Random-Frequency SAR Imaging Based on Compressed Sensing

J. Yang et al.

IEEE Trans. Geo. Remot. Sensing (2013. Feb.)

Presenter : Jin-Taek Seong

GIST, Dept. of Information and Communications, INFONET Lab.

Gwangju Institute of Science and Technology

Introduction

- The stepped-frequency waveform consists of sequences of single-frequency pulses
- The stepped-frequency waveform can be viewed as the frequency sampling of the total bandwidth
- The advantages of a single frequency
  - Simple hardware requirements
  - High resolution
- The drawback is
  - A long time period to transmit the signals, since the transmitter must scan over the radar bandwidth using a sequence of discrete frequencies
- Therefore, this leads to many limitations for the application of the stepped-frequency waveform in SAR.
- There has to be a tradeoff between the resolution and imaging range width.

Introduction

- If the targets are sparse or compressible, the required frequencies in the stepped-frequency SAR can be reduced significantly using a CS theory
- In this paper, a random-frequency SAR imaging scheme based on CS is proposed
  - Reconstruction the 2-D image of the sparse targets by transmitting a small number of random frequencies.
- A sparse transform structure is proposed for the reshaped 2-D reflectivity map.
- The main advantages of the proposed imaging scheme
  - 1) the available imaging range width can be enlarged significantly, while the range and azimuth resolutions are both maintained
  - 2) the required number of frequencies can be reduced
  - 3) random undersampling is very easy to implement for both range and azimuth

Stepped-Frequency Waveform (1/2)

- The stepped-frequency waveform uses a sequence of pulses to achieve an ultrawide bandwidth
- We denote the transmitted waveform as

$$s_t(n,t) = \text{rect}\left(\frac{t}{T_p}\right)\exp\left[j2\pi f_c(n)t\right] \tag{1}$$

- For a point reflector at range *R*, the echo signal is

$$s_e(n,t) = g \cdot \text{rect}\left(\frac{t-2R/c}{T_p}\right)\exp\left[j2\pi f_c(n)(t-2R/c)\right] \tag{2}$$

  - *g* is the reflectivity coefficient of the target
- The demodulation reference signal is

$$\begin{aligned} s(n,t) &= s_e(n,t) \cdot s^*_{\text{ref}}(n,t) \\ &= g \cdot \left(\frac{t-2R/c}{T_p}\right)\exp\left[j2\pi f_c(n)(t-2R/c)\right] \\ &\quad \cdot \exp\left[-j2\pi f_c(n)t\right] \\ &= g \cdot \text{rect}\left(\frac{t-2R/c}{T_p}\right)\exp\left[-j\frac{4\pi f_c(n)R}{c}\right] \end{aligned} \tag{4}$$

Stepped-Frequency Waveform (2/2)

- We consider that the frequency interval is equal to Δf, so that

$$f_c(n) = f_c + n\Delta f, \qquad n = 1, 2, \ldots N \tag{5}$$

- The demodulated signal can be rewritten as

$$s(n,t) = g \cdot \mathrm{rect}\left(\frac{t - 2R/c}{T_p}\right) \exp\left[-j\frac{4\pi(f_c + \Delta f n)R}{c}\right]. \tag{6}$$

