Ultrasound Imaging Super Resolution via Compressive Sensing

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***Abstract*— The appearance of granular 'noise' referred to as speckle that inherently exists in Ultrasound (US) imagery, which decreases the resolution of US image. Therefore, it is difficult to directly use common image processing methods in US imagery. Here by using the fact that the image is highly compressible in the wavelet domain and leverage new results of compressed sensing (CS) theory to make an accurate estimate of the original high-resolution of US image. Unfortunately, direct use of a wavelet compression basis not applicable in traditional CS approaches because of the coherency between the point-samples from the subsampling process and the wavelet basis. To overcome this problem, we include the subsampling low-pass filter into our measurement matrix, which decreases coherency between the basis. To invert the subsampling process, we use the appropriate reconstructing algorithm such as greedy, matching pursuit algorithm and obtain the high-resolution US image. The result is a simple and efficient algorithm that can generate high-resolution, high quality US images without the use of data sets. The experimental results show the proposed method is very effective and can get better reconstruction performances.**

Keywords-Compressive sensing; Super resolution; Ultrasound Imaging; Image processing; Signal processing

I. INTRODUCTION

In the last few decades, medical imaging has been rapidly gaining prominence in diagnostic radiology techniques such as CT scan, SPECT, magnetic resonance imaging (MRI), Ultrasound (US), digital radiography and others. Among these imaging techniques, ultrasound imaging is popular noninvasive and low cost method to observe the dynamical behavior of organs. It uses a transducer to produce ultrasound waves which travel through body tissues. The return sound waves vibrate the transducer which turns into electrical pulses that travel to the ultrasonic image scanner where signals processed and transformed into a digital image [1].Higher frequencies generates better resolution of the image but this limits the depth of the penetration. However, the presence of noise is imminent due to the loss of proper contact or air gap between the transducer probe and body [2]. Particularly, speckle noise degrades the edges definitions and fine details, and limits the contrast by making it a low resolution image. The challenge is to design methods which can selectively reduce noise and recover high frequency information that was lost during the image processing in US system. To overcome these challenges we use super resolution (SR) algorithm. The main goal of the SR algorithm is to recover lost information and recover the original high-resolution image as closely as possible.

Conventional approaches to generate a SR image require set of low-resolution US images, typically aligned with sub-pixel accuracy to solve for the missing high-frequency information [3]. However, at present, there has been growing interest in an area of research also known as image upsampling [4] or hallucination [5] which recovers the information from a single, low-resolution image. The single image super resolution (SISR) problem is particularly important for US imaging since only single, low-resolution image generated and the upsampling must be incorporated as a post process.

In our work, we consider the problem of US SISR and utilizes a novel algorithm for reconstructing the noiseless high resolution image based on the CS [6] [7]. CS brings the possibility of reconstructing a sparse image with fewer measurements than Nyquist sampling theory requires. The key idea is to obtain high resolution US image that will be sparse in a transform domain (e.g., wavelet) and using compressed sensing theory to solve the sparse coefficients from the low-resolution image. Furthermore, recovering an approximation of high-resolution image from the wavelet transform, we can compute the final result in the spatial domain.

In this paper, we first focus on SR problem within the algorithm of CS, which allows us to reconstruct the high resolution image using a simple greedy algorithm such as Regularized Orthogonal Matching pursuit (ROMP) [8]. Second, we implement a unique way of using wavelet basis for image compression in our algorithm by incorporating Gaussian noise from down sampling process into our technique. This allows us to improve the incoherence between the compression and sampling basis and yields better results than current approaches.

This paper organized as follows: Section 2 describes the compressed sensing theory. Section 3 discuss about US image super resolution via CS. Experimental results are discussed in section 4 and section 5 concludes the paper.

