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Compressive Holography

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Short summary: A gabor hologram is to reconstruct a 3D object from measurements acquired at a detector. In this seminar, we introduce this gabor hologram and see a relation between the measurements and the 3D object. Then, we consider why compressive sensing can be applied to the gabor hologram.

I. A GABOR HOLOGRAM

A gabor hologram, which is called an in-line hologram, is formed in interference between a plan wave *A* and a 3D object with scattering density $\eta(x', y', z')$ and measurements acquired at a 2D detector are given by

$$I(x,y) = |A + E(x,y)|^2 = |A|^2 + |E(x,y)|^2 + A^* E(x,y) + AE(x,y)^* \tag{1}$$

where A^* is the conjugate matrix of *A* and the scattered field *E* from the 3D object is defined under the Born approximation as

$$E(x,y) = \iiint dx' dy' dz' \eta(x', y', z') h(x-x', y-y', z-z'). \tag{2}$$

where *h* is the product of exp(*ikz*) which represents the phase delay at a distance z and the inverse Fourier transform of the propagation transfer function $\exp\left(iz\sqrt{k^2 - k_x^2 - k_y^2}\right)$, i.e.,

$$h = e^{ikz} \mathcal{F}^{-1}\left\{ e^{iz\sqrt{k^2 - k_x^2 - k_y^2}} \right\} \tag{3}$$

Since *A* is the plan wave, it is reasonable that *A* is assumed to be one. Then, the equation given in (1) is simplified to

$$
\begin{aligned}
I(x,y) &= \left|A+E(x,y)\right|^2 = 1+\left|E(x,y)\right|^2 + E(x,y) + E(x,y)^* \\
&= 1+\left|E(x,y)\right|^2 + 2\,\mathrm{Re}\left\{E(x,y)\right\} \\
&\approx 2\,\mathrm{Re}\left\{E(x,y)\right\} + e(x,y)
\end{aligned} \tag{4}
$$

where $e(x,y)$ is considered as model error and one can be ignored. Then, the equation given in (4) describes a mapping function between the 3D object and the intensity captured by the 2D detector.

II. CONNECTIONS BETWEEN THE GABOR HOLOGRAM WITH COMPRESSIVE SENSING

In the literature, compressive sensing (CS) has attracted attentions since CS offers a new paradigm for signal acquisition; it takes samples by a linear projection such as **y=Ax**, where **y** is the measurement vector, **A** is the measurement matrix, and **x** is the sparse signal that we wish to reconstruct, while these samples are being compressed. Simply speaking, CS suggests that signals are reconstructed from what we believed as incomplete information. To apply CS to a certain field, we show that sampling procedures of this field is modeled as the linear projection. Thus, in this section, we aim to see that sampling procedures of the Gabor hologram can be modeled as the linear projection.

Let a 3D object be denoted by $\eta(x',y',z')$ that we reconstruct. Let $\Delta_x = \Delta_y = \Delta$ be the sampling spacing in the *x*-axis and the *y*-axis respectively. Let Δ_z be the sampling spacing in the *z*-axis as well. Let *N* be the number of pixels along each dimension of the 2D detector. Then, the measurements acquired at the 2D detector are expressed by

$$
E(n_1\Delta, n_2\Delta) = \frac{1}{N^2}\sum_{l}\sum_{m_1}\sum_{m_2}\left[\sum_{m_3}\sum_{m_4}\eta_{m_3,m_4,l}\,e^{-i2\pi\frac{m_1m_3+m_2m_4}{N}}\right]e^{ikl\Delta_z}e^{il\Delta_z\sqrt{k^2-m_1^2\Delta_k^2-m_2^2\Delta_k^2}}e^{-i2\pi\frac{n_1m_1+n_2m_2}{N}} \tag{5}
$$

where $\eta_{m_3,m_4,l} = \eta(m_3\Delta, m_3\Delta, l\Delta_z)$. Note that the term in the square bracket is the 2D discrete Fourier transform of the 3D object. Hence, the above equation is simplified to

$$E\left(n_1\Delta, n_2\Delta\right) = \frac{1}{N^2}\sum_{l}\sum_{m_1}\sum_{m_2}\mathcal{F}_{2D}\left\{\eta_{m_3,m_4,l}\right\}e^{ikl\Delta_z}e^{il\Delta_z\sqrt{k^2-m_1^2\Delta_k^2-m_2^2\Delta_k^2}}e^{-i2\pi\frac{n_1m_1+n_2m_2}{N}} \quad (6)$$