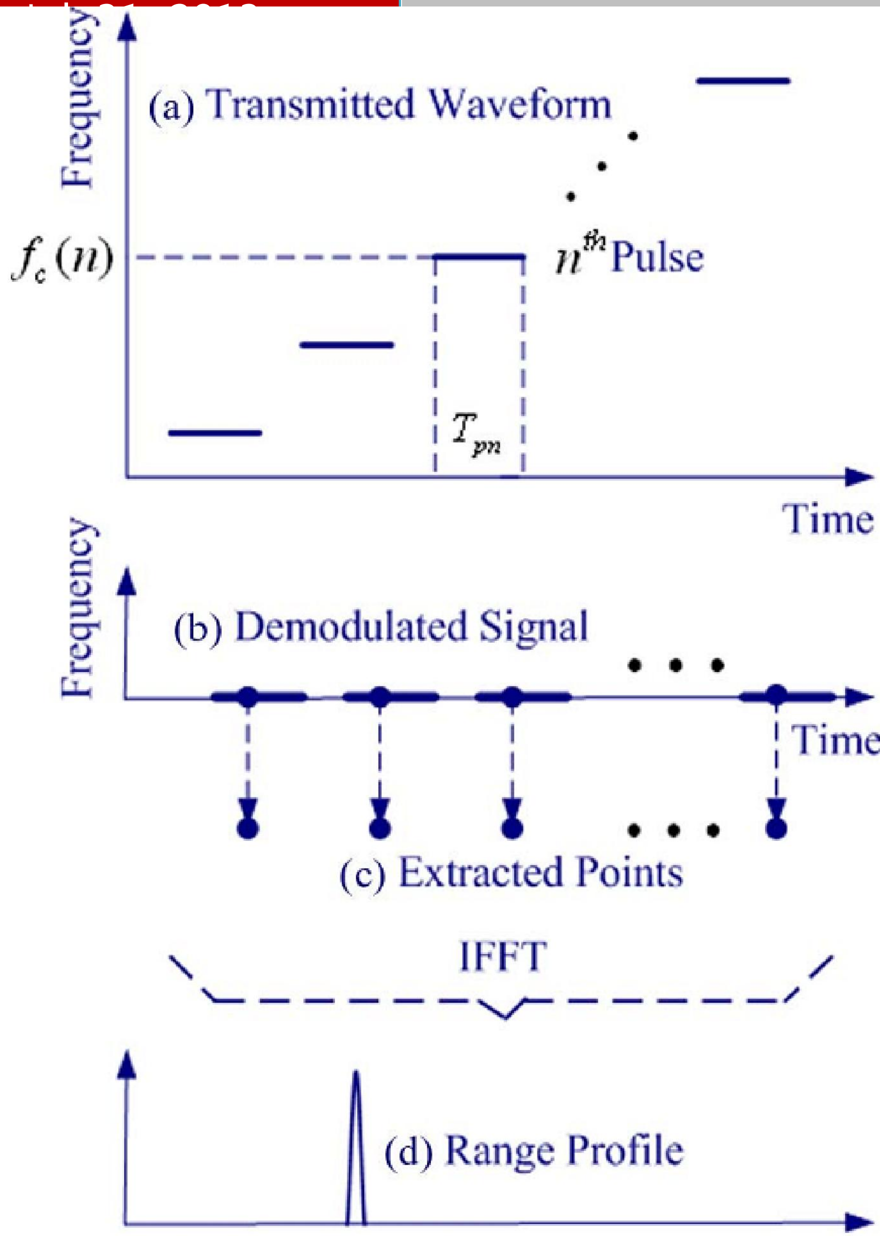

Fig. 1. Processing of stepped-frequency waveform. (a) Transmitted waveform. (b) Demodulated signal. (c) Extracted data points. (d) Range profile.

Limitations of Stepped-Frequency Waveform Applied to SAR (1/2)

- Sampling with Δf results in periodic repetition in the time domain, and the repetition period is $1/\Delta f$
- The corresponding repetition period for range is $c/(2\Delta f)$, so the nonaliasing range width is limited to

$$R_w < \frac{c}{(2\Delta f)} \tag{8}$$

- For a fixed pulse time interval, to avoid overlapping of the echoes, the maximum range width is

$$D_1 = \frac{\Delta t c}{2} \tag{9}$$

- For a given frequency step, the maximum nonaliasing range width is

$$D_2 = \frac{c}{2\Delta f} = \frac{Nc}{2B} \tag{10}$$

- Therefore, the **maximum available range width** is

$$D = \min\{D_1, D_2\}. \tag{12}$$

Limitations of Stepped-Frequency Waveform Applied to SAR (2/2)

- The equivalent azimuth sampling interval is $N\Delta tV$, where V is the radar velocity.

$$r_a = N\Delta tV. \tag{13}$$

- The range resolution is

$$r_r = \frac{c}{2B}. \tag{14}$$

- The available imaging range width and the range resolution and azimuth resolution must be traded off against each other
- To let the available range width become wider, Δt and N should be bigger, B should be smaller, but all of these requirements will decrease the resolution in both the range and azimuth dimensions.

