INFONET, GIST
Journal Club (2013. 08. 01)

Compressive Sensing for Spread Spectrum Receivers

Authors:	Karsten Fyhn, Tobias L. Jensen, Torben Larsen
Publication:	IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, May.2013
Speaker:	Hyeongho Baek

Short summary: Compressive sensing enables the receiver to sample below the Shannon-Nyquist sampling rate, which may lead to a decrease power efficiency and production cost. This paper investigates the use of CS in a general Code Division Multiple Access (CDMA) receiver. Furthermore, they numerically evaluate the proposed receiver in terms of bit error rate under different signal to noise ratio conditions and compare it with other receiver structures.

I. INTRODUCTION

- As wireless communication devices are becoming more and more widespread and ubiquitous, the need for power efficiency and low production cost becomes paramount.
- Recently, a new concept termed CS has been attracting more and more attention in the signal processing community. If a signal is sparse in some arbitrary basis, it may be sampled at a rate lower than the Nyquist frequency.
- In the spread spectrum area, some researchers have studied the general use of CS for spread spectrum communication systems.
- In their work they apply CS to a general CDMA system. And they show that a random demodulation implementation may be used to subsample the CDMA signal, but they also develop a simplified version of the RD which performs equally well for CDMA signals but is simpler and cheaper to implement.

II. SIGNAL MODEL

Each slot contains an independent CDMA signal and the slots decoded sequentially and independently of each other.

For one slot, define a discrete QPSK baseband signal, $\mathbf{x} \in \mathbb{C}^{N \times 1}$ as:

$$\mathbf{x} = \mathbf{\Psi}\boldsymbol{\alpha} \tag{1}$$

where $\mathbf{\Psi} \in S_{\Psi} \subset \{\pm 1\}^{N \times N}$ is an orthogonal or near orthogonal dictionary, containing spreading waveforms for transmission, S_{Ψ} is the subset of $\{\pm 1\}^{N \times N}$ that contains orthogonal or near-orthogonal dictionaries and $\boldsymbol{\alpha} \in \{\pm 1 \pm j, 0\}^{N \times 1}$ is a sparse vector, that selects which spreading waveform(s) and what QPSK constellation point(s) to send.

Each node has a unique CDMA sequence assigned, which it uses to transfer information and each node does not know which neighbors it has, but it knows all possible CDMA sequences. Note that in this signal model $\boldsymbol{\alpha}$ is defined so that all users have identical amplitude.

In cases where the number of active nodes or users in a network is smaller than the total number of possible users, the vector $\boldsymbol{\alpha}$ may be assumed sparse, which is the enabling factor for CS.

At the receiver the following signal is observed:

$$\mathbf{y} = \mathbf{\Theta}(\mathbf{x} + \mathbf{w}) = \mathbf{\Theta}\mathbf{\Psi}\boldsymbol{\alpha} + \mathbf{\Theta}\mathbf{w} \tag{2}$$

Where $\mathbf{\Theta}$ is a measurement matrix, which we shall treat later, and $\mathbf{w} \in \mathbb{C}^{N \times 1}$ is Additive White Gaussian Noise (AWGN). Notice here that we take into account noise folding as the noise is folded down into the compressed domain together with the signal. This makes the noise has an impact on the demodulation performance, because each time the sampling rate is reduced by one half, the Signal to Noise Ratio (SNR) is decreased by $3dB$.

A. *Spread Spectrum Dictionary of Gold Sequences*

In spread spectrum signals, a possible dictionary $\mathbf{\Psi}$ is a set of Gold sequences, as used in e.g. GPS technology. A set of Gold sequences is a special dictionary of binary sequences with very low auto and cross-correlation properties.

When using such a CDMA dictionary, the received signal must be sampled at a rate corresponding to the chip rate, where a chip is one entry in the received Gold sequences. If $\boldsymbol{\alpha}$ is sparse the information rate of the signal is much lower and it may be possible to decrease the sampling rate by using CS.

III. COMPRESSIVE SENSING

CS is novel sampling scheme, developed to lower the number of samples required to obtain some desired signal.

Denote by $\mathbf{\Theta}_{\kappa} \in \mathbb{R}^{M \times N}$ a CS measurement matrix, where $\kappa \in \mathbb{N}$ is the subsampling ratio when compared to the Nyquist rate and $M = N / \kappa$. This measurement matrix is then responsible for mapping the N -dimensional signal $\mathbf{x}$ to a M -dimensional signal $\mathbf{y}$. Normally this would make it impossible to recover the original signal, but under the assumption that $\mathbf{x}$ is sparse in some basis, it is possible to reconstruct the original signal from the sampled, M -dimensional signal $\mathbf{y}$.

A. *Compressive Spread Spectrum Measurement Matrix*

In most CS literature a choice of measurement matrix or structure must be made. The Random Demodulator (RD) sampling structure is one of the most well-known measurement matrix structures developed, which is well suited for practical implementation. In the RD a Pseudo-Random Noise (PRN) sequence is mixed with the received signal. Because a spread spectrum transmitter has already spread the signal before transmission, we show that the RD structure can be improved so that the mixing with a PRN sequence at the receiver may be skipped.

The proposed measurement matrix may therefore be defined similarly to the definition of the RD matrix in “Beyond Nyquist : Efficient Sampling of Sparse Bandlimited Signals”.

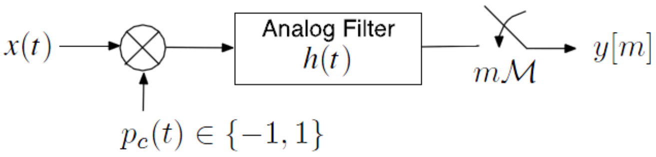


Figure 1 Pseudo-random demodulation scheme

In their work, the measurement matrix is based on two matrices, $\mathbf{D}$ and $\mathbf{H}$. First, let $\varepsilon_0, \varepsilon_1, \cdots, \varepsilon_N \in \{\pm 1\}$ be the chipping sequence used in the RD for a signal of length N. The mapping $\mathbf{x} \to \mathbf{Dx}$ signifies the demodulation mapping with the chipping sequence, where $\mathbf{D}$ is the diagonal matrix:

$$\mathbf{D} = \begin{bmatrix} \varepsilon_0 & & & \\ & \varepsilon_1 & & \\ & & \ddots & \\ & & & \varepsilon_N \end{bmatrix}$$

Second, the $\mathbf{H}$ matrix denotes the accumulate-and-dump action performed after mixing. Let M denote the number of samples taken. Then each sample is the sum of N/M consecutive entries of the demodulated signal. An example with $M = 3$ and $N = 6$ is :

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & & & & \\ & & 1 & 1 & & \\ & & & & 1 & 1 \end{bmatrix}$$

The reason for applying a chipping sequence is to spread the signal across the frequency spectrum, so that information is aliased down into the lower frequency area, which is left untouched by the low-pass filtering.

