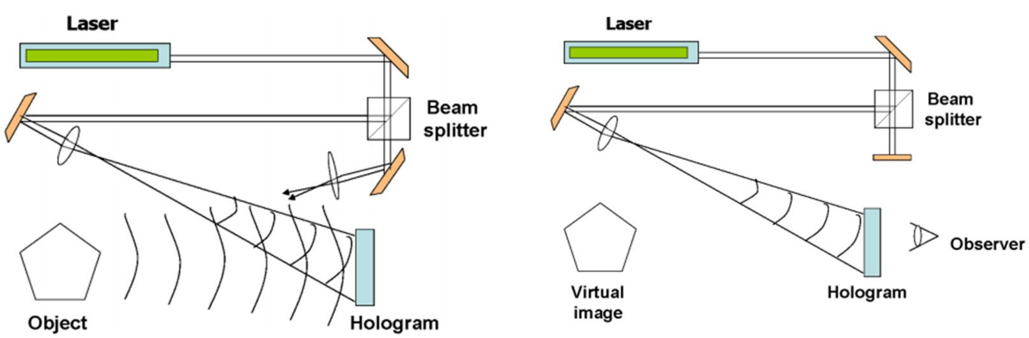
Title: Direct recording of holograms by a CCD target and numerical reconstruction

U.Schnars and W.Juptner

Presenter: Nitin Rawat
Date: 4/9/2013

Here they introduced a method that uses a charge-coupled device detector (CCD) as a holographic recording medium. However, using CCD's for recording holograms is advantageous and no chemical or physical developing is necessary. Reconstruction can be performed by digital image processing.

In this Note, the mathematical reconstruction is done directly with the digitally sampled Fresnel hologram form the CCD.



The holographic process is described mathematically as follows:

$$O(x,y) = o(x,y)\,\mathrm{e}^{i\varphi_o(x,y)} \quad \text{... (1.1)}$$

Is the complex amplitude of the object wave with real amplitude o and phase φ_o and

$$R(x,y) = r(x,y)\,\mathrm{e}^{i\varphi_R(x,y)} \quad \text{... (1.2)}$$

Is the complex amplitude of the reference wave with real amplitude r and phase φ_R

$$I_H(x,y) = \left|O(x,y) + R(x,y)\right|^2$$

$$= (O(x,y) + R(x,y))(O(x,y) + R(x,y))^*$$

$$= R(x,y)R^*(x,y) + O(x,y)O^*(x,y)$$

$$+ O(x,y)R^*(x,y) + R(x,y)O^*(x,y) \quad \text{... (1.3)}$$

$$I_H(x,y) = \left|R\right|^2 + \left|O\right|^2 + R^*O + RO^* \quad \text{... (1.4)}$$

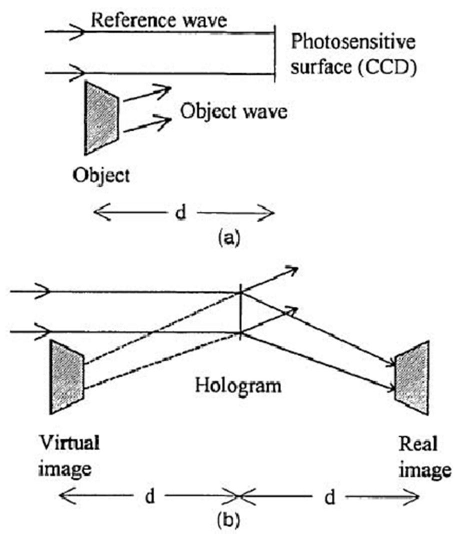


Fig. 1. Off-axis holography with a plane reference wave: (a) recording, (b) reconstruction.

Fig.1 (a) shows the recording geometry. A plane reference wave and the diffusely reflected object wave are interfering at the surface of a photosensitive medium.

In optical holography the object wave can be reconstructed by illumination of the processed hologram with a plane wave, similar to that used in the process of recording.

Looking through the hologram, one notice a virtual image of the object at the position of the original object $[Fig.1(b)]$ .If a screen is placed at a distance *d* behind the hologram, a real image is formed.

Mathematically the amplitude and phase distribution in the plane of the real image can be found by the Fresnel-Kirchhoff integral.

Fresnel integral of the digitized hologram intensity $I_H(k,l)$:-

$$\Psi(m,n) = A\exp\left[\frac{i\pi}{\lambda d}\left(m^2\Delta\xi^2 + n^2\Delta\eta^2\right)\right]\times FFT\left\{R_D(k,l)I_H(k,l)\times\exp\left[\frac{i\pi}{\lambda d}\left(k^2\Delta x^2 + l^2\Delta y^2\right)\right]\right\}_{m,n}$$

Where k, l, m, n $(-N/2 \le k,l \le N/2)$ are integers; FFT is the fast Fourier transform operator; d is the distance between the hologram and the observation plane; and $A = \exp(i2\pi d/\lambda)/(i\lambda d)$ is a constant.