Random-Frequency SAR Imaging Based on CS-SAR Imaging Model (1/3)

- The radar data are the superposition of the echoes of all scatterers in the area illuminated by the radar's beam (i.e., the scene)
- The received signal of the *n*th pulse in the *m*th sequence can be expressed as

$$s(m,n) = \iint_G g(x,y)\cdot \exp\left[-j\frac{4\pi f_c(n) R(m,n,x,y)}{c}\right] dxdy$$

  - x and y are the coordinates of the target
  - *g*(x, y) is the reflectivity coefficient of the target at (x, y)
  - *R*(m, n, x, y) is the range of the target at (x, y)
  - *G* is the area illuminated by the beam

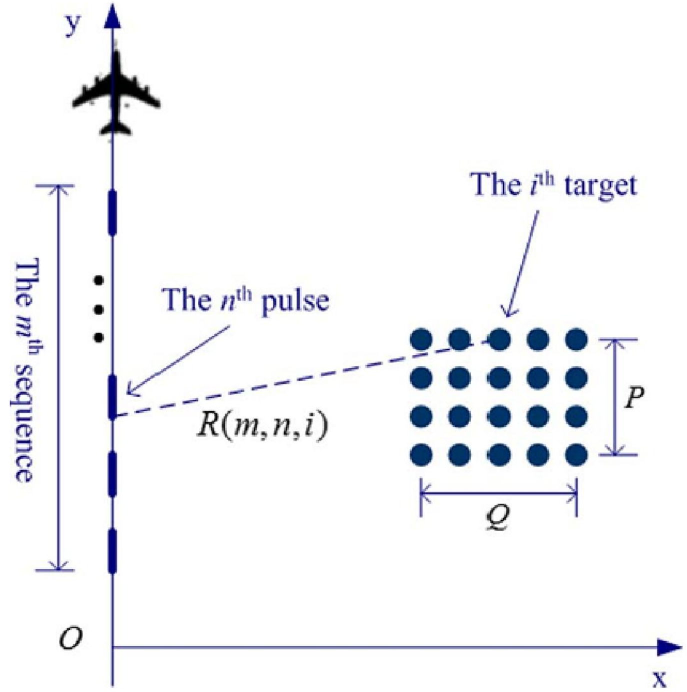

Fig. 2. Geometry model for SAR imaging.

Random-Frequency SAR Imaging Based on CS-SAR Imaging Model (2/3)

- The scene consists of a set of point scatterers on a grid, and the interval of the grids should be smaller than the radar resolution
- The reflectivity coefficients of the scatterers can be denoted as a 2-D matrix

$$\mathbf{G} = \begin{bmatrix} g(1,1) & \cdots & g(1,Q) \\ \vdots & \ddots & \vdots \\ g(P,1) & \cdots & g(P,Q) \end{bmatrix}$$

- The 2-D reflectivity coefficient matrix should be reshaped to a column vector, i.e., g is a PQ×1 vector
- The discrete expression of the radar data of the *n*th pulse in the *m*th sequence is

$$s(m,n) = \sum_{i=1}^{PQ} g(i) \cdot \exp\left[-j\frac{4\pi f_c(n) R(m,n,i)}{c}\right]$$

Random-Frequency SAR Imaging Based on CS-SAR Imaging Model (3/3)

- The linear equation can be expressed in matrix form as

$$\mathbf{s} = \mathbf{A}\mathbf{g} + \mathbf{n}$$

  - where *s* is an $MN \times 1$ vector, *A* is an $MN \times PQ$ matrix, *g* is a $PQ \times 1$ vector, and *n* is the noise term. *M* is the total number of sequences; *N* is the number of frequencies in one sequence.