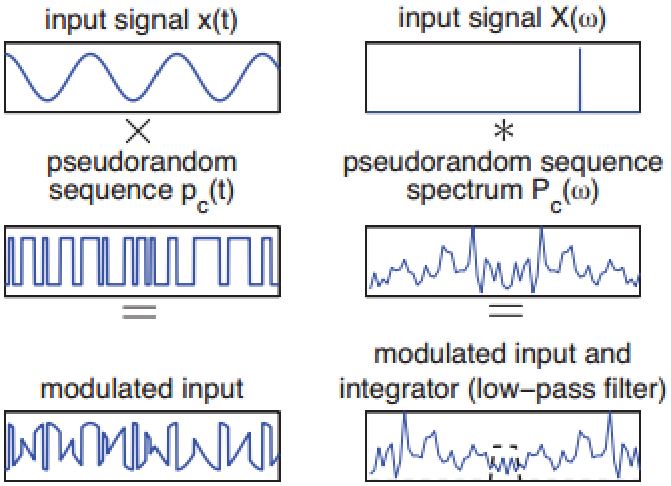


Figure 2 Action of the demodulator on a pure tone. The demodulation process multiplies the continuous-time input signal by a random sequence wave. The action of the system on a single tone is illustrated in the time domain(left) and the frequency domain (right). The dashed line indicates the frequency response of the lowpass filter.

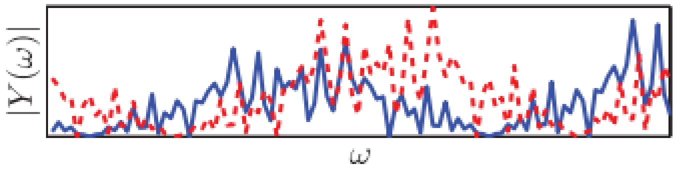


Figure 3 Signatures of two different tones. The random demodulator furnishes each frequency with a unique signature. This image enlarges the filter's passband region of the demodulator's output for two input tones (solid and dashed). The two signatures are nearly orthogonal.

In the proposed receiver this mixing is unnecessary because the signal has already been spread at the transmitter. The proposed receiver may therefore be simplified to:

$$\mathbf{y} = \mathbf{H}\mathbf{x} \tag{3}$$

This is significantly simpler to implement in hardware than the RD. The use of a CDMA dictionary introduces a random-like dictionary matrix, which spreads the signal out so that each sample contains a little bit of the original information signal. Therefore, the sampling process may be rewritten as:

$$\mathbf{y} = \mathbf{H}\mathbf{x} = \mathbf{H}\boldsymbol{\Psi}\boldsymbol{\alpha} = \boldsymbol{\Theta}\boldsymbol{\alpha} \tag{4}$$

Here, the measurement matrix becomes $\boldsymbol{\Theta} = \mathbf{H}\boldsymbol{\Psi}$

B. *Subspace Pursuit*

To reconstruct the signal a reconstruction algorithm must be chosen. Many different approaches have been developed, but two main classes of reconstruction algorithms are in widespread use: l_1 minimization and greedy algorithms. Often, l_1 minimization provides the best solution, but if the matrices $\mathbf{\Psi}$ and $\mathbf{\Theta}$ are very large, it is much more efficient to use the simpler greedy algorithms. Therefore, we choose to use greedy algorithms in this work.

Recall that $\mathbf{\Theta}_\kappa$ is a measurement matrix with N columns and N/κ rows and define $\mathbf{A} = \mathbf{\Theta}_\kappa \mathbf{\Psi}$. Then we define the Subspace Pursuit algorithm as in Algorithm 1. In each algorithm iteration, the pseudo-inverse is calculated as the least-squares solution as this is less computationally demanding.

Algorithm 1 Subspace Pursuit Algorithm [3]

```
Input:
Sparsity S, measurement and dictionary matrices combined $\mathbf{A}$ and received, sampled signal $\mathbf{y}$
Initialization:
$T^0 = \{$indices of the S largest absolute magnitude entries in the vector $\mathbf{A}^T\mathbf{y}$ $\}$
$\mathbf{y}_r^0 = \mathbf{y} - \mathbf{A}_{T^0}\mathbf{A}_{T^0}^T\mathbf{y}$
Repeat
    $l \leftarrow l+1$
    $T^l \leftarrow T^{l-1} \cup \{$indices of the S largest absolute magnitude entries in the vector $\mathbf{A}^T\mathbf{y}_r^{l-1}$ $\}$
    $T^l \leftarrow \{$indices of the S largest absolute magnitude entries in the vector $\mathbf{A}_{T^l}^\dagger\mathbf{y}$ $\}$
    $\mathbf{y}_r^l \leftarrow \mathbf{y} - \mathbf{A}_{T^l}\mathbf{A}_{T^l}^\dagger\mathbf{y}$
Until $\|\mathbf{y}_r^l\|_2 > \|\mathbf{y}_r^{l-1}\|_2$, $l \geq S$
```

To demonstrate the performance of the Subspace Pursuit algorithm with the Gold dictionary, they have performed numerical experiments to find the phase transition in the noise-less case for various choices of measurement matrices.

