Sparsity driven ultrasound imaging

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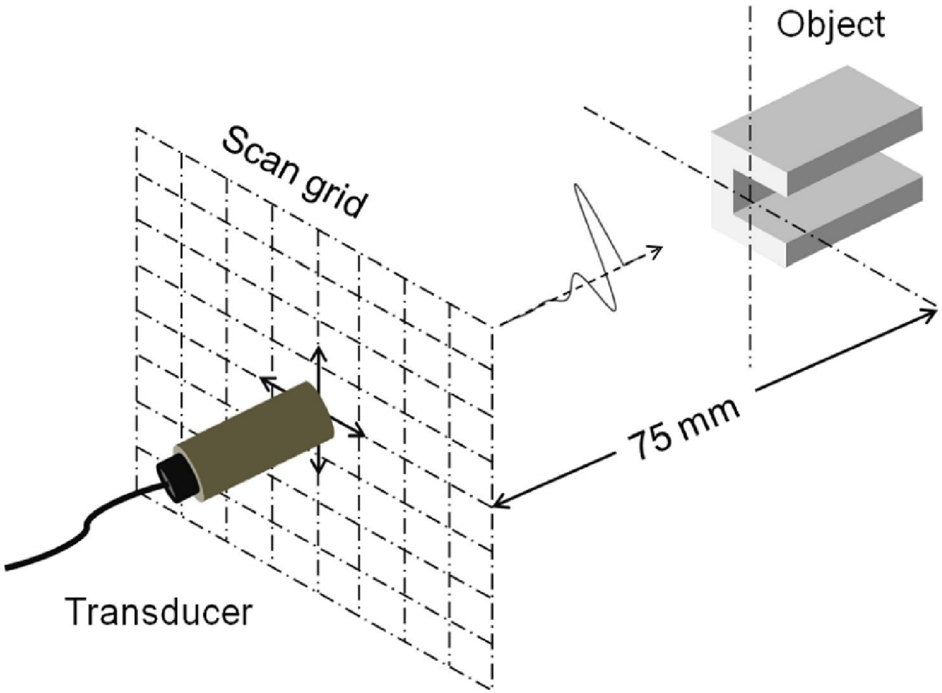
Overview of Scenarios

- A broadband single-element unfocused transducer performs a raster scan in a plane parallel to the cross section of the object.
- At each scan position, the transducer sends an acoustic pulse and then detects the echo.
- For all experiments, the initial distance between the object and transducer was set to be 75 mm.



Introduction

- A new model-based framework for ultrasound imaging that estimates a complex-valued reflectivity field is presented.
- The benefits are:
  - Providing improved resolution and reduced diffraction artifacts.
  - Overcoming challenging observation scenarios involving sparse and reduced apertures.
- The framework is based on a regularized reconstruction of the underlying reflectivity field using a wave-based linear model of the ultrasound observation process.
- The physical model is coupled with nonquadratic regularization functions, exploiting prior knowledge that the underlying field should be sparse.
- These nonquadratic functions enable the preservation of strong physical features, i.e., strong scatterers or boundaries.

Observation model for ultrasound scattering

- The free space Green's function is used to model the scattered field in space in response to a point source of excitation,

$$G(|\mathbf{r}' - \mathbf{r}|) = \frac{\exp(jk(|\mathbf{r}' - \mathbf{r}|))}{4\pi|\mathbf{r}' - \mathbf{r}|},$$

- This can linearize the Lippmann–Schwinger equation using Born approximation to obtain the following observation model:

$$y(\mathbf{r}') = c \int G^2(|\mathbf{r}' - \mathbf{r}|) f(\mathbf{r})\, dr$$

  - where **y**(·) denotes the observed data, **f**(·) denotes the unknown complex-valued reflectivity fields
  - Note that squaring the Green's function captures the two-way travel from the transducer to the target and back

Observation model for ultrasound scattering

- The model is discretized and the presence of measurement noise is taken to be additive to obtain the following discrete observation model:

$$\mathbf{y} = \mathbf{T}\mathbf{f} + \mathbf{n}$$

  - where **y** and **n** denote the measured data and the noise, respectively, at all transducer positions; **f** denotes the sampled unknown reflectivity field; and **T** is a matrix representing the discretized version of the observation kernel.