- The detailed form is

$$\mathbf{s} = \begin{bmatrix} s(1,1) \\ \vdots \\ s(1,N) \\ s(2,1) \\ \vdots \\ s(2,N) \\ \vdots \\ s(M,1) \\ \vdots \\ s(M,N) \end{bmatrix} = \mathbf{A}\mathbf{g} + \mathbf{n} = \begin{bmatrix} \mathbf{a}(1,1)^{\mathrm{T}} \\ \vdots \\ \mathbf{a}(1,N)^{\mathrm{T}} \\ \mathbf{a}(2,1)^{\mathrm{T}} \\ \vdots \\ \mathbf{a}(2,N)^{\mathrm{T}} \\ \vdots \\ \mathbf{a}(M,1)^{\mathrm{T}} \\ \vdots \\ \mathbf{a}(M,N)^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} g(1) \\ g(2) \\ \vdots \\ g(PQ) \end{bmatrix} + \mathbf{n}.$$

Random-Frequency SAR Imaging Based on CS-CS Imaging Scheme

- In order to apply a CS scheme, a reduced set of elements in *s* is selected randomly, and a reduced set of rows in *A* is also selected accordingly. It means that a small number of frequencies are selected randomly.
- The CS measurement can be expressed as

$$\mathbf{s}' = \mathbf{A}'\mathbf{g} + \mathbf{n}'$$

- The targets can be reconstructed as

$$\min \|\mathbf{g}\|_1 \quad s.t. \quad \|\mathbf{A}'\mathbf{g} - \mathbf{s}'\|_2 \leq \varepsilon.$$

- Assume that *L* samples are selected from the total of *MN* samples; then, the uniform pulse time interval becomes $(MN/L)\Delta t$, so that the maximum available range width becomes

$$D' = \frac{MN}{L}\frac{\Delta tc}{2}.$$

Random-Frequency SAR Imaging Based on CS-Sparsity of Targets

- A Priori Sparse Targets: it means that the targets consist of a small number of dominant scatterers
- Sparsely Representable Targets: we can find a transform to make most of the coefficients in the transform domain
- The CS imaging scheme combined with the sparse transform (see the appendix) can be expressed as

$$\mathbf{s}' = \mathbf{A}'\mathbf{g} + \mathbf{n}' = \mathbf{A}'(\tilde{\mathbf{\Psi}}_r \tilde{\mathbf{\Psi}}_c)^{-1} \tilde{\mathbf{\Psi}}_r \tilde{\mathbf{\Psi}}_c \mathbf{g} + \mathbf{n}'.$$

- This equation can be rewritten as

$$\mathbf{s}' = \mathbf{A}'(\tilde{\mathbf{\Psi}}_r \tilde{\mathbf{\Psi}}_c)^{-1} \mathbf{x} + \mathbf{n}'.$$

- We can solve for the transform coefficients using

$$\hat{\mathbf{x}} = \min \|\mathbf{x}\|_1 \quad s.t. \quad \left\| \mathbf{A}'(\tilde{\mathbf{\Psi}}_r \tilde{\mathbf{\Psi}}_c)^{-1} \mathbf{x} - \mathbf{s}' \right\|_2 \leq \varepsilon.$$

- Then, the reflectivity coefficients can be obtained by

$$\hat{\mathbf{g}} = (\tilde{\mathbf{\Psi}}_r \tilde{\mathbf{\Psi}}_c)^{-1} \hat{\mathbf{x}}.$$