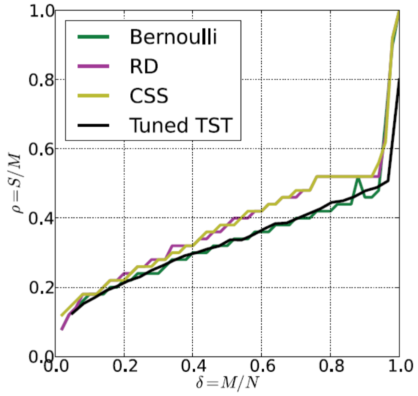


Figure 4 Phase Transition Diagrams for the three different measurement matrices (Rademacher, RD and CSS measurement matrix). The black line is the phase transition line for the Tuned Two Stage Thresholding(TST) algorithm from “Optimally tuned iterative reconstruction algorithms for compressed sensing”

IV. CONCLUSION

In this work they apply CS to a general CDMA system and they show that it is possible to use a very simple measurement scheme at the receiver side to enable subsampling of the CDMA signal.

V. DISCUSSION

After meeting, please write discussion in the meeting and update your presentation file.

Appendix

Reference

[1] W. Dai and O. Milenkovic, “Subspace pursuit for compressive sensing signal reconstruction,” IEEE Trans.Inf. Theory, vol. 55, no. 5, pp. 2230–2249, May 2009.
[2] A. Maleki and D. L. Donoho, “Optimally tuned iterative reconstruction algorithms for compressed sensing,” IEEE J. Sel. Topics Signal Process.,vol. 4, no. 2, pp. 330–341, Apr. 2010.
[3] W. Dai and O. Milenkovic, “Subspace pursuit for compressive sensing signal reconstruction,” IEEE Trans. Inf. Theory, vol. 55, no. 5, pp. 2230–2249, May 2009.

Active illumination single-pixel camera based on compressive sensing.

Filipe Magalhaes et al.

APPLIED OPTICS. (2011.02)

Presenter : Eunseok Jung

GIST, Dept. of Mechatronics , BiO-scopy Lab.

Gwangju Institute of
Science and Technology

Contents

- Background
  - Active illumination concept
  - Single pixel camera
- Experiment & result
- Conclusion

Background

- Active illumination

  - Direction, intensity, and pattern of illumination are controlled by commands or signals.

- Single pixel camera

  - Single pixel camera, developed originally at Rice University, is one of the paramount examples of CS.

  - It can be seen as an optical computer comprising:

    = digital micromirror device(DMD) (1024x768 micromirros)

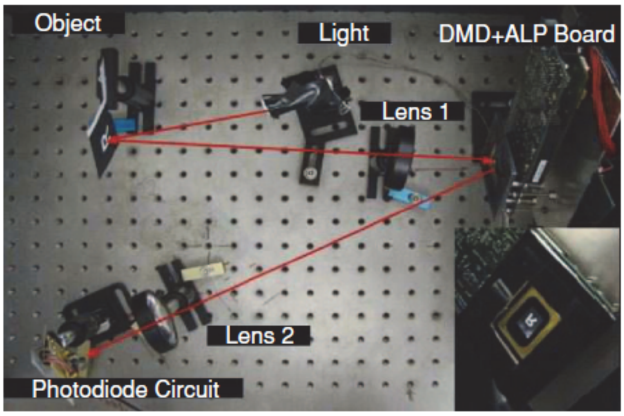
    = two lenses

    = single photodetector

    = A/D converter.

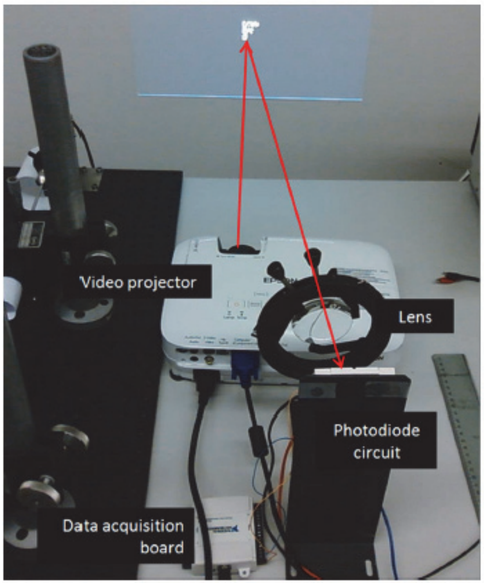
Background

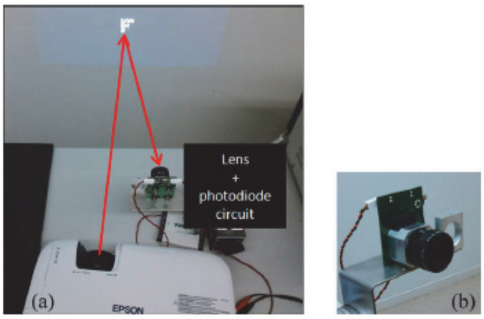
- Single pixel camera



- The process which include changing DMD pattern and taking signal from PhotoDiode will be repeated until M values are acquired.
- Each of these values (output voltage of the photodiode) can be interpreted as the inner product of the desired image x with a measurement basis $\Phi(m) = 1, 2, \ldots, M$.
- The resolution of the reconstructed image is limited by the pixel arrangement of the DMD.

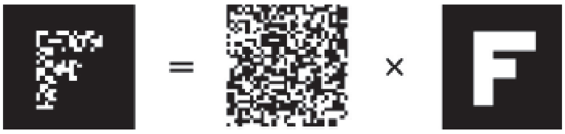
Active illumination Single-pixel camera





Instead of the DMD, Active illumination single-pixel camera setup used video projector to incorporate the random measurement matrix into the system.

Active illumination Single-pixel camera



- The video projector was used to project the result of the product between the image to be reconstructed and the random measurement patterns.
- Each of the output voltages of the photodiode amplifier circuit is representative for the inner product between the pattern used for that measurement and the image to be reconstructed.

Result

- Traditional Single-pixel camera(Rice university version)

Ideal image with 64*64 pixels

Haar wavelet →

(a) (b)

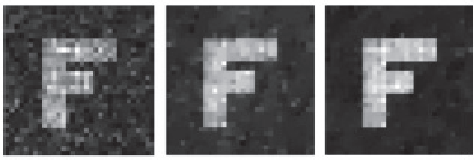
Fig. 9. Images (64 × 64 pixels) reconstructed using the best K-term Haar wavelet approximation: (a) $K = 400$ and (b) $K = 675$ [11].

(a) (b)

Fig. 10. CS reconstruction from: (a) $M = 820$ measurements ⇒ 20% and (b) $M = 1600$ measurements ⇒ 39% [11].

