INFONET Seminar Application Group 2014/08/14

Neural Networks and Their Applications

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Review of Science instruments, 1993

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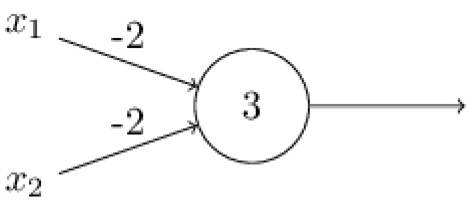
Contents

Introduction to Deep Learning

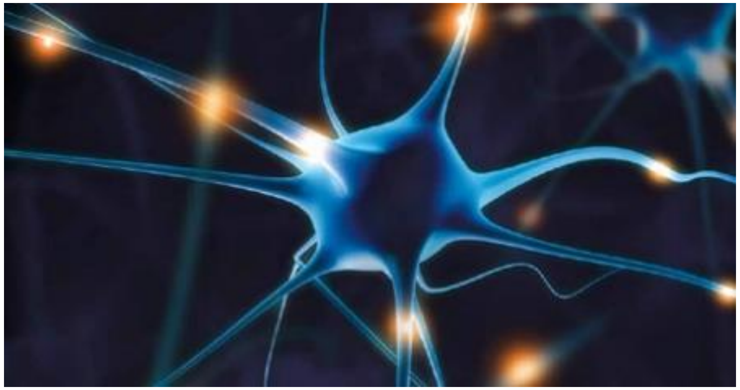
What is an Artificial Neural Network?

- Computational model inspired by a biological central nervous system(brain)
- Purpose of ANN – use its capabilities to perform a machine learning and pattern recognition tasks like humans.
- ANN consist of connected neurons
- Perceptron is a type of an artificial neuron



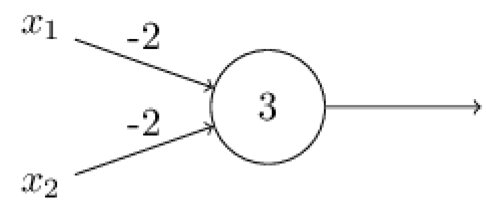
Pic. 1. Schematics of a perceptron


Pic. 2. Human brain and active central nervous system


Pic. 3. Active neuron

Perceptron and Sigmoid neuron (1)

- A perceptron takes several binary *inputs* x_1, x_2 and produces a single binary *output*
- *Weights* – real numbers expressing importance of respective inputs to the output
- *Bias* – threshold value

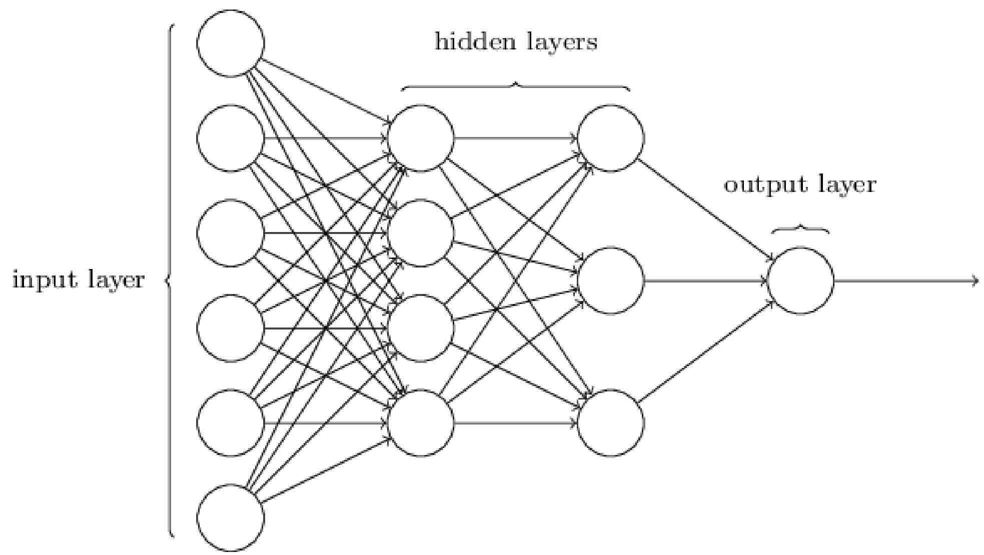


Pic. 4. Schematics of a perceptron

$$\text{output} = \begin{cases} 0 \text{ if } \sum_j w_j x_j \le \text{ threshold} \\ 1 \text{ if } \sum_j w_j x_j > \text{ threshold} \end{cases}$$