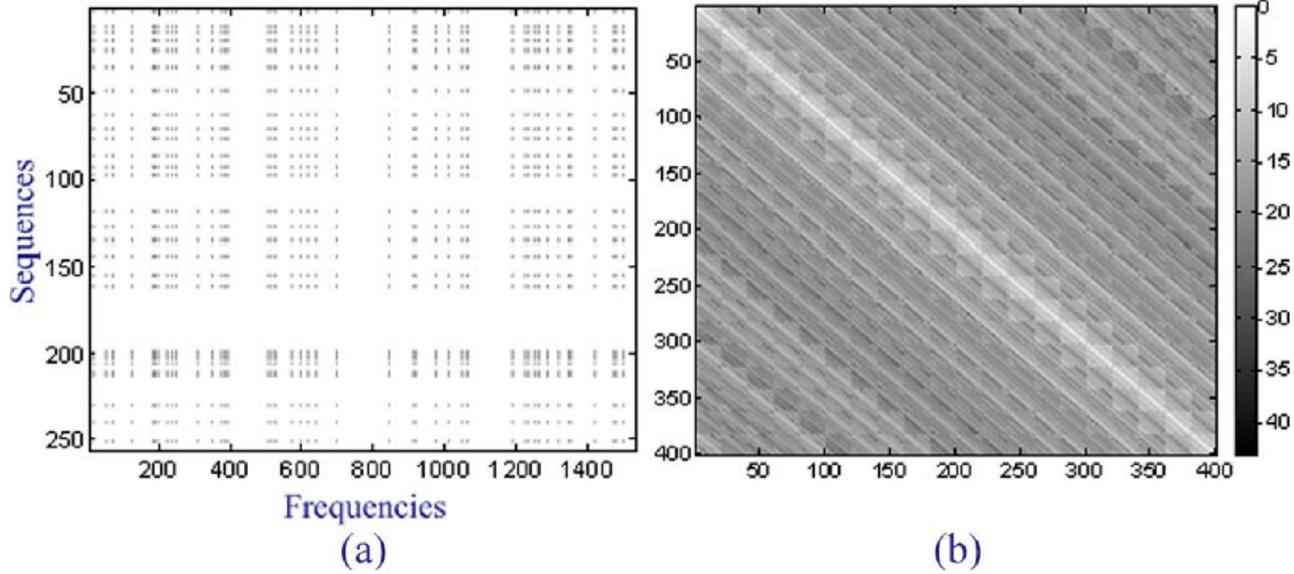

Fig. 3. Selected samples and mutual coherences for Option 1. (a) Selected samples. (b) Mutual coherences of the sensing matrix (scaled in decibels).