Result

- Active illumination single-pixel camera



(a) (b) (c)

Fig. 14. First results obtained (32 × 32 pixels ⇒ $N = 1024$) with the active illumination single-pixel camera: (a) 205 measurements ⇒ 20% (peak signal-to-noise ratio (PSNR) = 11.08 dB), (b) 410 measurements ⇒ 40% (PSNR = 12.30 dB), and (c) 717 measurements ⇒ 70% (PSNR = 13.21 dB).

1. Sharp edge was increased when use active illumination single pixel camera.
2. This system is cheaper and more compact than Rice university one.
3. It doesn't need active illumination source.

Thank you

Resource Allocation in Cognitive Radio Relay Networks

Authors: J.C Liang et. al.

Publication: IEEE JSAC, Mar. 2013
Speaker: Asif Raza

Short summary: In this paper authors formulate the problem of Resource Allocation (RA) in Cognitive Radio (CR) networks with relay stations. The problem takes into account the issues like: fluctuations of usable spectrum resource, channel quality variations caused by frequency selectivity, and interference caused by different transmit power levels. They propose easy to implement heuristic algorithms. The simulation results reveal that presented solutions show good proportional fairness among CR users and improvement in system throughput by power control.

I. INTRODUCTION

Resource allocation in CR relay networks is considered in the paper. The relay nodes are simply Medium Access Control (MAC) repeaters.

Main challenges in designing RA algorithm are:

- Fluctuations in available spectrum resource: The number of usable resource may differ in each area. It may cause great variations in available band-width and jitters between packets. Therefore they adopt proportional fair scheduling which allocates resource to CR MSs proportionally to their capabilities such as transmission rates determined by channel quality.
- Instability of wireless channels: The frequency selectivity would cause variations of Signal-to-Noise Ratio (SNR). As a result, the quality of each channel would differ on each node. The RA algorithm needs to take into account the instability of channel quality.
- Power control and interference among nodes: the number of usable channels are determined by the activity of PR networks. With proper power control, spatial reuse can be achieved and two adjacent CR RSs can transmit over the same channel.

II. SYSTEM DESCRIPTION

The network design considered is shown in the following figure.

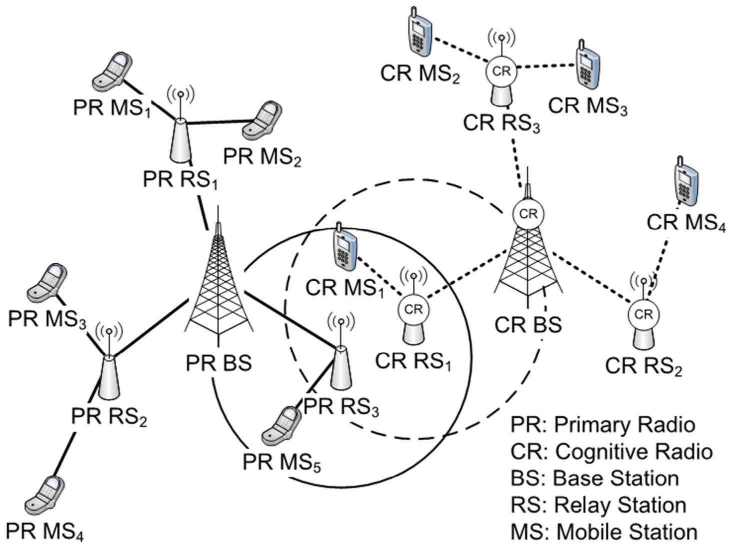


Fig. 1. An illustration of a cognitive radio relay network.

The network drawn with solid and dashed lines corresponds to Primary Radio (PR) network and Cognitive Radio (CR) network. The RA is performed by the Cognitive Radio Base Station (CR BS) in a centralized manner. The transmission of CR network is time-divided into frames and synchronized with the PR network. In each frame there are multiple time-slots not used by PR. The CR BS gathers available channels within the vicinity of each node and the quality of each channel. The CR BS then schedules the usage of frequency bands and time-slots for its downstream Cognitive Radio Relay Stations (CR RSs) and Cognitive Radio Mobile Stations (CR MSs). Each CR MS can connect to CR BS directly or through CR RS. The RA is done in downlink transmission only. The notations used are shown in table 1. The authors starts RA problem formulation by considering fixed transmit power and then extend the formulation to incorporate adjustable transmit power.

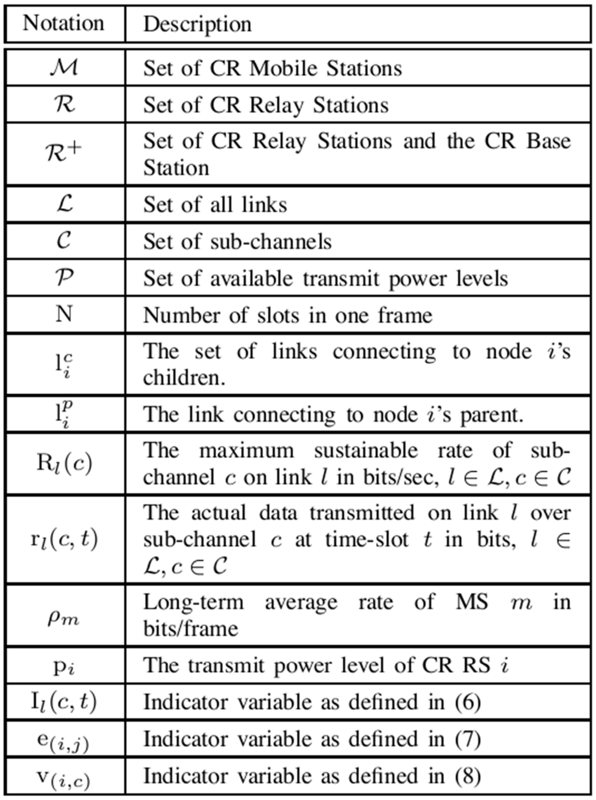
III. FIXED TRANSMIT POWER

A. *Problem Formulation:* proportional fair scheduling for cognitive relay networks with fixed transmit power in relay stations is considered. (Given parameters same as in system description section)

$$\mathrm{O}(\lambda) = \max \sum_{m \in \mathcal{M}} \frac{\lambda_m(t)}{\rho_m(t)} \text{ where } \lambda_m = \sum_{c \in \mathcal{C}} \sum_{t=0}^{N-1} r_{1_m^p}(c,t) \qquad (1)$$

Where $\lambda_m(t)$ is scheduled rate of CR MS *m* in frame *t*, and $\rho_m(t)$ is the long-term average rate of CR MS *m* until frame *t*.