II. COMPRESSED SENSING THEORY

The theory of CS heavily depends on signal or image sparsity and can efficiently extract the most efficient information from a small number of measurements, i.e., to reduce the collection of inessential data [6] [7]. CS demonstrates that a small non-adaptive linear measurements of a compressive image have enough information to reconstruct it perfectly [6] [8]. If we represent our desired high-resolution image as a *n*-dimensional vector $x \in R^n$ where *n* is large. We want to estimate this high-resolution signal from the low-resolution input $y \in R^m$, where $m << n$ .Let us consider that signal y has been acquired from the original through a linear down sampling measurement process, represented as:

$$y = \Theta x \tag{1}$$

where x is an n x 1 high-resolution image vector in spatial domain, Θ is a sampling matrix that performs the linear measurements on x. Our goal is to recover the high-resolution x using only y as input

Initially, this seems like an impossible feat since the *m* samples of y yield a (*n* − *m*) dimensional subspace of possible solutions for the original x that would match our given observations. In order to know one of those possible solutions for our desired y we apply a key assumption of CS that transformed version of signal, $\hat{x}$ is *k* -sparse under some basis Φ ,it means that at most *k* non-zero coefficients in that basis (e.g., $\|\hat{x}\|_0 \le k$, where $\|.\|_0$ denotes the l_0 quasi-norm). This is not an unreasonable assumption, since we know that the high-resolution image will be a real world image, and so it will be compressible in a transform domain, e.g., wavelet transform. We can now write our measurement process from Eq. (1) as:

$$y = \Theta\Phi\hat{x} = V\hat{x} \tag{2}$$

where $V = \Theta\Phi$ is a general *m*×*n* measurement matrix. If we can have for $\hat{x}$ given the measured y , to get our desired high resolution signal x we could apply the inverse transform $\Phi^* x$ .Unfortunately, conventional techniques such as least square,inversion approach for solving for $\hat{x}$ do not work since Eq. (2) is severely under-determined. However, in paper[9] proof show that in CS if $m \ge 2k$ and V meets certain properties of the restricted isometry property (RIP)[10],that is:

$$(1-\delta_k)\|\alpha\|_2 \le \|V\alpha\|_2 \le (1+\delta_k)\|\alpha\|_2, (0 < \delta_k < 1) \tag{3}$$

where α represents random k-sparse vector. In general, the RIP states that a measurement matrix will be valid if every possible set of *Z*-sparse vector columns of V forms an approximate orthogonal set. In effect, we want the sampling matrix Θ to be as incoherent to the compression basis Φ as possible. Examples of matrices that have been proven to meet RIP include Gaussian matrices, partial Fourier matrices and Bernoulli matrices [11].

Then Eq.(2) can be solved uniquely for the sparest $\hat{x}$ that satisfies the equations in paper[9].Therefore the sparse solution for Eq.(2) is found by solving the following l_0 norm minimization problem,

$$\min\|\hat{x}\|_0 \; s.t. y = V\hat{x} \tag{4}$$

where $\|\hat{x}\|_0$ describes the l_0 norm, the number of non-zero entries in $\hat{x}$. The solution to the problem in Eq. (4) is combinatorial in nature with prohibitive computational load in practical applications. Convex relaxation of the l_0 problem to the following l_1 problem,

$$\min\|\hat{x}\|_1 \; s.t. y = V\hat{x} \tag{5}$$

The l_1 optimization of Eq. (5) will solve correctly for $\hat{x}$ [19] as long as the number of samples $m = O(k \log n)$ and the matrix V meets the RIP [10] with this parameters $(2k, \sqrt{2}-1)$ .This can be done with methods such as linear programming [9] and basis pursuit [7].

Greedy reconstruction algorithm: Orthogonal Matching Pursuit (OMP) was one of the first algorithms explored to solve Eq. (4) which is simple and fast [12] to overcome the large running time of l_1 since there is no known polynomial-time algorithm for linear programming [10] even its optimization is more efficient than the l_0 .However, OMP has a major drawback because of its weaker guarantee of exact recovery than the l_1 methods. To bypass these limitations, a modification to OMP called Regularized Orthogonal Matching Pursuit (ROMP) was proposed which recovers multiple coefficients in each iteration, thereby accelerating the algorithm and making it more robust to meeting the RIP [10]. In this technique, we use the ROMP algorithm for signal reconstruction.