The last term forms the inverse 2D Fourier transform. Then, the equation is again simplified to

$$E\left(n_1\Delta, n_2\Delta\right) = \mathcal{F}_{2D}^{-1}\left\{\sum_{l}\mathcal{F}_{2D}\left\{\eta_{m_3,m_4,l}\right\}e^{ikl\Delta_z}e^{il\Delta_z\sqrt{k^2-m_1^2\Delta_k^2-m_2^2\Delta_k^2}}\right\} \quad (7)$$

Now, we aim to rewrite the above equation as a matrix-matrix multiplication form. For this end, we begin by defining matrices by

$$\mathbf{B} = diag\left(\mathbf{F}_{2D}, \mathbf{F}_{2D}, \cdots, \mathbf{F}_{2D}\right) \quad (8)$$

where $\mathbf{F}_{2D}$ is the 2D DFT matrix whose size is $N^2 \times N^2$, and the matrix $\mathbf{G}_{2D}$ denotes the 2D inverse DFT matrix. Then, we consider the term in the 2D inverse DFT matrix in (7). Then, it is easy to see that

$$\begin{aligned}
\sum_{l}\mathcal{F}_{2D}\left\{\eta_{m_3,m_4,l}\right\}e^{ikl\Delta_z}e^{il\Delta_z\sqrt{k^2-m_1^2\Delta_k^2-m_2^2\Delta_k^2}} &= \sum_{l}\mathbf{F}_{2D}\eta_{m_3,m_4,l}\,p_{m_1,m_2,l} \\
&= \mathbf{F}_{2D}\,p_{m_1,m_2,l}\eta_{m_3,m_4,1} + \mathbf{F}_{2D}\,p_2\eta_{m_3,m_4,2} + \dots \\
&= \begin{bmatrix} p_{m_1,m_2,1} & p_{m_1,m_2,2} & \cdots \end{bmatrix}\begin{bmatrix} \mathbf{F}_{2D} & & \\ & \mathbf{F}_{2D} & \\ & & \ddots \end{bmatrix}\begin{bmatrix} \eta_{m_3,m_4,1} \\ \eta_{m_3,m_4,2} \\ \vdots \end{bmatrix} \\
&= \mathbf{P}_{m_1,m_2}\mathbf{B}\boldsymbol{\eta}
\end{aligned} \quad (9)$$

where $p_{m_1,m_2,l} = e^{ikl\Delta_z}e^{il\Delta_z\sqrt{k^2-m_1^2\Delta_k^2-m_2^2\Delta_k^2}}$ and $\boldsymbol{\eta}$ is the 3D object. By combining the above equation with the equation given in (7), we finally get

$$E\left(x, y\right) = \left[\mathbf{G}_{2D}\mathbf{P}\mathbf{B}\boldsymbol{\eta}\right]_{x,y} \quad (10)$$

Thus, the intensities acquired at the 2D detector is given by

$$\begin{aligned} I(x,y) &= 2\,\mathrm{Re}\left\{ \left[\mathbf{G}_{2D}\mathbf{PB}\boldsymbol{\eta} \right]_{x,y} \right\} + e(x,y) + 1 \\ &\approx 2\,\mathrm{Re}\left\{ \left[\mathbf{G}_{2D}\mathbf{PB}\boldsymbol{\eta} \right]_{x,y} \right\} \end{aligned} \tag{11}$$

Then, they solve a below total variation (TV) optimization problem to reconstruct the 3D object.

$$\boldsymbol{\eta}_{sol} = \arg\min \left\| \boldsymbol{\eta} \right\|_{TV} \text{ such that } E(x,y) = \left[\mathbf{G}_{2D}\mathbf{PB}\boldsymbol{\eta} \right]_{x,y} \text{ for all } x, y. \tag{12}$$

where $\left\| \boldsymbol{\eta} \right\|_{TV}$ is the total variation of $\boldsymbol{\eta}$.

III. Experiment Results

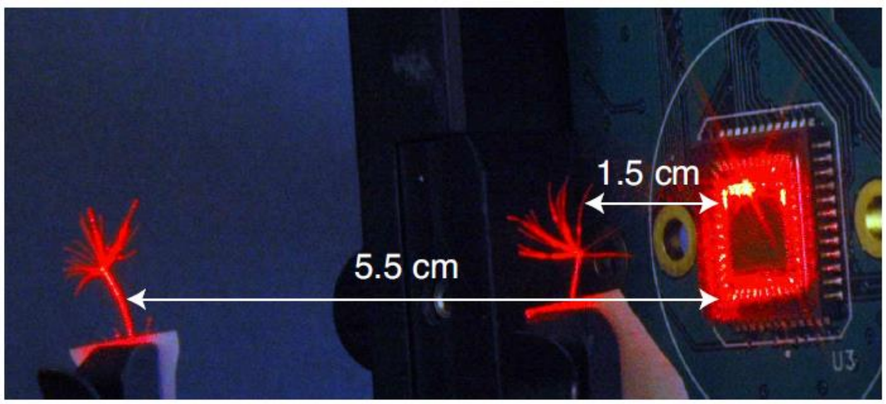

Figure 1 experimental approach

They illuminated two seed parachutes of common dandelions with a collimated, spatially filtered Helium-Neon laser of 632.8 nm wave length. One object is placed 1.5 cm away from the detector array, and the other dandelion is placed 5.5 cm away from the detector.