TABLE I
LIST OF NOTATIONS



Notation	Description
$\mathcal{M}$	Set of CR Mobile Stations
$\mathcal{R}$	Set of CR Relay Stations
$\mathcal{R}^+$	Set of CR Relay Stations and the CR Base Station
$\mathcal{L}$	Set of all links
$\mathcal{C}$	Set of sub-channels
$\mathcal{P}$	Set of available transmit power levels
N	Number of slots in one frame
1_i^c	The set of links connecting to node i's children.
1_i^p	The link connecting to node i's parent.
$\mathrm{R}_l(c)$	The maximum sustainable rate of sub-channel c on link l in bits/sec, $l \in \mathcal{L}, c \in \mathcal{C}$
$\mathrm{r}_l(c,t)$	The actual data transmitted on link l over sub-channel c at time-slot t in bits, $l \in \mathcal{L}, c \in \mathcal{C}$
ρ_m	Long-term average rate of MS m in bits/frame
p_i	The transmit power level of CR RS i
$\mathrm{I}_l(c,t)$	Indicator variable as defined in (6)
$\mathrm{e}_{(i,j)}$	Indicator variable as defined in (7)
$\mathrm{v}_{(i,c)}$	Indicator variable as defined in (8)

$$\begin{aligned}&\sum_{c\in\mathcal{C}}\sum_{t=0}^{\tau} r_{1_i^p}(c,t) \ge \sum_{k\in 1_i^c}\sum_{c\in\mathcal{C}}\sum_{t=0}^{\tau} r_k(c,t)\ \forall i\in\mathcal{R},\ 0\le\tau\le N-1\\&\sum_{c\in\mathcal{C}}\sum_{t=0}^{N-1} r_{1_i^p}(c,t) = \sum_{k\in 1_i^c}\sum_{c\in\mathcal{C}}\sum_{t=0}^{N-1} r_k(c,t) \qquad \forall i\in\mathcal{R}\end{aligned}\tag{2}$$

$$\max_{c\in\mathcal{C}} \mathrm{I}_{1_i^p}(c,t) + \max_{k\in 1_i^c, c\in\mathcal{C}} \mathrm{I}_k(c,t) \le 1\ \ \forall i\in\mathcal{R},\ 0\le t\le N \tag{3}$$

$$\sum_{k\in 1_i^c} \mathrm{I}_k(c,t) + \sum_{j\in\{\mathcal{R}^+\setminus i\}}\ \sum_{m\in 1_j^c} e_{(i,j)}\mathrm{I}_m(c,t) \le 1 \quad \forall i\in\mathcal{R}^+,\ \forall c\in\mathcal{C},\ 0\le t\le N \tag{4}$$

$$r_j(c,t) \le \mathrm{I}_j(c,t)\mathcal{R}_j(c) \quad \forall j\in\mathcal{L},\ \forall c\in\mathcal{C},\ 0\le t\le N \tag{5}$$

$$\max_{k\in 1_i^c, 0\le t\le N} \mathrm{I}_k(c,t) \le v_{(i,c)} \quad \forall i\in\mathcal{R}^+,\ \forall c\in\mathcal{C} \tag{6}$$

The problem is integer programming problem which is NP-hard and computationally infeasible to solve. By relaxing the integrality constraint the computationally lighter upper bound of original problem is defined as: let $w_l(c) = \frac{1}{N}\sum_{t=0}^{N-1} \mathrm{I}_l(c,t)$

Maximize: $$\mathrm{O}(\lambda) = \sum_{m \in \mathcal{M}} \frac{\lambda_m^*}{\rho_m} \tag{1}$$

Subject to: $$\lambda_m^* = \mathrm{N} \sum_{c \in \mathcal{C}} w_{1_m^p}(c) R_{1_m^p}(c) \tag{2}$$

$$\sum_{c \in \mathcal{C}} w_{1_i^p}(c) R_{1_i^p}(c) \geq \sum_{k \in 1_i^c} \sum_{c \in \mathcal{C}} w_k(c) R_k(c) \qquad \forall i \in \mathcal{R} \tag{3}$$

$$\sum_{k \in 1_i^c} w_k(c) + \sum_{j \in \left\{\mathcal{R}^+ \backslash i\right\}} e_{(i,j)} \sum_{m \in 1_j^c} w_m(c) \leq \frac{2}{R+1} \forall i \in \mathcal{R}^+, \forall c \in \mathcal{C} \tag{4}$$

$$\max_{k \in 1_i^c} w_k(c) \leq v_{(i,c)} \qquad \forall i \in \mathcal{R}^+, \forall c \in \mathcal{C} \tag{5}$$

Algorithm 1 Greedy Algorithm for PFSCRN-FTP

```
Let $T$ be the set of CR RSs that have been scheduled.
Let $U$ be the set of CR RSs that interfere with each other.
Let $M$ be the candidate CR MSs to be scheduled.
$M \leftarrow \phi$
for all $c \in \mathcal{C}$ do
  $T \leftarrow \phi$
  for all $r \in \mathcal{R}$ such that $\mathrm{v}_{(r,c)} = 1$ and $r \notin T$ do
    $U \leftarrow \phi$
    for all $i \in \mathcal{R}$ do
      if $\mathrm{e}_{(r,i)} = 1$ and $\mathrm{v}_{(i,c)} = 1$ then
        $T \leftarrow i, \quad U \leftarrow i$
      end if
    end for
    for all $u \in U$ do
      $m_{(u,c)} = \arg \max_{m \in u's\ children} \frac{\mathrm{R}_{1_m^p}(c)}{\rho_m}$
      $M \leftarrow M \cup \{m_{(u,c)}\}$
    end for
  end for
end for
Sort elements in $M$ in descending order of contribution
to the objective function
Denote by $n_{(l,c)}$ the available time-slots on link $l$ over
sub-channel $c$
for all $m_{(u,c)} \in M$ do
  $D_m = \mathrm{R}_{m_{(u,c)}}(c) n_{(\mathrm{l}^p_{m_{(u,c)}},c)}$
  $SchedData \leftarrow 0$
  while $SchedData < D_m$ and CR BS has available
  sub-channels do
    $c' \leftarrow \arg \max_{c \in \mathcal{C}\ and\ \mathrm{v}_{(BS,c)}=1} \mathrm{R}_{\mathrm{l}_u^p}(c)$
    Allocate available time-slots of sub-channel $c'$ on link
    $\mathrm{l}_u^p$ to CR RS $u$
    $SchedData \leftarrow SchedData + \mathrm{R}_{\mathrm{l}_u^p}(c')$
  end while
  Allocate time-slots of sub-channel $c$ on link $\mathrm{l}^p_{m_{(u,c)}}$ to
  $m_{(u,c)}$
end for
```