III. SUPER RESOLUTION ULTRASOUND COMPRESSED SENSING

In our methodology, we utilize wavelets as our compression basis Φ since they are good at representing images sparsely than non-localized bases for example Fourier. However, downsampling matrix Θ in SR process involves point-sampled measurements, which lead to measurement matrix V that does not meet the RIP conditions due to incoherency. Intuitively, we can see that the better a basis is at representing confined features (such as wavelet), the more coherent it will be to point sampling because it can represent small spatial features (e.g., point samples) with only a less coefficients, by definition. Therefore, we propose to modify Eq. (2) depends on the observation that is filtering the high resolution image before downsampling. In other words, we can write our desired high-resolution image as x_r (the sharp version), which is then filtered by matrix β to result in a blurred, high resolution version $x_b = \beta x_r$ .This blurred version is then downsampled by Eq. (1):

$$y = \Theta x_b = \Theta\beta x_r \tag{6}$$

In this approach, we choose a Gaussian filter β . Since it act as a multiplication by a Gaussian in the frequency domain we can define $\beta = f^h G f$, which makes Eq. (6):

$$y = \Theta f^h G f x_r \quad (7)$$

where f is the Fourier transform matrix, f^h is the inverse fourier transform and G is a Gaussian matrix with values of the Gaussian function along its diagonal and zeros elsewhere. With this formulation in hand, we can now solve for x_r by posing it as a compressed sensing problem by assuming that its transform $\widehat{x}_r$ is sparse in the wavelet domain:

$$\min \|\widehat{x}_r\|_0 \; s.t. y = \Theta f^h G f \Phi \widehat{x}_r \quad (8)$$

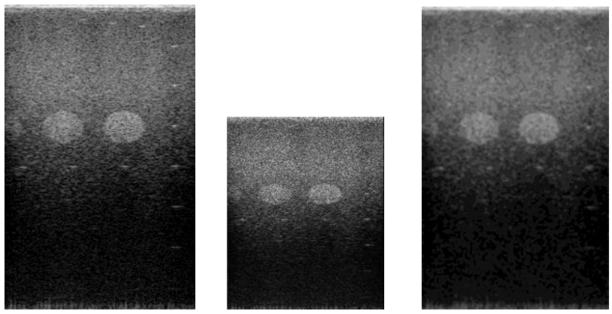
An approximate solution to this optimization problem can be achieved using greed methods. However, in order to solve $q = Vx$, where V^* = V^T with the assumption that $\|V^T V A\| \approx \|A\|$ [10] greedy algorithms require having both the forward matrix V and backward matrix V^*.We have shown the forward Matrix $V = \Theta\Phi\beta$,where, the Gaussian matrix in β cannot be inverted through transpose, i.e., $G^{-1} \neq G^T$,we use the backwards matrix of the form $V^* = \Phi^T \beta^{-1} \Theta^T = \Phi^T f^h G^{-1} f \Theta^T$.

We note that G^{-1} is also a diagonal matrix that is supposed to have the inversion of the Gaussian function of G along its diagonal. Whereas, we must be careful because of the well-known problem of noise amplification while inverting the Gaussian, we use a linear Weiner filter to invert the Gaussian function [13] to avoid noise, which means that our inverse matrix G^{-1} has diagonal elements of form $G_{i,i}^{-1} = G_{i,i} / (G_{i,i}^2 + \gamma)$.

The increases of the incoherence between the measurement and compression basis must be verified for proposed algorithm by computing the coherence with and without the blurring filter β . The coherence can be found by taking the maximum inner product between any two basis elements scaled by √*n* [14]. When the Gaussian filter is introduced the coherence drops to 131.8.On the other hand, Without the Gaussian filter, the coherence is 243.6 for n = 380×600.It's found that with Gaussian filter coherence reduction is enough to us apply the CS framework to this problem and get the high-quality results shown in the paper. With this formulation in place, we are now ready to solve the CS problem of Eq. (8).