(a) (b) (c)

Figure 2 (a) is the illumination and scattered field were captured in the Gabor hologram, both (b) and (c) are photographs of the two seed parachutes.

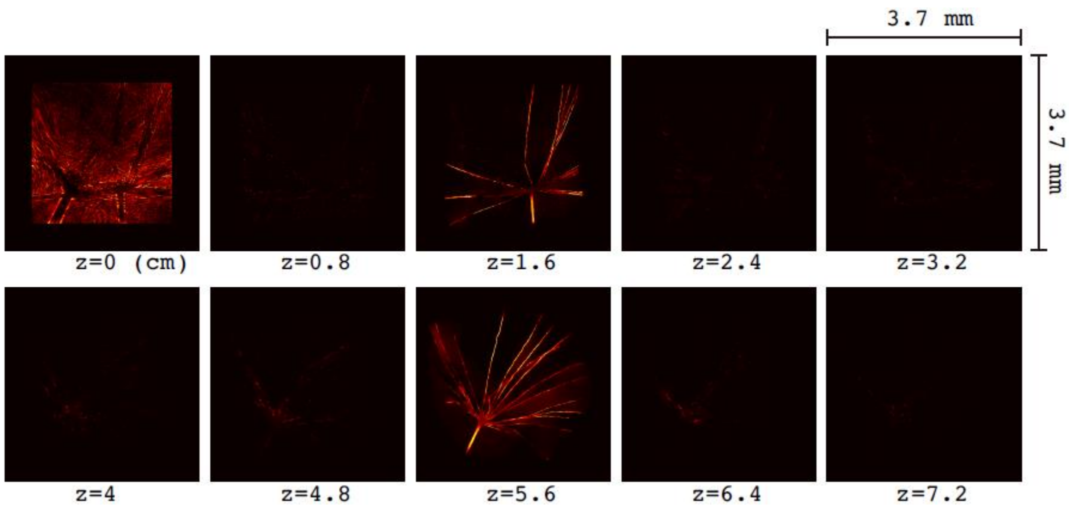

Figure 3 the 3D date cube estimated from the Gabor recording by solving the above TV-minimization problem.

As we see Figure 3, the stem and the petals, which represent the high frequency features in the images, are reconstructed well. In addition, depending on the distance between the detector plan and the first parachute and the distance between the parachutes are also accurately estimated

On the other hands, the stem and the petals are ambiguous, and two parachutes are not explicitly represented according to the distance.

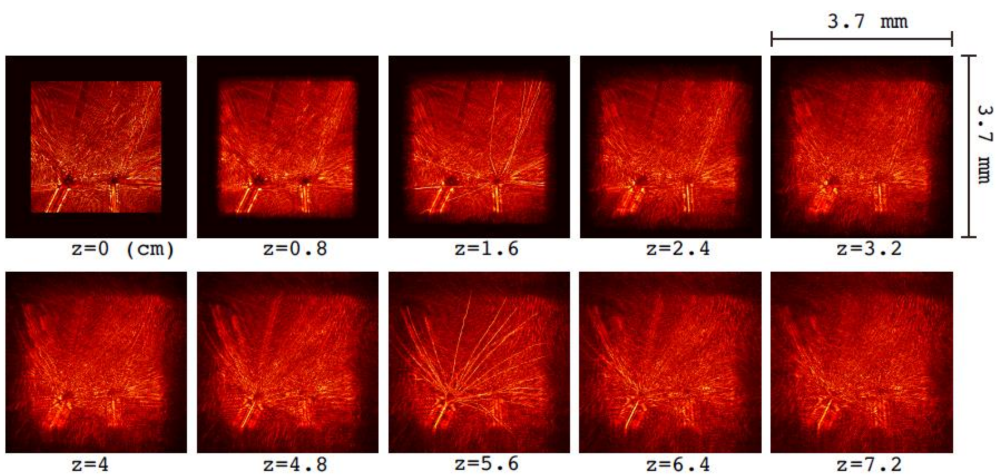

Figure 4 the 3D date cube estimated from the Gabor recording by digitally back propagation.

IV. DISCUSSION

After meeting, please write discussion in the meeting and update your presentation file.