- Given the noisy observation model, the imaging problem is to find an estimate of **f** based on the measured data **y.**

Sparsity-driven ultrasound imaging-Imaging problem formulation

- The conventional ultrasound imaging method of synthetic aperture focusing technique (SAFT) essentially corresponds to using $\mathbf{T}^H$ to reconstruct the underlying field **f**,

$$\hat{\mathbf{f}}_{\text{SAFT}} = \mathbf{T}^H \mathbf{y}$$

- The proposed method produces an image as the solution of the following optimization problem, which will be called sparsity-driven ultrasound imaging (SDUI):

$$\hat{\mathbf{f}}_{\text{SDUI}} = \underset{f}{\operatorname{argmin}}\, J(\mathbf{f})$$

  – where the objective function has the following form:

$$J(\mathbf{f}) = ||\mathbf{y} - \mathbf{T}\mathbf{f}||_2^2 + \lambda_1 ||\mathbf{f}||_p^p + \lambda_2 ||\mathbf{D}|\mathbf{f}|||_p^p$$

  – **D** is a discrete approximation to the derivative operator or gradient, $\boldsymbol{\lambda}_1$, $\boldsymbol{\lambda}_2$ are scalar parameters

Sparsity-driven ultrasound imaging-Solution of the optimization problem (1/2)

- The following smooth approximation is used as

$$||\mathbf{z}||_p^p \approx \sum_{i=1}^{K} \left(\left| (\mathbf{z})_i \right|^2 + \epsilon \right)^{p/2}$$

- Using the approximation, we obtain a modified cost function,

$$J_m(\mathbf{f}) = ||\mathbf{y} - \mathbf{T}\mathbf{f}||_2^2 + \lambda_1 \sum_{i=1}^{N} \left(\left| (\mathbf{f})_i \right|^2 + \epsilon \right)^{p/2} + \lambda_2 \sum_{i=1}^{M} \left(\left| (\mathbf{D}|\mathbf{f}|)_i \right|^2 + \epsilon \right)^{p/2}.$$

- The quasi-Newton method is employed.
- The gradient of the cost function is expressed as

$$\nabla J_m(\mathbf{f}) = \tilde{\mathbf{H}}(\mathbf{f})\mathbf{f} - 2\mathbf{T}^H \mathbf{y}$$

Sparsity-driven ultrasound imaging-Solution of the optimization problem (2/2)

- The Hessian is

$$\tilde{\mathbf{H}}(\mathbf{f}) \overset{\Delta}{=} 2\mathbf{T}^H\mathbf{T} + p\lambda_1\Lambda_1(\mathbf{f}) + p\lambda_2\Phi^H(\mathbf{f})\mathbf{D}^T\Lambda_2(\mathbf{f})\mathbf{D}\Phi(\mathbf{f})$$

- They use $\tilde{\mathbf{H}}(\mathbf{f})$ as an approximation to the Hessian in the following quasi-Newton iteration:

$$\hat{\mathbf{f}}^{(n+1)} = \hat{\mathbf{f}}^{(n)} - \left[\tilde{\mathbf{H}}\left(\hat{\mathbf{f}}^{(n)}\right)\right]^{-1} \nabla J_m\left(\hat{\mathbf{f}}^{(n)}\right)$$

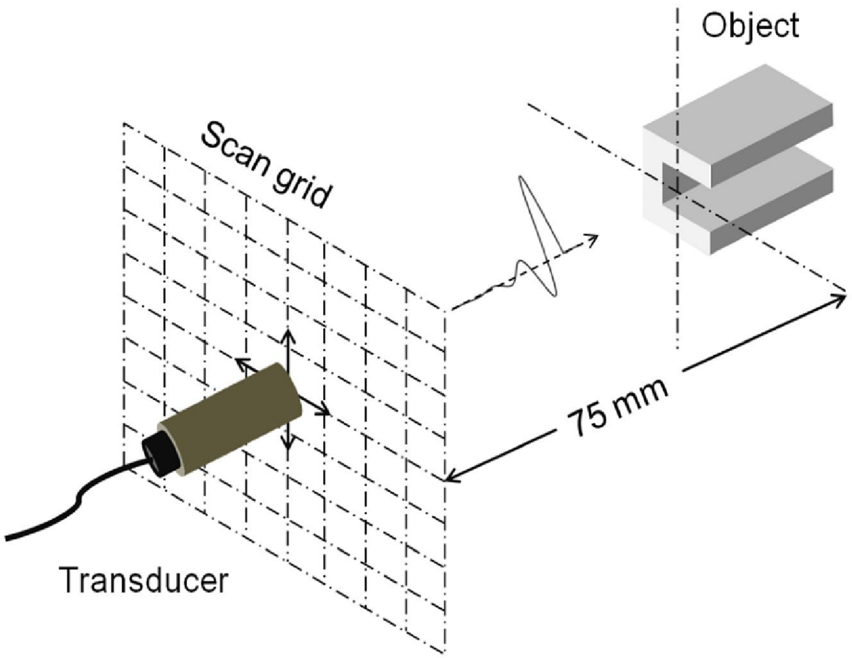
- The following fixed point iterative algorithm can be obtained:

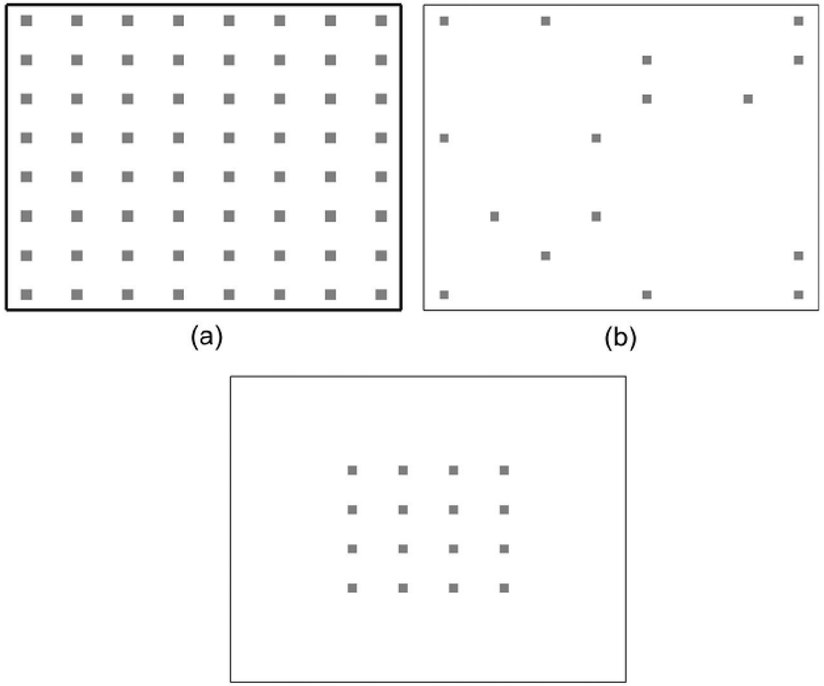
$$\tilde{\mathbf{H}}\left(\hat{\mathbf{f}}^{(n)}\right)\hat{\mathbf{f}}^{(n+1)} = 2\mathbf{T}^H\mathbf{y}$$

- The iteration runs until $\| \hat{\mathbf{f}}^{(n+1)} - \hat{\mathbf{f}}^{(n)} \|_2^2 / \| \hat{\mathbf{f}}^{(n)} \|_2^2 < \delta$