The algorithm has two parts:

- Find and schedule available sub-channels for each CR RS.
- Allocate sub-channels and time-slots of all hops in a greedy-based approach.

Resolve conflicts among interfering CR RSs and schedule their available sub-channels: The algorithm scans through all the sub-channels. For each sub-channel c, it scans through each CR RS r, and record all CR RSs that interfere with r in the set U. All of the CR RSs in the set U have access to sub-channel c, but are interfered with each other. The CR RS m with maximum value of $\frac{R_{1_m^p}(c)}{\rho_m}$ in U can access the sub-channel c at a given time.

Allocate the resource between the CR BS and the CR RSs: the CR MSs are sorted in descending order of their contributions to the objective function. For each m in the sorted order and the CR RS u that m attaches to, the amount of service that m receives is initialized to the number of bits that can be transmitted using all the available time-slots between m and u over the scheduled sub-channel c (line 23). The sub-channels between the CR BS and u are allocated in the order of transmission rate. That is, the sub-channel c' with highest rate $R_{1_u^p}(c')$ is allocated first, and then the second highest sub-channel is allocated, and so on.

IV. VARIABLE TRANSMIT POWER

Better spectrum utilization and system throughput can be achieved with proper control of CR RS transmit power. A scheduler decides power level of a CR RS from pool of discrete power levels. The decision normally involves the interference indicator between two CR RSs, e(i, j), and the maximum sustainable rate Rl (c) on an CR RS-CR MS link l over a sub-channel c become functions of the transmit power p.

A. Problem Formulation: Let p_i denotes the transmit power level of CR RS *i*. (Given parameters are same as in fixed power transmission).

$$\text{Objective: } \mathrm{O}(\lambda) = \max \sum_{m \in \mathcal{M}} \frac{\lambda_m(t)}{\rho_m(t)} \text{ where } \lambda_m = \sum_{c \in \mathcal{C}} \sum_{t=0}^{N-1} r_{1_m^p}(c,t) \qquad (1)$$

$$\text{Subject to: } p_i \in \mathcal{P}, \forall i \in \mathcal{R} \qquad (2)$$

$$\begin{aligned} &\sum_{c \in \mathcal{C}} \sum_{t=0}^{\tau} r_{1_i^p}(c,t) \geq \sum_{k \in 1_i^c} \sum_{c \in \mathcal{C}} \sum_{t=0}^{\tau} r_k(c,t) \ \forall i \in \mathcal{R}, 0 \leq \tau \leq N-1 \\ &\sum_{c \in \mathcal{C}} \sum_{t=0}^{N-1} r_{1_i^p}(c,t) = \sum_{k \in 1_i^c} \sum_{c \in \mathcal{C}} \sum_{t=0}^{N-1} r_k(c,t) \qquad \forall i \in \mathcal{R} \end{aligned} \qquad (3)$$

$$\max_{c\in\mathcal{C}} \mathrm{I}_{1_i^p}(c,t) + \max_{k\in 1_i^c, c\in\mathcal{C}} \mathrm{I}_k(c,t) \le 1 \quad \forall i\in\mathcal{R},\ 0\le t\le N \tag{4}$$

$$\sum_{k\in 1_i^c} \mathrm{I}_k(c,t) + \sum_{j\in\left\{\mathcal{R}^+\backslash i\right\}} \sum_{m\in 1_j^c} e_{(i,j)}(p_i)\mathrm{I}_m(c,t) \le 1 \qquad \forall i\in\mathcal{R}^+,\ \forall c\in\mathcal{C},\ 0\le t\le N \tag{5}$$

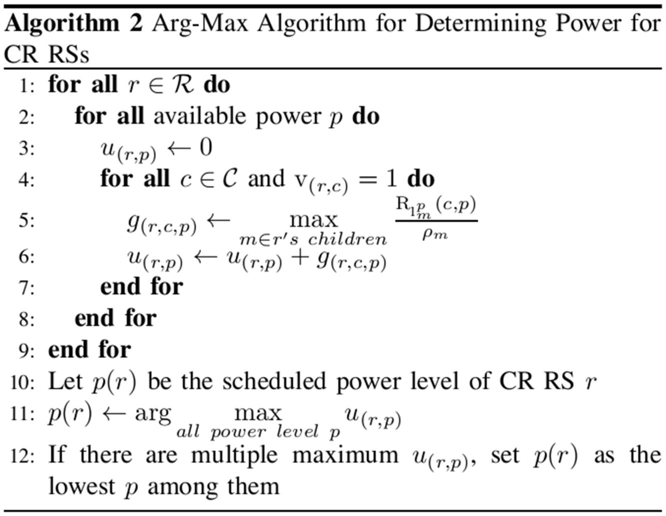
$$r_j(c,t) \le \mathrm{I}_j(c,t)\mathcal{R}_j(c,p_i) \qquad \forall i\in\mathcal{R}^+,\ \forall j\in 1_i^c,\ \forall c\in\mathcal{C},\ 0\le t\le N \tag{6}$$

$$\max_{k\in 1_i^c, 0\le t\le N} \mathrm{I}_k(c,t) \le v_{(i,c)} \quad \forall i\in\mathcal{R}^+,\ \forall c\in\mathcal{C} \tag{7}$$

The problem formulation same as fixed power transmission case except that interference indicator e(i,j)(p) between two CR RSs (i, j) and the maximum sustainable rate R_l(c, p) on an CR RS and CR MS link *l* over sub-channel c are functions of the CR RS transmit power as shown in constraint 5 and 6 respectively.