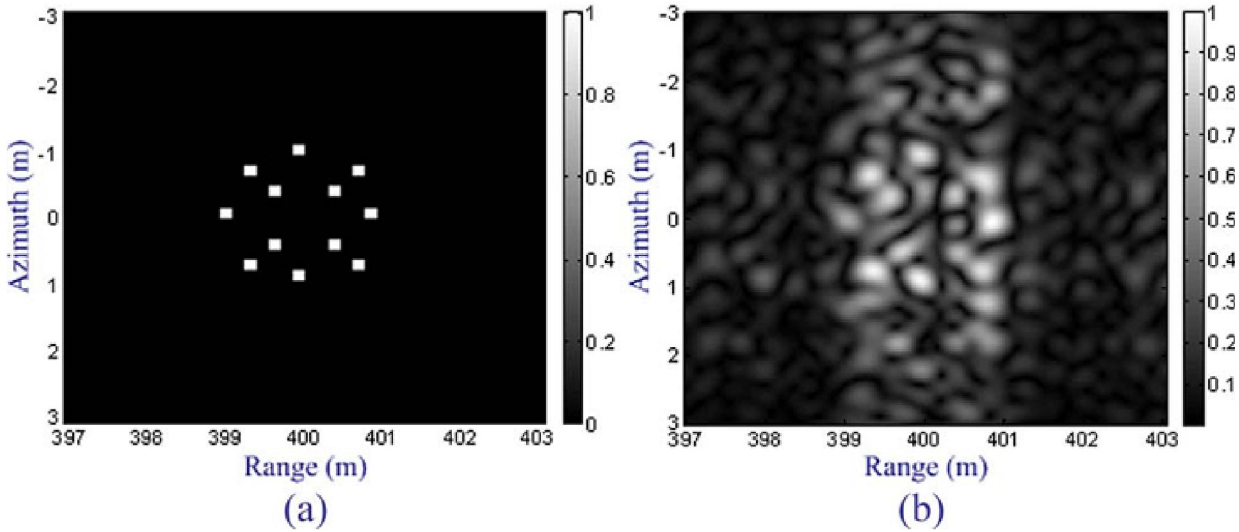

Fig. 6. Results of Option 1. (a) CS reconstruction result of the 1/192 data. (b) Imaging result of the 1/192 data using the Omega-K algorithm.

Experimental Results

- In order to show the validity, an experiment is carried out for stepped-frequency and random-frequency SAR imaging
- A stepped-frequency radar is mounted on a rail to acquire data
- The rail is controlled by a microcomputer, and the radar can move on the rail with a preset velocity

(a)

(b)

(c)

TABLE II
EXPERIMENT PARAMETERS

Bandwidth	512MHz
Pulse Time Interval	1e-3s
Radar Velocity	0.05m/s
Squint Angle	0°
Range Resolution	0.293m
Azimuth Resolution	0.05m
Number of Frequencies	512
Number of Sequences	480
Scene Azimuth Points	40
Scene Range Points	20
Selected Samples for CS	1024

