Compressed sensing with off-axis frequency-shifting holography

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Short summary: This work reveals an experimental microscopy acquisition scheme successfully combining compressed sensing (CS) and digital holography in off-axis and frequency-shifting conditions. The authors propose a CS-based imaging scheme for sparse gradient images, acquiring a diffraction map of the optical field with holographic microscopy and recovering the signal from as little as 7% of random measurements.

I. COMPRESSED SENSING

- A signal $g \in R^N$ has a sparse representation if it can be written as a linear com bination of a small set of vectors taken from some basis Ψ, such as $g = \sum_{i=1}^{N} c_i \Psi_i$, with $\|c\|_1 \approx S$ where $S << N$.
- If such a sparsifying transform Ψ exists in the spatial domain, it is possible to reconstruct an image g from partial knowledge of its Fourier spectrum [1].

II. SYSTEM OVERVIEW

- g represents the local optical intensity in the object plane.
- We denote by $f \in C^N$ the associated complex optical field, satisfying $g = |f|^2$.
- The radiation field propagates from the object to the detector plane in Fresnel dif fraction conditions (d>>$(x,y)_{max}$).
- Thus, the optical field in the object plane f is linked to the field F in the detect ion plane by a Fresnel transform, expressed in the discrete case as

$$F = \mathrm{F}(f): C^N \to C^N,$$

$$F_p = \frac{1}{N}\sum_{n=1}^{N} f_n e^{i(\alpha n^2 - 2\pi np/N)} \quad (1),$$

where *p* and *n* denote pixel indices and $\alpha \in R^+$ is the parameter of the quadratic phase factor $e^{i\alpha n^2}$ describing the curvature in the detection plane of a wave emitted by a point source in the object plane.

III. Optical configuration [2]

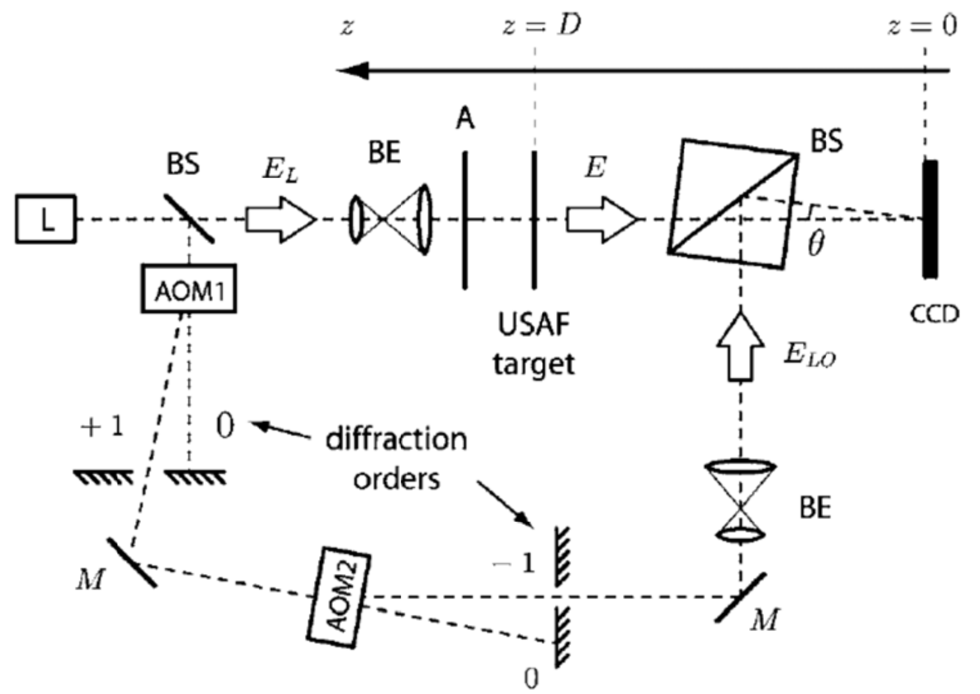

Fig. 1. USAF target digital holography setup: L, main laser; BS, beam splitter; AOM1 and AOM2, acousto-optic modulators; BE, beam expander; *M*, mirror; A, light attenuator; USAF, transmission target; CCD, CCD camera.

- It consists of an off-axis frequency-shifting digital holography scheme.
- The interfered object beam by the reference beam is recorded at the array detector.
- $F \in C^M$ is calculated from a four-phase measurement.

Phase-shifting digital holography [3].