B. *Proposed Greedy Algorithm for PFSCRN-ATP:* an algorithm for power level selection, shown in Algorithm 2 below, is based upon following criteria:

- Interference to other CR RSs is minimized.
- Spectrum should be fully utilized



Algorithm 2 Arg-Max Algorithm for Determining Power for CR RSs

```
for all $r \in \mathcal{R}$ do
    for all available power $p$ do
        $u_{(r,p)} \leftarrow 0$
        for all $c \in \mathcal{C}$ and $\mathrm{v}_{(r,c)} = 1$ do
            $g_{(r,c,p)} \leftarrow \max_{m \in r's\ children} \frac{\mathrm{R}_{1_m^p}(c,p)}{\rho_m}$
            $u_{(r,p)} \leftarrow u_{(r,p)} + g_{(r,c,p)}$
        end for
    end for
end for
Let $p(r)$ be the scheduled power level of CR RS $r$
$p(r) \leftarrow \arg\max_{all\ power\ level\ p} u_{(r,p)}$
If there are multiple maximum $u_{(r,p)}$, set $p(r)$ as the lowest $p$ among them
```

A score is calculated for each combination of (CR RS, power level)(r, p). It is calculated based upon contribution, i.e., $\frac{R_{1_m^p}(c,p)}{\rho_m}$, of m^{th} CR RM node to (1) on each available sub-channel *c* of *r*. The contribution of CR MSs is summed up. The power level for CR RS is determined then as: Priority is given to the (r, p) combination with highest score while tie is broken by choosing the combination that can achieve the objective function with lowest power level.

After the transmit power of CR RSs are determined by using Algorithm 2, the scheduling problem is solved by using a PFSCRN-ATP as shown in Algorithm 3. It is modified version of Algorithm 1 with selected power levels.

Algorithm 3 Greedy Algorithm for PFSCRN-ATP

```
Let $p(r)$ be the scheduled power determined by Algorithm 2
Let $T$ be the set of CR RSs that have been scheduled.
Let $U$ be the candidate CR RSs to be scheduled.
Let $M$ be the candidate CR MSs to be scheduled.
$M \leftarrow \phi$
for all $c \in \mathcal{C}$ do
    $T \leftarrow \phi$
    for all $r \in \mathcal{R}$ such that $\mathrm{v}_{(r,c)} = 1$ and $r \notin T$ do
        $U \leftarrow \phi$
        for all $i \in \mathcal{R}$ do
            if $\mathrm{e}_{(r,i)}(p(r)) = 1$ and $\mathrm{v}_{(i,c)} = 1$ then
                $T \leftarrow i, \quad U \leftarrow i$
            end if
        end for
        for all $u \in U$ do
            $m_{(u,c)} = \arg \max_{m \in u's\ children} \frac{\mathrm{R}_{\mathrm{l}_m^p}(c, p(u))}{\rho_m}$
            $M \leftarrow M \cup \{m_{(u,c)}\}$
        end for
    end for
end for
Denote by $n_{(l,c)}$ the available time-slots on link $l$ over sub-channel $c$
for all $m_{(u,c)} \in M$ do
    $D_m = \mathrm{R}_{m_{(u,c)}}(c, p(u)) n_{(\mathrm{l}^p_{m_{(u,c)}}, c)}$
    $SchedData \leftarrow 0$
    while $SchedData < D_m$ and CR BS has available sub-channels do
        $c' \leftarrow \arg \max_{c \in \mathcal{C}\ and\ \mathrm{v}_{(BS,c)}=1} \mathrm{R}_{\mathrm{l}_u^p}(c)$
        Allocate available time-slots of sub-channel $c'$ on link $\mathrm{l}_u^p$ to CR RS $u$
        $SchedData \leftarrow SchedData + \mathrm{R}_{\mathrm{l}_u^p c'}()$
    end while
    Allocate time-slots of sub-channel $c$ on link $\mathrm{l}^p_{m_{(u,c)}}$ to $m_{(u,c)}$
end for
```

V. PERFORMANCE EVALUATION

The simulation environment of an IEEE 802.16j relay network has been constructed to evaluate the proportional fairness of proposed algorithms, and to compare the system throughput of each algorithm.

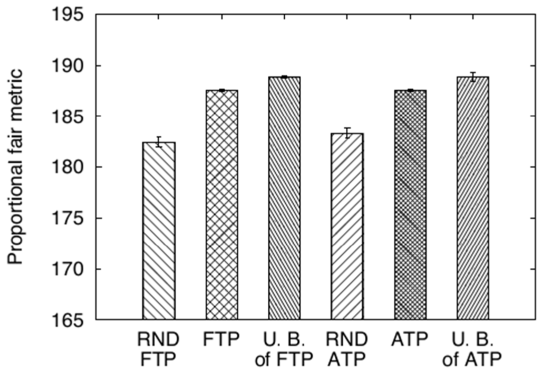
A. *Simulation Setup:* two scenarios for evaluation of performance of proposed algorithms.

Total CR BS =1 (located in center of the cell), Total CR RS = 4, Total CR MS = 20 or 40 in low system load or high system load case, Total 40 random topologies are generated,

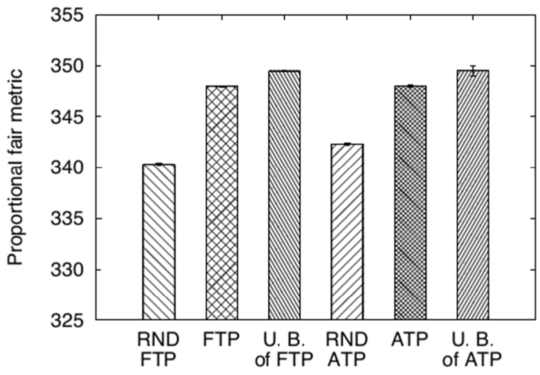
Total Frames = 2000, No of sub-channels = 64