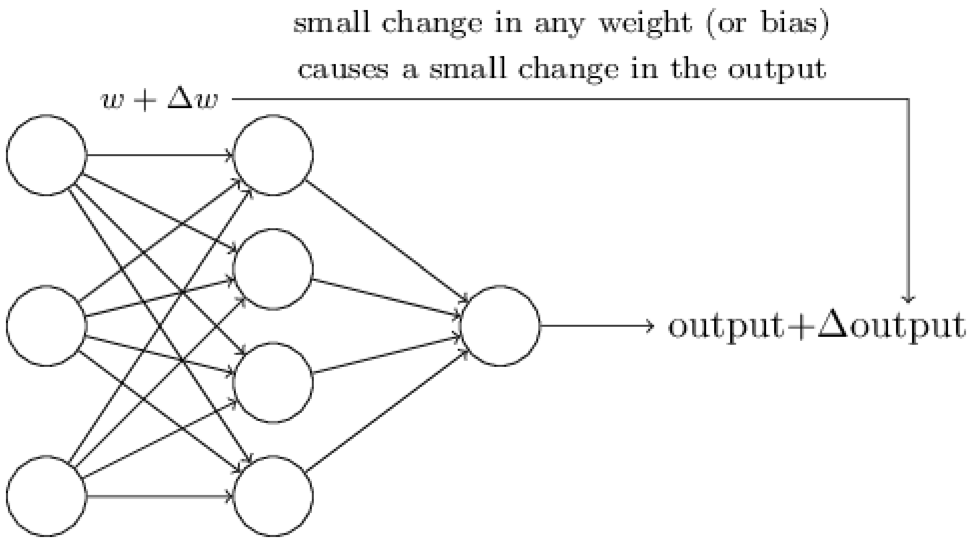
$$w \cdot x \equiv \sum_j w_j x_j$$

where w and x are vectors whose components are the weights and input



Pic. 5. Architecture of neural networks with 2 hidden layers

Perceptron and Sigmoid neuron (2)



