

# Digital holography for quantitative phase-contrast imaging

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## Short summary:

In the paper, they have presented a new application of digital holography for phase-contrast imaging. The technique uses a CCD camera for recording of a digital Fresnel off-axis hologram and a numerical method for hologram reconstruction. The method simultaneously provides an amplitude-contrast image and a quantitative phase-contrast image.

## I. INTRODUCTION

The holographic process is described mathematically as follows:

$$O(x, y) = o(x, y) e^{i\varphi_o(x, y)} \quad \dots (1.1)$$

Is the complex amplitude of the object wave with real amplitude  $o$  and phase  $\varphi_o$  and

$$R(x, y) = r(x, y) e^{i\varphi_R(x, y)} \quad \dots (1.2)$$

Is the complex amplitude of the reference wave with real amplitude  $r$  and phase  $\varphi_R$

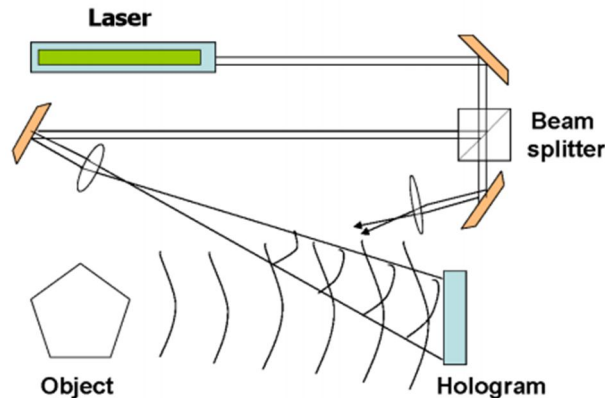


Fig. 1 Construction of a hologram

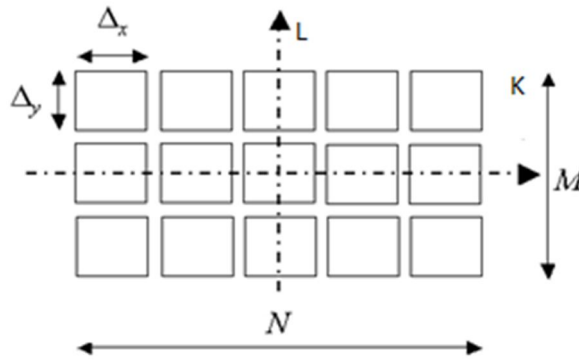
Both waves interfere at the surface of the recording medium. The intensity is calculated by

$$I_H(x, y) = |O(x, y) + R(x, y)|^2 \\ = (O(x, y) + R(x, y))(O(x, y) + R(x, y))^*$$

$$\begin{aligned}
&= R(x, y)R^*(x, y) + O(x, y)O^*(x, y) \\
&+ O(x, y)R^*(x, y) + R(x, y)O^*(x, y) \quad \dots (1.3)
\end{aligned}$$

$$I_H(x, y) = |R|^2 + |O|^2 + R^*O + RO^* \quad \dots (1.4)$$

## Digital Recording



**Fig. 2 Concept diagram of a surface-pixel sensor**

The digital recording will (depending on the directions  $x$  and  $y$  on the recording plane) consist of  $M \times N$  pixels. Each of these pixels is of a dimension  $\Delta_x \times \Delta_y$ . In CCD sensors, Matrices made up of photosensitive elements called pixels are generally square-shaped. In our case it is  $12 \times 12 \mu m$

In their case, the hologram intensity was recorded by a standard black and white CCD camera (Hitachi Denshi KP-M2).

The two neutral-density filters allow the adjustment of the object and the reference intensities.

A square image of area  $L \times L$  (Sensor size) = 4.83mm X 4.83mm containing  $N \times N = 512 \times 512$  pixels is acquired in the center of the CCD sensor, and a digital hologram is transmitted to a computer via a frame grabber.