In Eq. (8), we have shown the solution for the wavelet transform of the sharp, high-resolution image $\widehat{x}_r$ .We can take the inverse wavelet transform $\Phi^* x_r$ once we solve for $\widehat{x}_r$, to get the high-resolution image x_r . We use the ROMP greedy algorithm which is preferable over non-linear methods like linear programming [9] or basispursuit [7]. The algorithm is faster and can handle large vectors and matrices, which is vital when in images because the size of the matrices involved are $n \times n$.

The ROMP algorithm is similar to the OMP algorithm except the main difference is that instead of only selecting the largest coefficient, ROMP selects the continuous sub-group of coefficients with the largest energy, with the restriction that the largest coefficient in the group cannot be more than twice as big as the smallest member. These coefficients are then added to a list of non-zero coefficients and a least-squares problem is then solved to find the best approximation for these non-zero coefficients. The approximation error is then computed based on the measured results and the algorithm iterates again. In this work, we limit the number of coefficients added in each iteration to obtain the better results.



(a) (b) (c)

Figure 1: (a) Original image (380×600) (b) Down sampled image (190×300) and (c) Reconstructed image (380×600) obtained by using compressive sensing.

IV. EXPERIMENTAL RESULTS

We applied the proposed method In vivo and phantom ultrasound images. The ultrasound images are obtained using COSMOS US diagnostic system. The parameters used to generate the phantom image: the lateral beam width was 1.5, the pulse width 1.2 and the center frequency was 5 MHz.The size of phantom image is 380 *X* 600 the images used in this experiments are shown in figures 1

The proposed CS approach for SISR is compared with Nonlinear-diffusion(ND) [15], Bicubic [16] and Yang et al. [17], to estimate the quality of reconstructed US images, the peak signal to noise ratio (PSNR) used. The PSNR defined by:

$$PSNR = 10\log_{10}\frac{N_{\max}}{MSE} \quad (9)$$

where $N_{\max}$:The maximum fluctuations in the input image, $N_{\max} = (2^n - 1)$, $N_{\max}$ =255,when the components of a pixel are encoded on 8 bits;MSE:denotes the mean square error, given by:

$$MSE = \frac{1}{MN}\sum_{i=1}^{N}\sum_{j=1}^{M}\left|O(i,j) - R(\hat{i},j)\right|^2 \quad (10)$$

where $O(i, j)$:the original image, $R(i, j)$:the restored image.

Figure 2 shows us the comparison of PSNR used different reconstruction Algorithm as we can see from the curve, compared with the ND, bicubic and Yang.et.al, the performance of proposed algorithm is better.

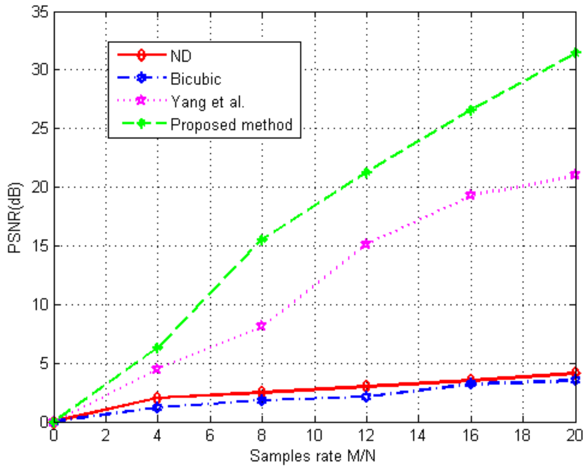
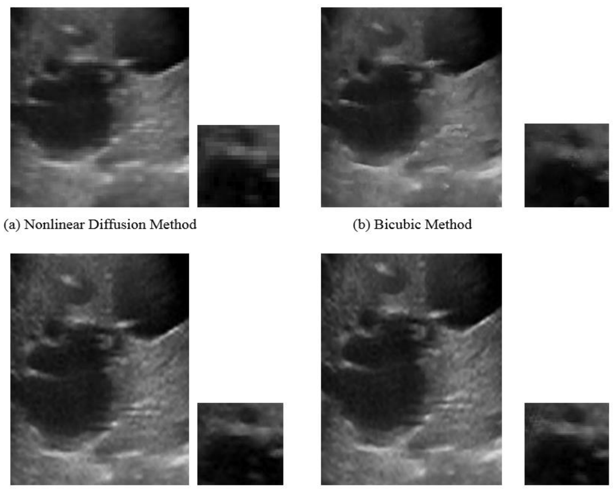