$\Delta x = \Delta y = L/N$ define the sampling intervals in the observation plane ($\Delta\xi$ and $\Delta\eta$) are related to the size of the CCD (L) and to the distance d by the following relation:

$$\Delta\xi = \Delta\eta = \lambda d / L$$

The reconstructed wave front is an array of complex numbers. An amplitude-contrast image and a phase-contrast image can be obtained by calculation of the intensity

$\left[\mathrm{Re}(\psi)^2 + \mathrm{Im}(\psi)^2\right]$ and the argument $\left\{a\tan\left[\mathrm{Re}(\psi)/\mathrm{Im}(\psi)\right]\right\}$ of $\psi(m,n)$, respectively.

$\Psi(m,n)$ is a matrix of N x N points that describes the amplitude and phase distribution of the real image.

$\Delta\xi$ and $\Delta\eta$ are the pixel sizes in the reconstructed image. If only the intensity distribution according to Eq.3 is of interest, the phase factor before the summation can be neglected.

In the experimental investigations a CCD array is placed at the position of the photosensitive surface (Fig.1).

The CCD array consists of 1024 x 1024 pixels. The pixel area(L) is $6.8\mu m \times 6.8\mu m$.

For computation the hologram is stored in a digital image processing system. The object in this experiment was a cube with a side of length of 11 mm, which was placed at a distance of 1 m from the target.

A helium-neon laser was used as a light source.

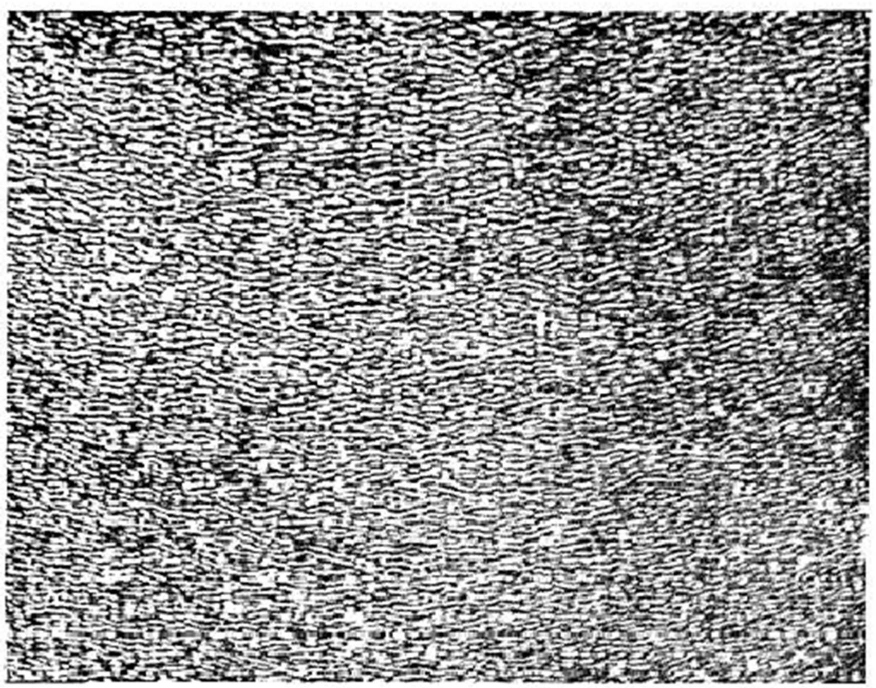


Fig. 2. Digitally sampled off-axis hologram.

Fig. 2 shows a part of a digitally sampled hologram. The original dimensions of the whole hologram were 7mm x 7mm, which are the dimensions of the CCD chip.

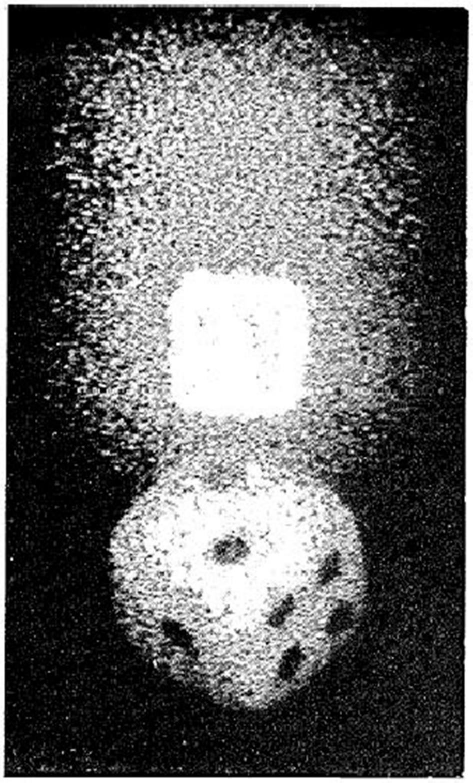


Fig. 3. Numerical reconstruction.

The numerical reconstruction according to $\Psi(m,n)$ is demonstrated in Fig.3. A real image of the cube together with the undiffracted reference wave is noticeable.

Because of the off-axis geometry, these two parts of the real image are separated. Furthermore, a speckle appearance on the reconstructed cube is noticeable. It is a result of the interaction between coherent light and the rough surface of the object.