Scenario 1 {purpose} Signal enhancement	**Scenario 2** {purpose} Range extension
CR RSs within 1200m of CR BS	CR RSs within 1500m of CR BS
CR MSs uniformly distributed in cell, with in 1800m of CR BS	CR MSs uniformly distributed with in 300m of randomly selected CR RS

B. *Proportional Fairness:*

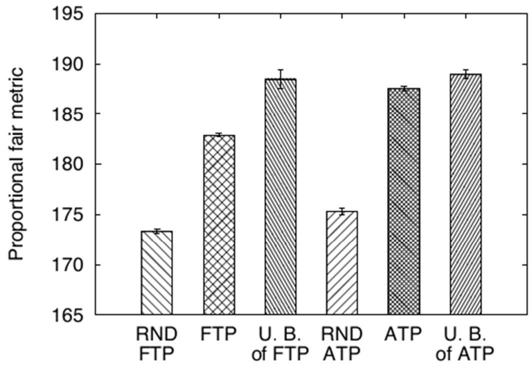


(a) Low system load

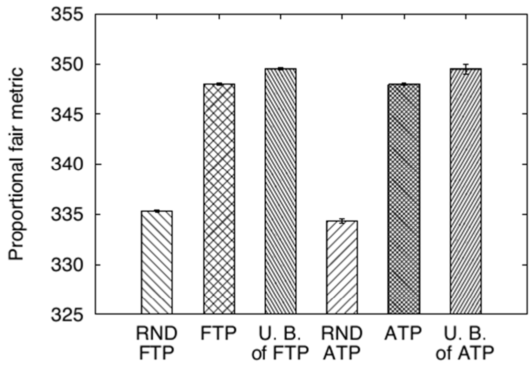


(b) High system load

Fig. 2. Proportional fair metrics of scenario 1 with 95% of confidenc interval. (RND: Random Scheduling; FTP: Fixed Transmit Power; U.B. Upper Bound; ATP: Adjustable Transmit Power.)



(a) Low system load



(b) High system load

Fig. 3. Proportional fair metrics of scenario 2 with 95% of confiden interval.(RND: Random Scheduling; FTP: Fixed Transmit Power; U.B.: Up Bound; ATP: Adjustable Transmit Power.)

The upper bounds of FTP case in both figures are derived as that presented in III-A. The upper bounds of ATP in both figures are derived by using the power levels determined by Algorithm 2. The random scheduling is lower than proposed algorithms because in scenario 1, the CR RSs are placed closer to the CR BS. The channel condition of each CR MS would be less variant. On the contrary, in scenario 2, the CR RSs are placed far apart from each other as well as from the CR BS, and the channel condition of each CR MS varies a lot. However proposed algorithms are insensitive to the changes of scenario and system load.

C. *System Throughput:* Overall system throughput is the sum of the services that each CR MS receives during the whole simulation period.

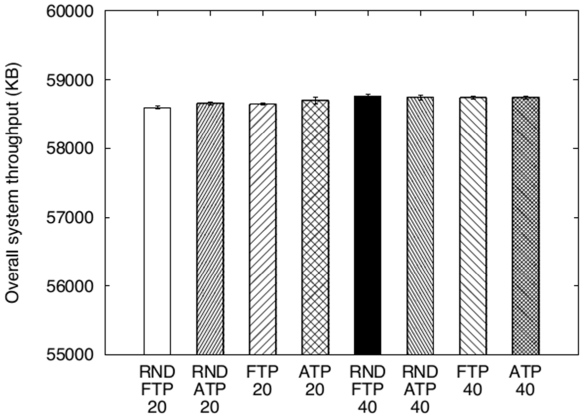


Fig. 4. Overall system throughput of scenario 1 with 95% of confidenc interval. (RND: Random Scheduling; FTP: Fixed Transmit Power; AT Adjustable Transmit Power. 20 stands for 20 CR MSs. 40 stands for 40 C MSs.)

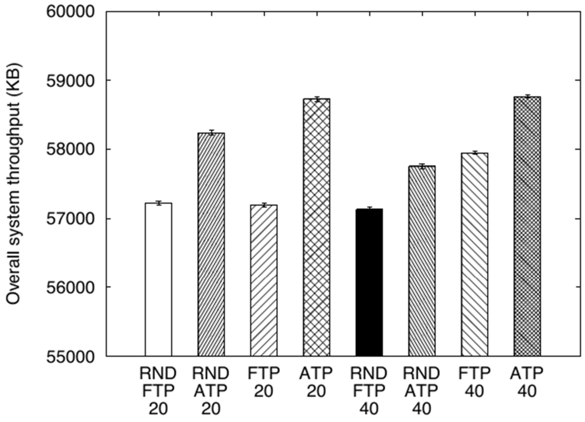


Fig. 5. Overall system throughput of scenario 2 with 95% of confidence interval. (RND: Random Scheduling; FTP: Fixed Transmit Power; ATP: Adjustable Transmit Power. 20 stands for 20 CR MSs. 40 stands for 40 CR MSs.)

In scenario 1, the overall system throughput of 20 and 40 CR-MSs does not differ much. The overall system throughput of FTP and ATP does not differ much, either. The reason is that CR RSs are placed closer to the CR BS. As a result, CR RSs will easily interfere with each other even with the lowest transmit power level. In such circumstance where cell coverage is small and CR MSs are evenly distributed, transmit power control will not help much. Also, in scenario 1, the channel condition of each CR MS does not differ much. Hence, the throughput does not differ much in all settings.

In scenario 2 the CR RSs are placed farther from the CR BS. Also, CR MSs tend to form hot-spot near the CR RSs. In systems where CR RSs are apart far enough and CR MSs are clustered in a certain extent, the interference among CR RSs will be highly controllable through adjustment of transmit power levels. Thus, as shown in Fig. 5, the performance of ATP is better than that of FTP.

D. *Algorithm Running Time:* The implemented algorithms are executed with 4 CR RSs, 40 CR MSs, 64 sub-channels, 48 time-slots per frame, and 2000 frames. The proposed FTP and ATP requires 0.38 ms and 0.75ms, respectively. however random FTP and random ATP require 0.14 ms and0.23ms, respectively. Although they are slightly less than proposed algorithms, yet proposed algorithms outperform random scheduling algorithms as shown in Figs. 2–5.