Title: Real-valued channel coding

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I. RS CODES

The Reed-Solumon (RS) codes assure maxized minimum distance because of their parity check matrix with Vandermande matrix form. However, the decoder of RS codes has very high complexity in finding error locations.

We note that their t-error correcting generator polynomial is defined by

$$g(x) = \left(x - \alpha^{b}\right)\left(x - \alpha^{b+1}\right)\cdots\left(x - \alpha^{b+2t-1}\right), \tag{1}$$

where α is a primitive element of GF($n = q^{m} - 1$) and b is a nonnegative integer.

II. ALTERNATIVE SOLUTION

The main idea is derived from extracting coefficients of polynomials. The encoding is similar with RS codes to keep the Vandermande matrix form.

- **Encoding**

The generator polynomial

$$G(x) = \prod_{m=0}^{r-1}\left(x - e^{-j2\pi\frac{m}{N}}\right) = (x-1)\left(x - e^{-j2\pi\frac{1}{N}}\right)\left(x - e^{-j2\pi\frac{2}{N}}\right)\cdots\left(x - e^{-j2\pi\frac{r-1}{N}}\right), \tag{2}$$

where r is the number of redundances and N is the code length. For a message vector $M = \left(m_0, m_1, ..., m_{k-1}\right)$ with a polynomial form $M(x) = m_0 + m_1 x + \cdots + m_{k-1} x^{n-k-1} = \sum_{l=0}^{k-1} m_l x^l$, the codewords are defined by

$$C(x) = M(x)G(x) = \prod_{m=0}^{r-1}\left(x - e^{-j2\pi\frac{m}{N}}\right)\sum_{l=0}^{k-1} m_l x^l \ , \tag{3}$$

where $m_i \in \{0,1\}$.

- **Decoding (Finding a syndrome vector)**

When we transmit a codeword $C=\left(c_0,c_1,...,c_{N-1}\right)$ over an channel with an error vector $E=\left(e_0,e_1,\cdots,e_{N-1}\right)$, the received message vector $R=C+E$ and p-th syndrome S_p for $p=0,1,...,r-1$ can be found by FFT as follow,

$$S_p=\mathcal{F}_{p,N}\left\{C\right\}=\sum_{l=0}^{N-1}R_l e^{j2\pi\frac{l}{N}p}\ , \tag{4}$$

where R_l is l-th message of R. It implies that the parity check matrix H is

$$H=\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & e^{-j2\pi\frac{1}{N}} & e^{-j2\pi\frac{2}{N}} & \cdots & e^{-j2\pi\frac{N-2}{N}} & e^{-j2\pi\frac{N-1}{N}} \\ 1 & e^{-j2\pi\frac{1}{N}2} & e^{-j2\pi\frac{2}{N}2} & \cdots & e^{-j2\pi\frac{N-2}{N}2} & e^{-j2\pi\frac{N-1}{N}2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & e^{-j2\pi\frac{1}{N}r-2} & e^{-j2\pi\frac{2}{N}r-2} & \cdots & e^{-j2\pi\frac{N-2}{N}r-2} & e^{-j2\pi\frac{N-1}{N}r-2} \\ 1 & e^{-j2\pi\frac{1}{N}r-1} & e^{-j2\pi\frac{2}{N}r-1} & \cdots & e^{-j2\pi\frac{N-2}{N}r-1} & e^{-j2\pi\frac{N-1}{N}r-1} \end{bmatrix}.$$

Namely, $S=HC$. We note that the parity check matrix H has Vandermande matrix form with maxized minimum distance.

- **Decoding (Finding an error vector)**

When binary errors are occurred in received messages, the all received messages can be recovered for a specfic code rate and block length.

Let us define a vector Z, which is a zero-padding vector of the syndrome vector S,

$$Z_p=\begin{cases} S_p & ,\text{if } \ p<r \\ 0 & ,\text{otherwise} \end{cases}$$

where $p=0,1...,N-1$. Then, we can find a binary error vector E, $E_n\in\left\{0,1\right\}$ for $n=0,1...,N-1$, by IFFT,

$$E_n=\begin{cases} 1, & Z_n\geq\varepsilon \\ 0, & \text{o.w} \end{cases}$$

where $\varepsilon=0.5$ and $E_n=\mathcal{F}_{n,N}^{-1}\left\{Z\right\}=\dfrac{1}{N}\sum_{p=0}^{N-1}Z_p e^{j2\pi\frac{n}{N}p}$.

- **Strength and weekness**

  - Strength

→All binary error vectors can be found. (For $n = 50, r = 25$, all codewords with errors can be exactly recoved)

→ Low complexity

  - Weekness

→This sheme can be applied to only binary input and output and binary errors.

- **Future works**

→ Connection with OFDM system

→Decoder using AMP algorithm