The digital hologram  $I_H(k, l)$  results from two-dimensional spatial sampling of  $I_H(x, y)$  by the CCD:

$$I_H(k, l) = I_H(x, y) \text{rect}\left(\frac{x}{L}, \frac{y}{L}\right) \times \sum_k^N \sum_l^N \delta(x - k\Delta_x, y - l\Delta_y) \quad \dots (1.5)$$

Where  $k$  and  $l$  are integers ( $-N/2 \leq k, l \leq N/2$ ) and  $\Delta_x$  and  $\Delta_y$  are the sampling intervals in the hologram plane i.e. pixel size:  $\Delta_x = \Delta_y = L/N$

## II. NUMERICAL RESULTS

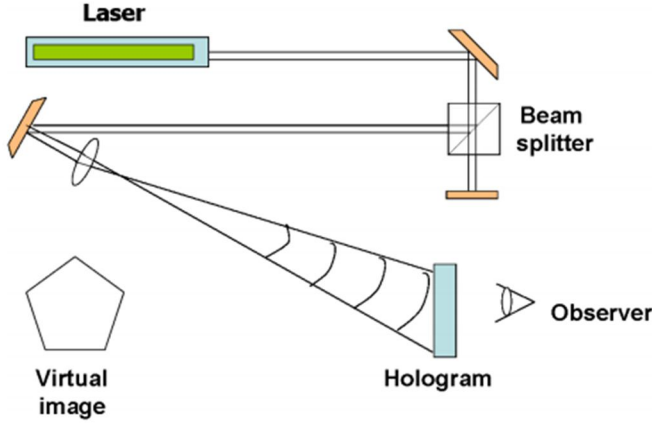


Fig.3 Reconstruction of a hologram

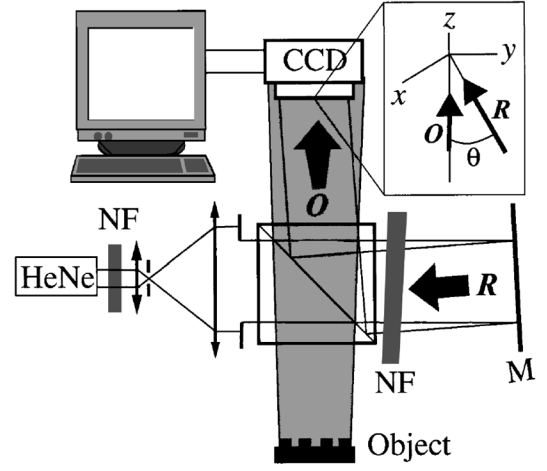


Fig.4 Experimental setup

A wave front  $\psi_H(x, y) = R(x, y)I(x, y)$  is transmitted by a hologram and propagates toward an observation plane, where a three-dimensional image of the object can be observed.

For reconstructing a digital hologram, a digital transmitted wave front  $\psi_H(k\Delta x, l\Delta y)$  is computed by multiplication of digital hologram  $I_H(k, l)$  by a digital computed reference wave,  $R_D(k, l)$ , called the digital reference wave.

If we assume that mirror M reflects a plane wave of wavelength  $\lambda$  then  $R_D$  can be calculated as follows:

$$R_D(k, l) = \underbrace{A_R}_1 \exp \left[ i \frac{2\pi}{\lambda} (k_x k \Delta x + k_y l \Delta y) \right] \quad \dots (1.6)$$

Where  $k_x$  and  $k_y$  are the two components of the wave vector and  $A_r$  is the amplitude.

$$k_x = -3.12 \times 10^{-3} \text{ and } k_y = -5.34 \times 10^{-3}$$

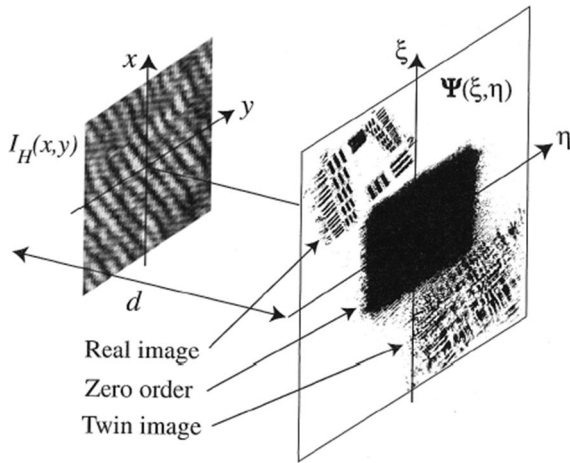
Taking into account the definition of hologram intensity [Eq. 1.4], we have