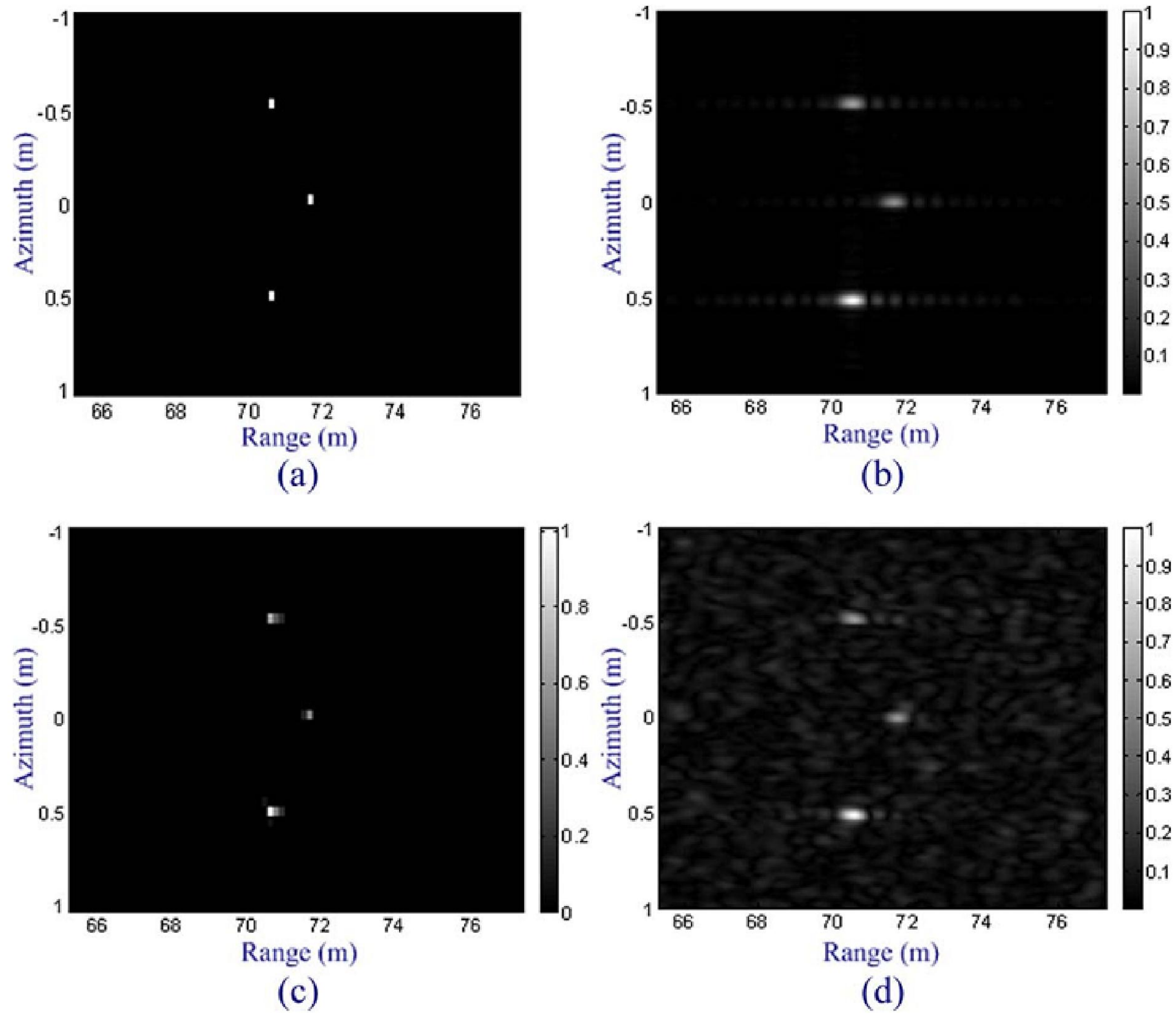

Fig. 12. Experimental results. (a) Position of the three corner reflectors. (b) Imaging result of the full data using the Omega-K algorithm. (c) CS reconstruction result of 1/240 data. (d) Imaging result of 1/240 data using the Omega-K algorithm.

Conclusions

- The theory of CS has been used to reduce the required frequencies in a stepped-frequency SAR system
- Based on the theory of CS, the traditional sampling requirements can be avoided, and the limitations of the stepped-frequency waveform applied in SAR are overcome
- The available imaging range width can be enlarged significantly, while the range and azimuth resolutions are maintained.
- The results of the CS imaging scheme are even better than the traditional results of the fully sampled data
- Future work will include fast reconstruction strategies and detailed investigations of the sparsity and compressibility of the targets
- Speckle noise will make the phase of the reflectivity map random, and it is difficult to find an effective sparse transform for a complex reflectivity map

Appendix – Transform of the 2D Matrix

- We begin with the sparse transform for the 2-D reflectivity matrix. The sparse transform can be applied to both the columns and rows of the 2-D matrix, and it can be expressed as.

$$\mathbf{X} = \mathbf{\Psi}_c \mathbf{G} \mathbf{\Psi}_r$$

  - where *X* contains the coefficients after the sparse transform, *G* is shown in (29), Ψ_c is the sparse transform matrix for the columns, the size of Ψ_c is $P \times P$, Ψ_r is the sparse transform matrix for the rows, and the size of Ψ_r is $Q \times Q$. Ψ_c and Ψr are full rank matrices.

- In the imaging scheme based on CS, the 2-D reflectivity matrix *G* is reshaped to a column vector *g*

$$\tilde{\mathbf{\Psi}}_c = \begin{bmatrix} \mathbf{\Psi}_c & & & \\ & \mathbf{\Psi}_c & & \\ & & \ddots & \\ & & & \mathbf{\Psi}_c \end{bmatrix} \qquad \tilde{\mathbf{\Psi}}_r = \begin{bmatrix} \mathbf{\Psi}_{r1} \\ \mathbf{\Psi}_{r2} \\ \vdots \\ \mathbf{\Psi}_{ri} \end{bmatrix}$$

- The sparse transform of the reshaped reflectivity vector can be expressed as

$$\mathbf{x} = \tilde{\mathbf{\Psi}}_r \tilde{\mathbf{\Psi}}_c \mathbf{g}$$