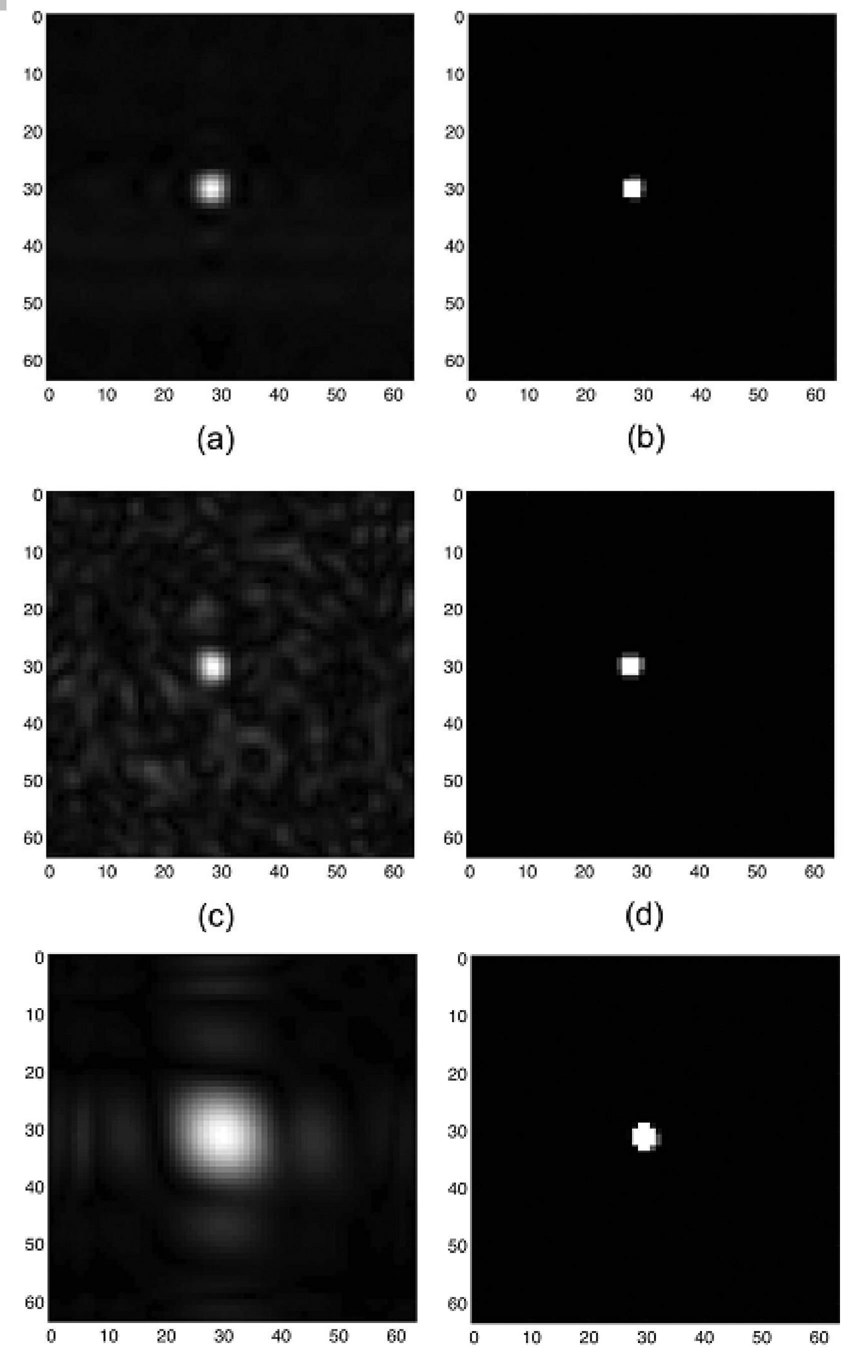
Experiments and Results

- Ultrasound experiments were carried out in a tank of water (2 X 1 X 1 m).
- Data acquisition scenarios are considered: (a) full aperture case, (b) sparse aperture case, and (c) reduced aperture case.
- A full scan forms a 64 X 64 grid with a total of 4096 scan locations.



