Preliminary Test for Information Theory
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1. There are two boxes. The first box contains 3 white balls and 7 black balls. The second box contains 4 white balls and 5 black balls. Kim takes a ball from the first box, makes note of its color, and places it into the second box. Then, Kim draws a ball from the second box. Let $X_{1}$ be the random variable for the first draw. Let $X_{2}$ be the random variable for the second draw.
A. Define the two random variables.
B. Obtain the conditional probability $\operatorname{Pr}\left\{X_{1}=\right.$ black ball $\mid X_{2}=$ white ball $\}$.
2. (Monty Hall Problem) There is a probability game called Monty Hall Problem. This problem has its origin at one of America's famous TV game show <Let's make a deal> in the 1960s. The name of the game show host is Monty Hall.

The problem is given as follow. There are Monty Hall and a guest. Monty Hall is the game show host. The guest participates in the game. The guest is given a chance to make a selection and he could win a nice Cadillac sedan. There are three doors. The guest is led by Monty Hall to choose a single door and get the gift behind the door. Behind one door there is a nice Cadillac sedan. Behind each of the other two doors, there is a goat.

Now the guest makes his selection. For example, Door 1 out of three doors is selected. Monty Hall is supposed to open the door, Door 1, that the guest has selected; but he tricks the guest by opening other door, say Door 3, and shows that there is a goat behind Door 3 . Then, he adds by asking the guest if the guest would like to switch his selection to Door 2? Of course, Monty Hall would never open up the Door behind which there is the Cadillac sedan.


The problem given for you to ponder is to analyze the following strategies for the guest? Which one is best for the guest? Please give your answer with the probabilistic analysis you learned in class.

1. Staying his selection to Door 1
2. Switching his selection to Door 2
3. Making a random guess between Door 1 and Door 2.
