\underline{x}} = \mathbf{\Psi}\hat{\underline{s}} \qquad (2)$$

By the way the samples of each sensor are highly correlated, the sensor to sensor compression which use the inter correlation can be achieved. In conclusion, original signal $\underline{x}$ can be reconstructed from much fewer measurements generated by performing CS in each sensor.

III. Simulation Results

We show meaningful results proceeding step by step. The simulation parameters we used in this paper are briefly described as follow. Signal sensed in each sensor has 10 nonzero coefficients in step 1(I_N) or step 2(D_N) and the signal length N is 100 respectively. The location of nonzero coefficients and the measurement matrix are randomly generated in each loop. In each measurements point, 1000 runs are implemented. We denoted the probability in figure 2, 3 by the total number of exact reconstruction to total loops ratio.

A. *Step 1 : considering measured signal as sparse signal in $\mathbf{I}_N$ domain*

Simply, we assume that measured signals from brain are directly sparse signal, $\underline{s}_1, \underline{s}_2$; and the number of sensors is two that are adjacent to each other. Since $\underline{s}_1, \underline{s}_2$ is measured from the same signal, $\underline{s}$, we can represent measured signal as $\mathbf{C}_1\underline{s}, \mathbf{C}_2\underline{s}$. And we use general class of measurement matrices satisfying a suitable RIP as measurement matrix ($\mathbf{F}_1, \mathbf{F}_2 \in \mathbb{R}^{M_s \times N}$) of CS in each sensor node. Let $\underline{s}, \underline{s}_1, \underline{s}_2 \in \mathbb{R}^N$ be k -sparse signal in $\Psi = \mathbf{I}_N$. Individual compressive sensing is performed with measurement matrix as $\mathbf{F}_1, \mathbf{F}_2$ at sensor node separately, $\underline{y}_s = \mathbf{F}_s\underline{x}_s$. Where $\underline{y}_1, \underline{y}_2$ are vectors of M_s dimension. Above entire procedure is represented as matrix equation.

$$\begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{F}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_2 \end{bmatrix} \begin{bmatrix} \underline{s}_1 \\ \underline{s}_2 \end{bmatrix} \quad (3)$$

Joint reconstruction from received measurement is achieved by solving the minimization problem at a center unit.

$$\hat{\underline{s}} = \arg\min \| \underline{s} \|_1 \ s.t. \underline{y} = \mathbf{F}\mathbf{\Psi}\underline{s}, \ \Psi = \begin{bmatrix} \mathbf{C}_1\mathbf{I}_N \\ \mathbf{C}_2\mathbf{I}_N \end{bmatrix} \quad (4)$$

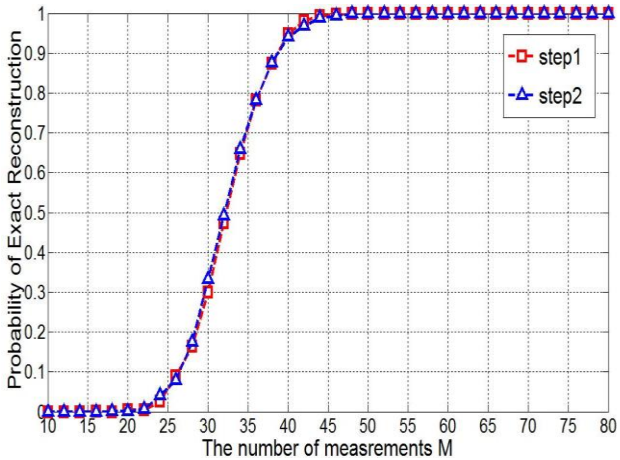

Fig.2. In situation that the sensed signal length is N=100 and sparsity is k=10 on each sensor, the total number of measurement(M) required to exactly reconstruct original signal $\underline{s}$ and $\underline{x}$ respectively. Step1(square), step2(triangle).

B. Step 2 : considering measured signal as sparse signal representing in $\mathbf{D}_N$ *domain*

Assume that a signal $\underline{x}$ in an N-dimensional vector space that is k-sparse in sparsity basis space, $\Psi = \mathbf{D}_N$. So $\underline{x}$ is compressible signal. Also each sensor measures $\underline{x}_1, \underline{x}_2$ that have inter-correlation. These signal experience compressive sensing.

$$\begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{F}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \underline{x} = \mathbf{D^T}_N \underline{s} \quad (5)$$

Determining the sparsest signal that explains the measurement $\underline{y}$ through the l_1 minimization is followed by $\underline{x} = \mathbf{D}^T{}_N \underline{s}$.

$$\hat{\underline{s}} = \arg\min \| \underline{s} \|_1 \ s.t. \underline{y} = \mathbf{F}\mathbf{\Psi}\underline{s}, \ \Psi = \begin{bmatrix} \mathbf{C}_1\mathbf{D^T}_N \\ \mathbf{C}_2\mathbf{D^T}_N \end{bmatrix} \quad (6)$$

C. Step 3 : The number of measurement required to recovery exactly analysis as the number of sensor increase.

We experiment required total measurement rate by increasing the number of sensor. Each sensor independently computes CS compressing $\begin{bmatrix} \underline{x}_1 & \underline{x}_2 & \cdots & \underline{x}_s \end{bmatrix}$ into $\begin{bmatrix} \underline{y}_1 & \underline{y}_2 & \cdots & \underline{y}_s \end{bmatrix}$ and transmit individual measurements to a center unit. Subsequently, a center unit recovery original $\underline{x}$ by joint reconstruction exploiting inter sensor correlation. The result of this step show the number of individual required measurements to be transmitted to a center unit decreases as increasing the number of sensor. Since limited for battery power on each sensor node, sensor networks must be energy efficient by minimizing communication. In terms of decreasing individual measurements to be transmitted, which a few sensor experience overhead for data quantity is alleviated on each sensor.

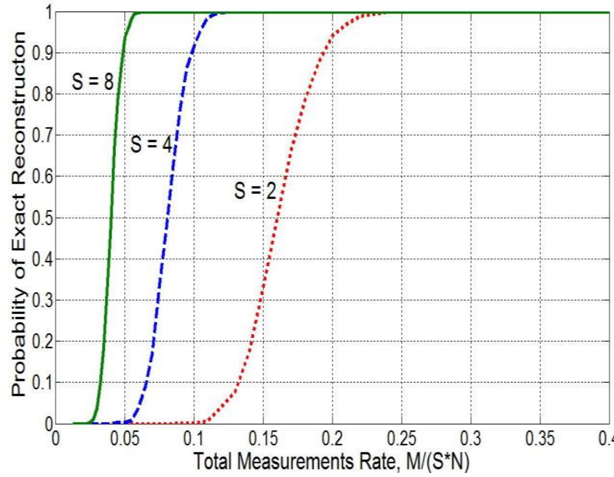

Fig.3. Total measurement rate for reconstructing the original signal as increasing the number of sensors (in case of s=2, 4, 8)

IV. Conclusion and Future Works

We experiment performance of proposed framework using CS for sensor network step by step. The results of above simulations show applying CS to sensor networks can achieve exploiting inter and intra correlation in measured signals without collaboration. And though the number of sensors is increase, total measurements required to reconstruct the original signal do not increase linearly. Therefore, the larger the number of sensors, each sensor transmits to a center unit the smaller measurements. The signals we have used in these simulations are different from natural signal, so in a more practical model the system might encode real signal and recover it. In order to build such system, we should find proper basis transform that has superior energy compaction.

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