$$\psi(k\Delta x, \Delta y) = R_D(k, l)I_H(k, l) \quad \dots (1.7)$$

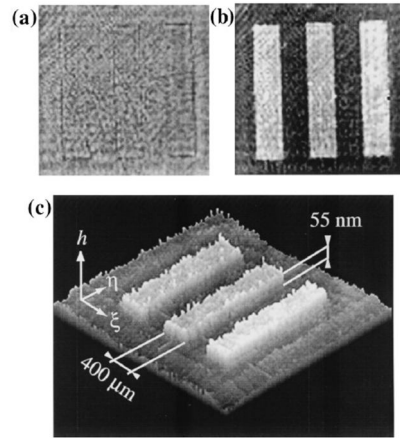
$$= \underbrace{R_D |R|^2 + R_D |O|^2}_{\text{Zero order of diffraction}} + \underbrace{R_D R^* O}_{\text{Twin image}} + \underbrace{R_D R O^*}_{\text{real image}}$$

To avoid an overlap of these three components of  $\psi$  during reconstruction, they recorded the hologram in the so-called off-axis geometry. For this purpose the mirror in the reference arm, M is oriented such that the reference wave **R** reaches the CCD at an incidence angle  $\theta$ . The value  $\theta$  must be sufficiently large to ensure separation between the real and the twin images in the observation planes. However,  $\theta$  must not exceed a given value so that the spatial frequency of the interferogram does not exceed the resolving power of the CCD.

$$\theta \leq \theta_{\max} = \text{arc sin} \left( \frac{\lambda}{2\Delta x} \right) \quad \dots (1.8)$$



Geometry for hologram reconstruction.  $oxy$ , hologram plane;  $O\xi\eta$ , observation plane;  $d$ , reconstruction distance;  $\Psi(\xi, \eta)$ , reconstructed wave front.



Reconstructed images obtained with a pure phase object: (a) amplitude contrast, (b) phase contrast, (c) three-dimensional perspective of the reconstructed height distribution (the vertical scale is not equal to the transverse scale).

The reconstructed wave front  $\psi(m\Delta\xi, n\Delta\eta)$ , at a distance  $d$  from the hologram plane, is computed by use of a discrete expression of the Fresnel integral:

$$\begin{aligned} \psi(m\Delta\xi, n\Delta\eta) = A \exp \left[ \frac{i\pi}{\lambda d} (m^2 \Delta\xi^2 + n^2 \Delta\eta^2) \right] \\ \times \text{FFT} \left\{ R_D(k, l) I_H(k, l) \exp \left[ \frac{i\pi}{\lambda d} (k^2 \Delta x^2 + l^2 \Delta y^2) \right] \right\}_{m,n} \quad \dots (1.9) \end{aligned}$$

Where  $m$  and  $n$  are integers ( $-N/2 \leq m, n \leq N/2$ ), FFT is the fast Fourier transform operator, and  $A = \exp(i2\pi d / \lambda) / (i\lambda d)$

$\Delta\xi$  and  $\Delta\eta$  are the sampling intervals in the observation plane and define the transverse resolution of the reconstructed image.

This transverse resolution is related to the size of the CCD ( $L$ ) and to the distance  $d$  by,

$$\Delta\xi = \Delta\eta = \lambda d / L \quad \dots (1.10)$$

The reconstructed wave front is an array of complex numbers. The amplitude and the phase contrast images can be obtained by calculation of the square modulus.

#### References

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- [3] E. Cuhe, P. Poscio, and C. Depeursinge, Proc. SPIE 2927, 61 (1996)