Figure 2: Comparison of PSNR for different reconstruction algorithm

Figure 3 represents recovered images by different methods. The amplification of the local regions of the recovery images is also shown in the right. From the result we can see that there are remarkable block effect in Non-diffusion method, Bicubic method and yang.et.al (shown in Fig. 3(a), (b) and (c)) recovers too smooth, distorted image and our prosed method can achieve



(a) Nonlinear Diffusion Method (b) Bicubic Method

(c) Yang et. al (d) Proposed algorithm

Figure 3(a-d): Reconstruction comparison of US images.

better result than other methods (shown in Fig. 3(d)) on detail preservation. To test our technique for clear visual comparison and accuracy by computing the root square error (RSE), a measure of the Euclidean distance, of their output to the original US high resolution image. In Figure 4 we plot RSE error variations for different magnifications level for various approaches and it is noted that our technique more information is available as the magnifications decreases so, the error reduced.

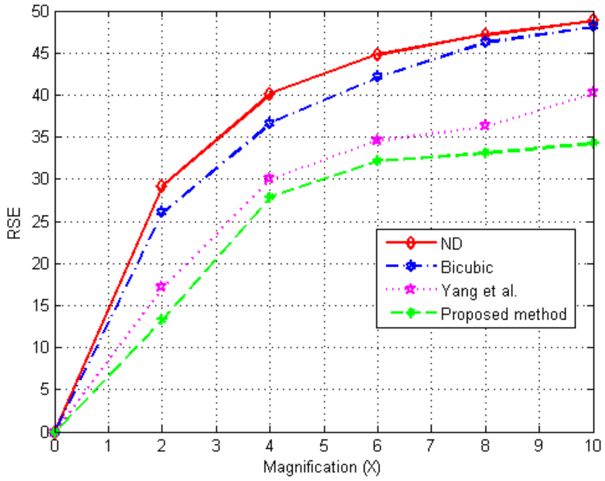


Figure 4: Reconstruction error comparison as a function of magnification level for the US phantom image.

The quantitative performance is measured with parameters like structural similarity (SSIM) [18] [19], mean structural similarity [20] and feature similarity (FSIM)[21] which compares the structure of two images by measuring the structural similarity, SSIM is related with the distortion of the visual sensing. The higher of SSIM and MSSIM are, much similar of the structure of the recovered image to the original image. Feature Similarity (FSIM) index is proposed based on the fact that human visual system (HVS) understands an image mainly according to its low level features [21]. It uses the phase congruency (PC) and gradient magnitude (GM) features of images as the evaluation index.

Table 1 shows the SSIM, MSSIM and FSIM of the reconstructed images obtained by different methods. From this we can see that the image recovered by our method is higher than that of the images obtained from the conventional methods mentioned in [15] [16] [17].

Methods	**FSIM**	**SSIM**	**MSSIM**
ND	0.7568	0.9543	0.6955
Bicubic	0.8231	0.9772	0.7593
Yang et al.	0.8467	0.9815	0.8067
Our Method	0.8538	0.9878	0.8114

Table 1: The numerical guidelines of the recovered images obtained from different methods.

V.CONCLUSIONS

In this work, we have demonstrated the single image super resolution problem within the compressive sensing framework and utilize greedy matching pursuit algorithm to solve for the high resolution image. The uniqueness of our approach is based on the fact that we are not using the dictionary data for training. As a consequence, we could significantly recover images and reconstruct it in high resolution than conventional methods commonly used for these applications. Experimental results show that our method not only gives better performance but also an effective reconstruction on ultrasound images.

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