- The reference beam is dynamically phase shifted with respect to the signal field; the phase shift in the experiment is linear in time (frequency shift).
- It is then possible to obtain the two quadratures of the field in an on-axis configuration even though the conjugate image alias and the true image overlap, because aliases can be removed by taking image differences.
- This shift produces time-varying interferograms on a two dimensional sensor.
- Intensity in the detector plane results from the interference of the signal field with the δf -shifted reference field:

$$I(t) = \left|E_s + E_r \exp i(2\pi \cdot \delta f \cdot t)\right|^2$$

  where Es and Er represent the complex amplitudes of the signal and the reference fields, respectively.
- L intensity $I_l = \left|E_s + E_r \exp i(\frac{2\pi l}{L})\right|^2$ measurements are performed at $t_l = \frac{l}{\delta f \cdot L}$, $l = 0,..., L-1$; $I_l = \left|E_s\right|^2 + \left|E_r\right|^2 + E_s^* E_r \exp i(\frac{2\pi l}{L}) + E_s E_r^* \exp - i(\frac{2\pi l}{L})$.
- Obtain Es by demodulating I:

$$E_s = \frac{1}{L \cdot E_r^*} \sum_{l=0}^{L-1} I_l \exp i(\frac{2\pi l}{L}) .$$

- The CCD camera records the hologram of the interference with frame rate f_{CCD} =12.5Hz, acquisition time T=1/12.5=80ms. They $\delta f = f_{CCD} / L$ shift the reference beam by combining two acousto-optic modulator (Crystal Technology: $f_{\mathrm{AOM}} \cong 80\mathrm{MHz}$), AOM1 and AOM2, working at $\Delta f + \delta f$ and $-\Delta f$, respectively, with Δf =80MHz. δf =3.125Hz is equal to one quarter of the CCD image frequency (L=4).

The k-space hologram at z=D from the CCD

- $\tilde{E}_s(k_x, k_y, z) = \tilde{K}(k_x, k_y, z)\tilde{E}_s(k_x, k_y, z = 0)$ where $\tilde{K}(k_x, k_y, z)$ is the k-space kernel function that describes the propagation from 0 to z.
- $\tilde{K}(k_x, k_y, z) = e^{iz(k_x^2 + k_y^2)/k}$ where $k = 2\pi / \lambda$ is the optical wave vector.

- Consider the Fresnel propagation of Es from z=0 to z=D, which can be formally expressed as an x and y convolution product (symbol $\otimes$):

$$E_s(x,y,z) = P(x,y,z) \otimes E_s(x,y,z=0),$$

where $P(x,y,z) = \dfrac{e^{i\cdot kz}}{i\lambda z}\exp\left[i\dfrac{k}{2z}(x^2+y^2)\right]$.

Combination of “Off axis” and “phase shift”.

- With off-axis holography, it is possible to record with a single hologram, the two quadratures of the object complex filed. However, the object field of view is reduced, since one must avoid the overlapping of the image with the conjugate image alias.
- With phase-shifting holography, it is possible to obtain the two quadratures of the field even though the conjugate image alias and the true image overlap, because aliases can be removed by taking image differences.
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IV. Conventional approach

- F can be backpropagated numerically to the target plane with the standard convol ution method when all measurements $F \in C^N$ are available.
- In this case, the complex filed in the object plane f is retrieved from a discrete i nverse Fresnel transform of F; $f = \mathrm{F}^{-1}(F)$,

$$f_p = \frac{1}{N}\sum_{n=1}^{N} F_n e^{-i(\alpha n^2 - 2\pi np/N)} \tag{2}$$

V. Proposed CS approach

- In the CS, the signal reconstruction consists of solving a convex optimization pro blem that finds the candidate $\hat{g}$ of minimal complexity satisfying $\hat{F}|_\Gamma = F|_\Gamma$, whe re $F|_\Gamma \subseteq F$ is a partial subset of measurements in the set Γ.
- We want to recover the intensity image of the object $g = \left\{|f|^2 : f \in C^N\right\}$ from a small number of measurements $F|_\Gamma \in C^M$, where $M << N$.

- Partial measurements in the detection plane can be written as $F|_{\Gamma} = \Phi f$, where the sampling matrix Φ models a discrete Fresnel transformation and a random undersampling with a flat distribution.

- To find the best estimator $\hat{g}$, we solve the following convex optimization problem:

$$\hat{g} = \arg\min_{g \in R^N} \left\| \Psi^{\mathrm{T}} g \right\|_1 \text{ subject to } \hat{F}|_{\Gamma} = F|_{\Gamma} .$$

- Since the test image is piecewise constant with sharp edges (such as most microscopy images), it can be sparsely represented computing its gradient.

- In image processing, a suitable norm to constrain the gradient of an image was introduced as the total variation (TV) which measures the l-1 norm of the gradient magnitudes over the whole image, $\|g\|_{\mathrm{TV}} = \|\nabla g\|_1$.