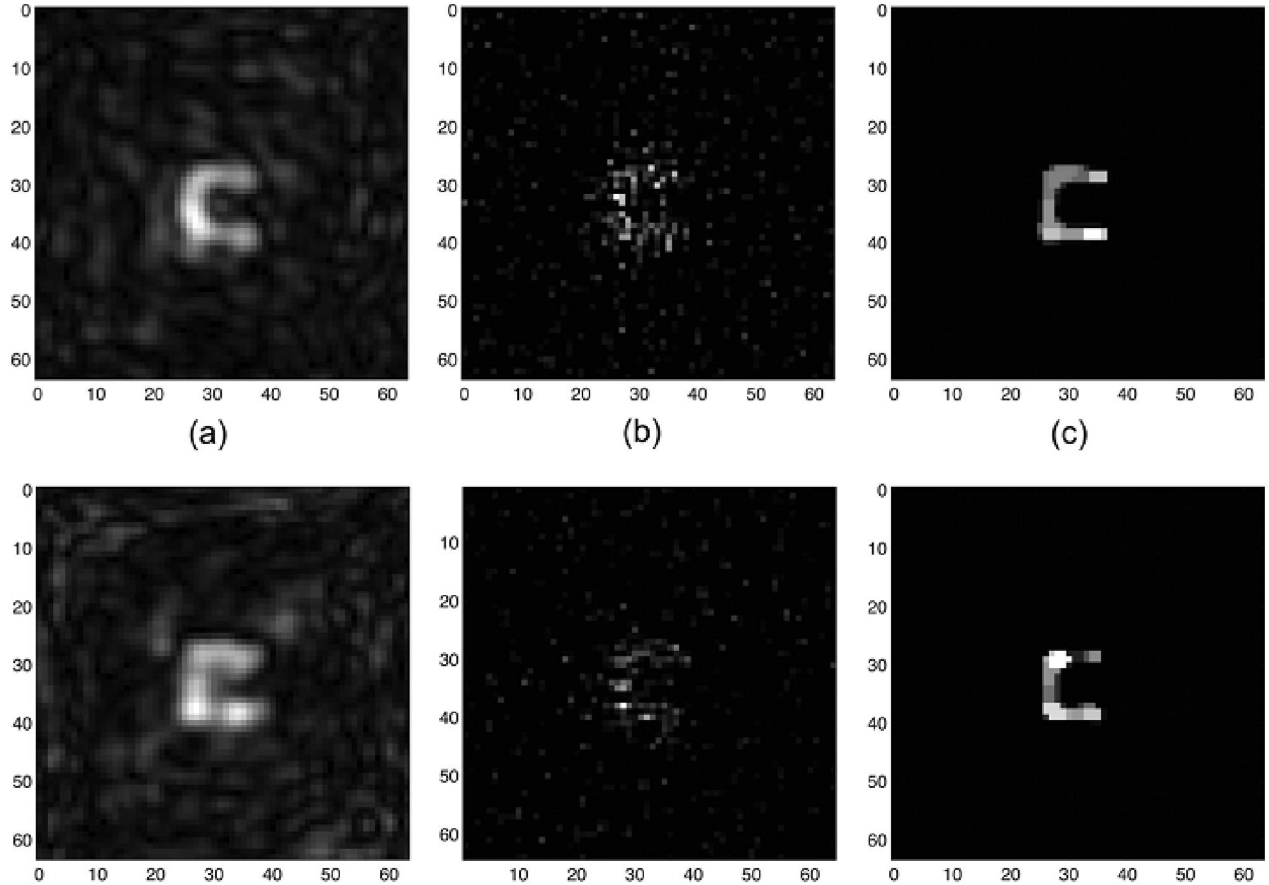


- Images of the 3.2 mm steel rod using full, sparse, and reduced aperture data
  - Reconstructions by SAFT using (a) full data, (c) 6.25% sparse data, and (e) 6.25% reduced data
  - Reconstructions by the SDUI method using (b) full data with λ_1=500, λ_2=100, (d) 6.25% sparse data with λ_1=25, λ_2=5, and (e) 6.25% reduced data λ_1=170, λ_2=5



(a) (b) (c) (d) (e) (f)

- Effect of the gradient-based regularization
  - Images of the channel using sparse aperture data. Reconstructions by SAFT using (a) 14.06% and (d) 6.25% sparse data
  - Reconstructions by the SDUI method with $\lambda_2=0$ using (b) 14.06% sparse data with $\lambda_1=20$, (e) 6.25% sparse data with $\lambda_1=5$
  - Reconstructions by the SDUI method using with (c) $\lambda_1=600$, $\lambda_2=20$ and (f) $\lambda_1=250$, $\lambda_2=10$