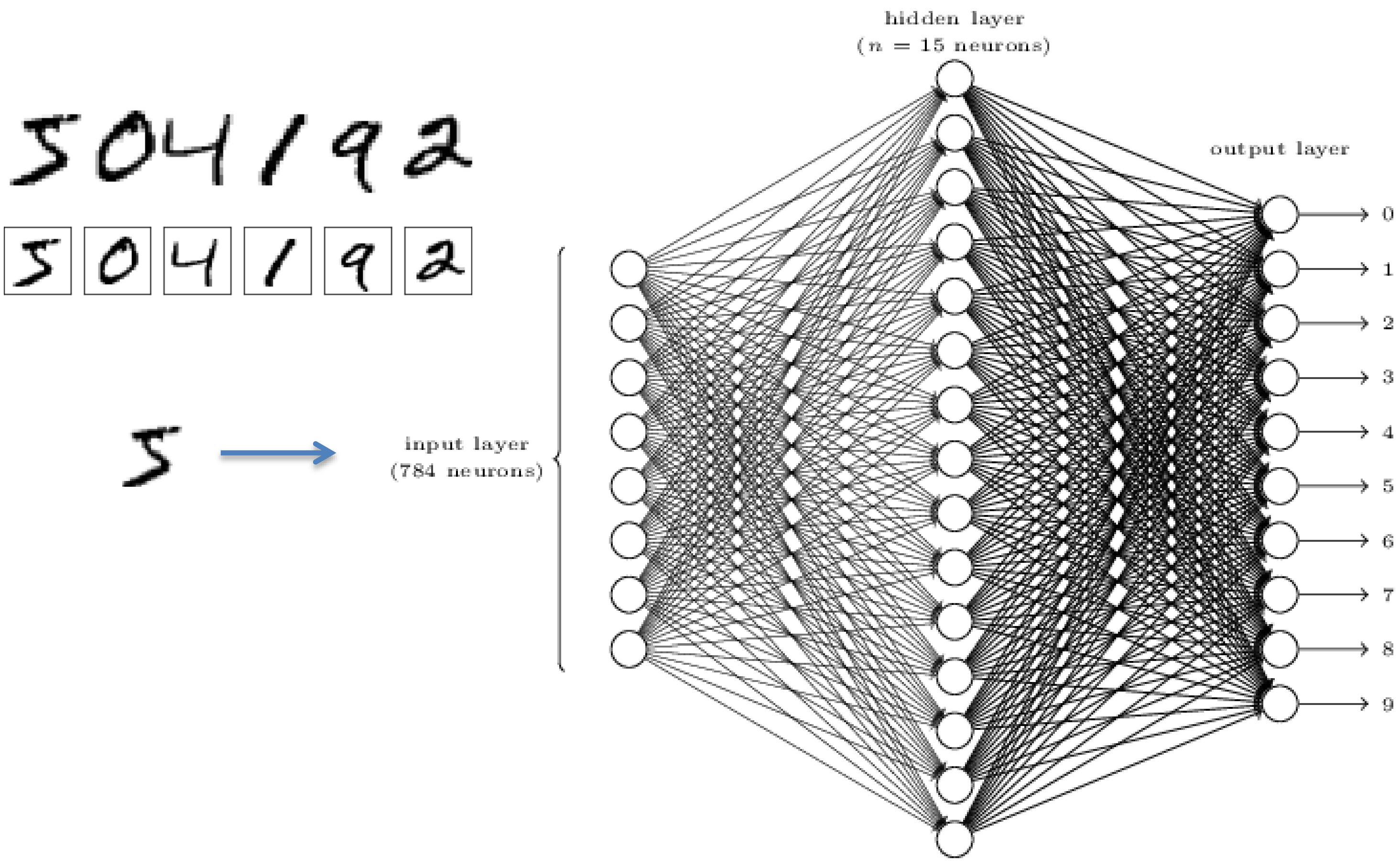
Pic. 6. Schematics of a perceptron

Sigmoid neurons are similar to perceptron's, but modified so that small changes in their weights and bias cause only a small change in their output.

$$\sigma(z) \equiv \sigma(w \cdot x + b) \equiv \frac{1}{1+e^{-z}} \equiv \frac{1}{1+\exp(-\sum_j w_j x_j - b)}$$

sigmoid function

Example: hand written digit recognition



Pic. 7. hand written digit recognition

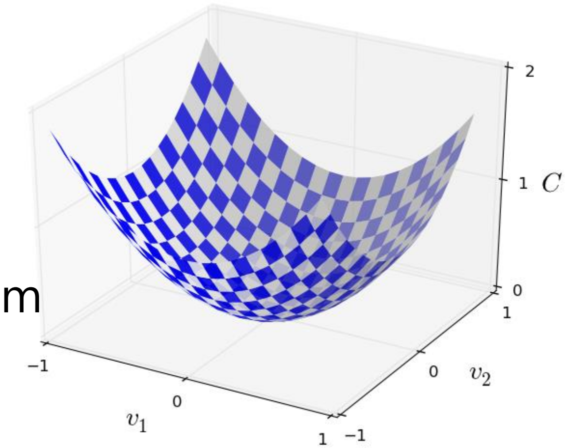
- We denote training input x as a $28 \times 28 = 784$ dimensional vector (gray scale pixels).
- Corresponding desired output by $y = y(x)$

Learning with gradient descent (1)

$$C(w,b) \equiv \frac{1}{2n}\sum_x \|y(x)-a\|^2$$

- where w denotes all weights in the network, b all the biases, n is the total number of training inputs. a is the vector of outputs from the network when x is input
- C is a cost function used to quantify how well algorithm can find weights and biases (i.e. *mean squared error*)
- Training a neural network means to find set of weights and biases which makes the quadratic cost function as small as possible

Learning with gradient descent (2)