- The incoherence property holds for the two bases adopted here which are the Fresnel spectrum and the TV. Moreover, random measurements in the spectral domain satisfy the RIP condition.

- Hence for an overwhelming percentage of Fresnel coefficients sets Γ with cardinality obeying $|\Gamma| = M \geq KS \log N$, for some constant K, $\hat{g}$ is the unique solution to the problem,

$$\hat{g} = \arg\min_{g \in R^N} \|\nabla g\|_1 \text{ subject to } \hat{F}|_{\Gamma} = F|_{\Gamma} .$$

- The reconstruction of g with robustness to noise:

  $\hat{g} = \arg\min_{g \in R^N} \|\nabla g\|_1$ subject to $\left\| \hat{F}|_{\Gamma} - F|_{\Gamma} \right\|_2 \leq \delta$, for some $\delta \leq C\varepsilon$,
  which depends on the noise energy.

VI. Results

Standard convolution method (eq. 2) and CS approach.

- For the CS approach, Fresnel coefficients are undersampled randomly
- Figure 3(b) shows the CS reconstruction result from only 7% of the pixels used in the standard approach.
- Figure 3(d) illustrates the residual (Euclidean distance $|\hat{g} - g|$) from standard holographic and CS reconstructions. The global normalized error is $\|\hat{g} - g\|_2$ =0.005.

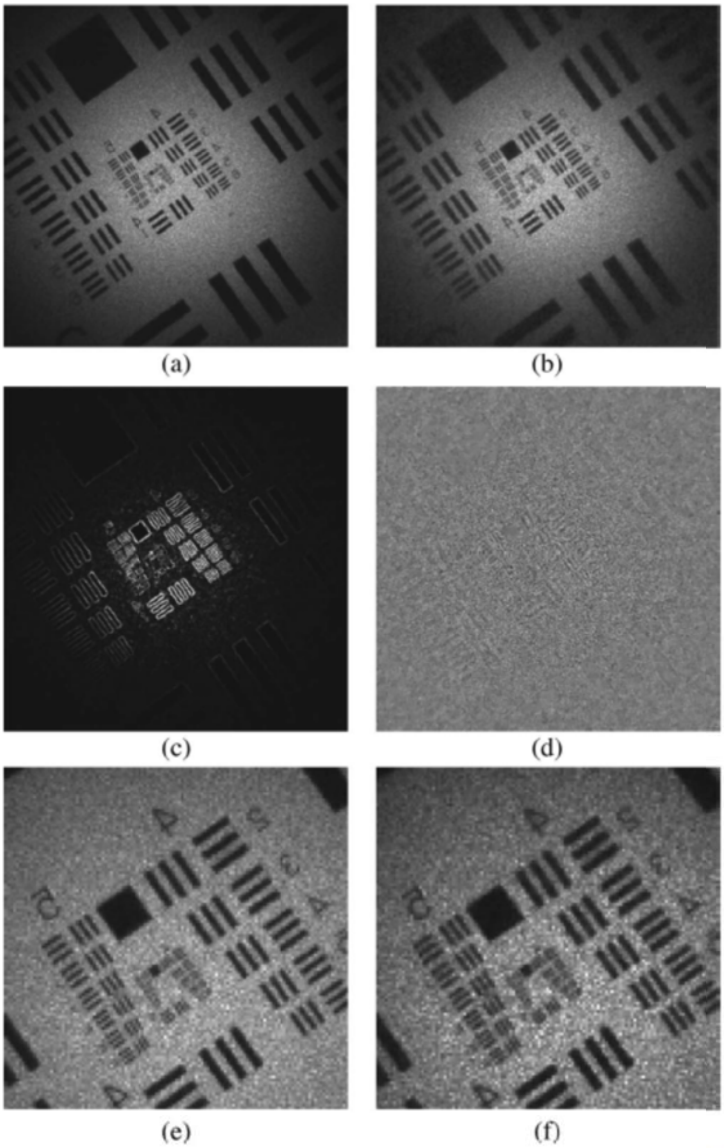

Fig. 3. (a) Standard holography, as described in Eq. (1). (b) CS reconstruction, using 7% of the Fresnel coefficients. (c) Gradient of g. (d) Residual from (a) and (b). (e), (f) Magnified views from (a) and (b).

Reference

[1] E. Candes and J. Romberg, “Practical signal recovery from random projections,” Proc. SPIE, 5674, Jan. 2005.
[2] M. Gross and M. Atlan, “Digital holography with ultimate sensitivity,” Opt. Lett., Vol. 32, no. 8, Apr. 2007.
[3] F. Le Clerc and L. Collot, “Numerical heterodyne holography with two-dimensional photodetector arrays,” Opt. Lett., vol. 25, no. 10, May 2000.