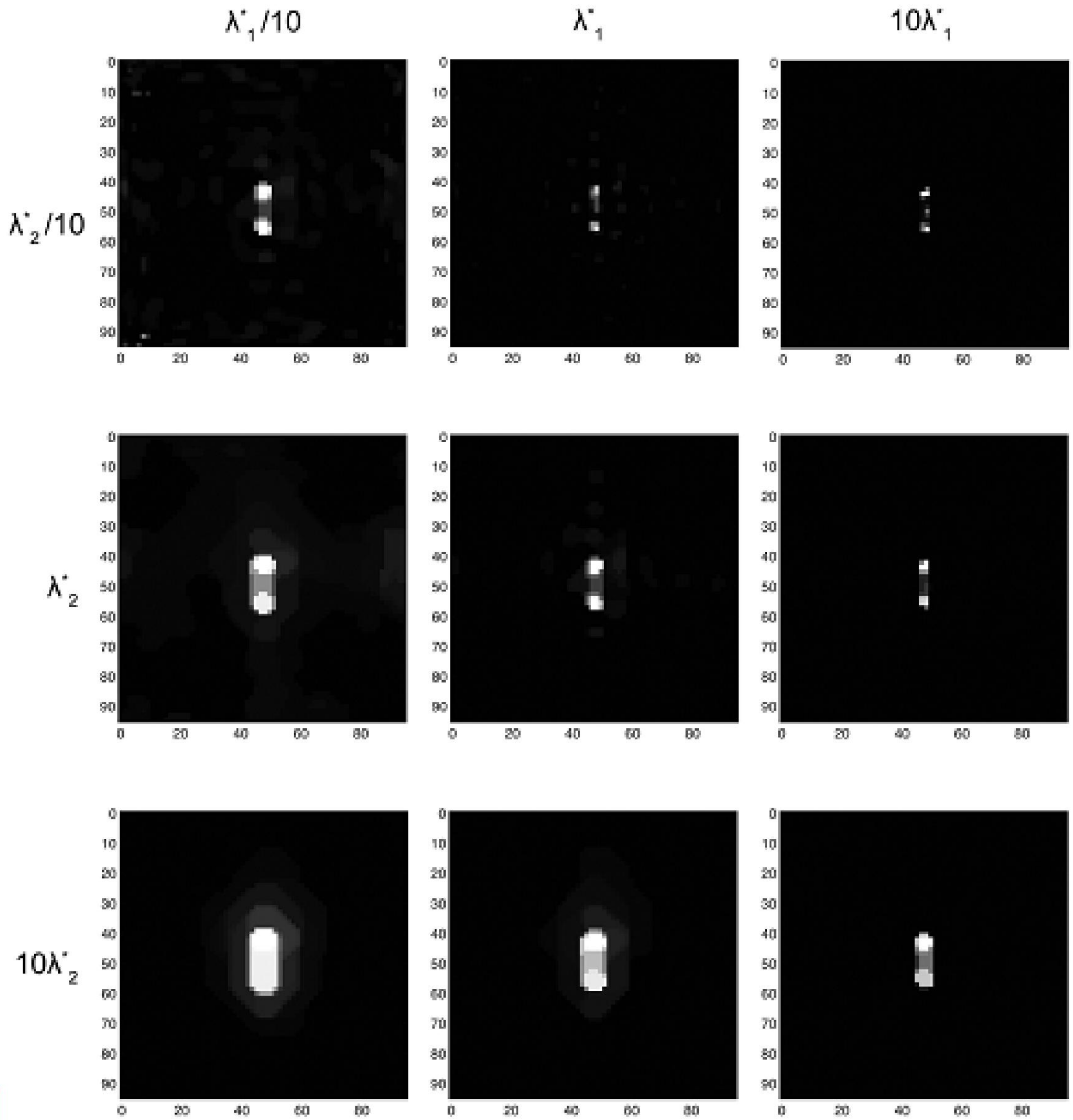


(a) (b) (c)
(d) (e) (f)

Selection of regularization parameters

- Recall that λ_1 scales the term that emphasizes preservation of strong scatterers, whereas λ_2 scales the gradient of the image and emphasizes smoothness and sharp transitions.
  - If the object features of interest are below the size of a nominal resolution cell, that is they should appear as "points," then they can be emphasized by choosing $\lambda_1 \gg \lambda_2$. This case leads to sparse reconstructions and can produce super-resolution.
  - If instead the object features of interest span multiple pixels, and thus form regions, these homogeneous regions can be recovered with sharp boundaries by choosing $\lambda_1 \ll \lambda_2$

- SDUI reconstructions of the 3.2 mm steel and the 3.2 mm aluminum rod separated by 10 mm reconstructed from 6.25% reduced aperture data for various choices of the regularization parameters.



Conclusions

- A new method for ultrasound image formation has been described that offers improved resolvability of fine features, suppression of artifacts, and robustness to challenging reduced data scenarios.
- The resulting nonlinear optimization problem was solved through efficient numerical algorithms exploiting the structure of the SDUI formulation.
- Results obtained from sparse aperture data scenarios suggest that SDUI can alleviate the motion artifact problem.
- The performance of the SDUI could be likely enhanced using multi-frequency data where the choice of number of frequency components and the appropriate weightings will be key factors to consider.