- Lets assume that C is a function of two variables

v_1 and v_2

- Our goal to find where C achieves its global minimum

$$\Delta C \approx \frac{\partial C}{\partial v_1}\Delta v_1 + \frac{\partial C}{\partial v_2}\Delta v_2$$

$$\Delta v \equiv (\Delta v_1, \Delta v_2)^T, \qquad \nabla C \equiv \left(\frac{\partial C}{\partial v_1}, \frac{\partial C}{\partial v_2}\right)^T.$$

$$\Delta C \approx \nabla C \cdot \Delta v \longrightarrow \Delta v = -\eta \nabla C \longrightarrow \Delta C \approx -\eta \nabla C \cdot \nabla C = -\eta \|\nabla C\|^2$$

$$v \to v' = v - \eta \nabla C$$

$$w_k \to w'_k = w_k - \eta \frac{\partial C}{\partial w_k}, \qquad b_l \to b'_l - \eta \frac{\partial C}{\partial w_l}$$

Now we can replace vector of changes with weights and biases

Backpropagation algorithm (1)

- Backpropagation lets us to compute the partial derivatives ∂Cx/∂w and ∂Cx/∂b for a single training example.
- it shows how changing the weights and biases in a network changes the cost function (overall behavior of the network).

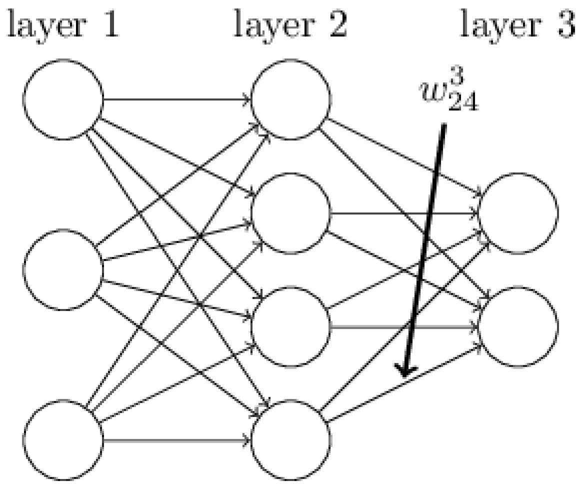
$$\delta^L = \nabla_a C \odot \sigma'(z^L) \qquad \text{(BP1)}$$

$$\delta^l = ((w^{l+1})^T \delta^{l+1}) \odot \sigma'(z^l) \qquad \text{(BP2)}$$

$$\frac{\partial C}{\partial b^l_j} = \delta^l_j \qquad \text{(BP3)}$$

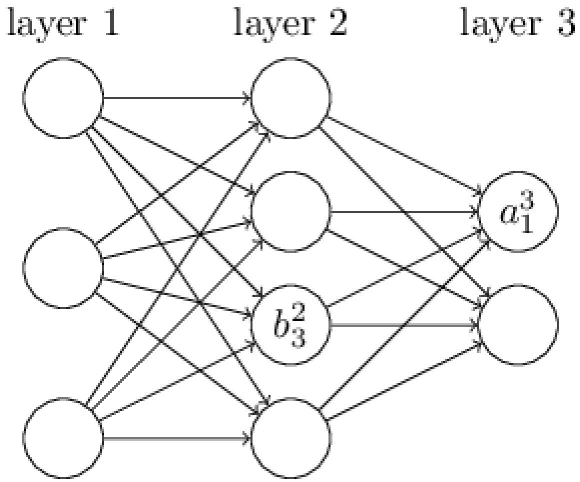
$$\frac{\partial C}{\partial w^l_{jk}} = a^{l-1}_k \delta^l_j \qquad \text{(BP4)}$$

Backpropagation algorithm (2)



Pic. 8. Schematics of a perceptron

Notations:
w^l_{jk} is the weight from the k^{th} neuron in the $(l-1)^{th}$ layer to the j^{th} neuron in the l^{th} layer



Pic. 9. Schematics of a perceptron

Notations:
we use b^l_j for the bias of the j^{th} neuron in the l^{th} layer.
And we use a^l_j for the activation of the j^{th} neuron in the l^{th} layer.

Backpropagation algorithm (3)

The activation a_j^l of the j^{th} neuron in the l^{th} layer is related to the activation in the $(l-1)^{th}$ layer by the equation:

$$a_j^l = \sigma\left(\sum_k w_{jk}^l a_k^{l-1} + b_j^l\right),$$

Sum of the all neurons in the $(l-1)^{th}$ layer

$$a^l = \sigma(w^l a^{l-1} + b^l)$$

In the matrix form

$$z^l \equiv w^l a^{l-1} + b^l$$

weighted input

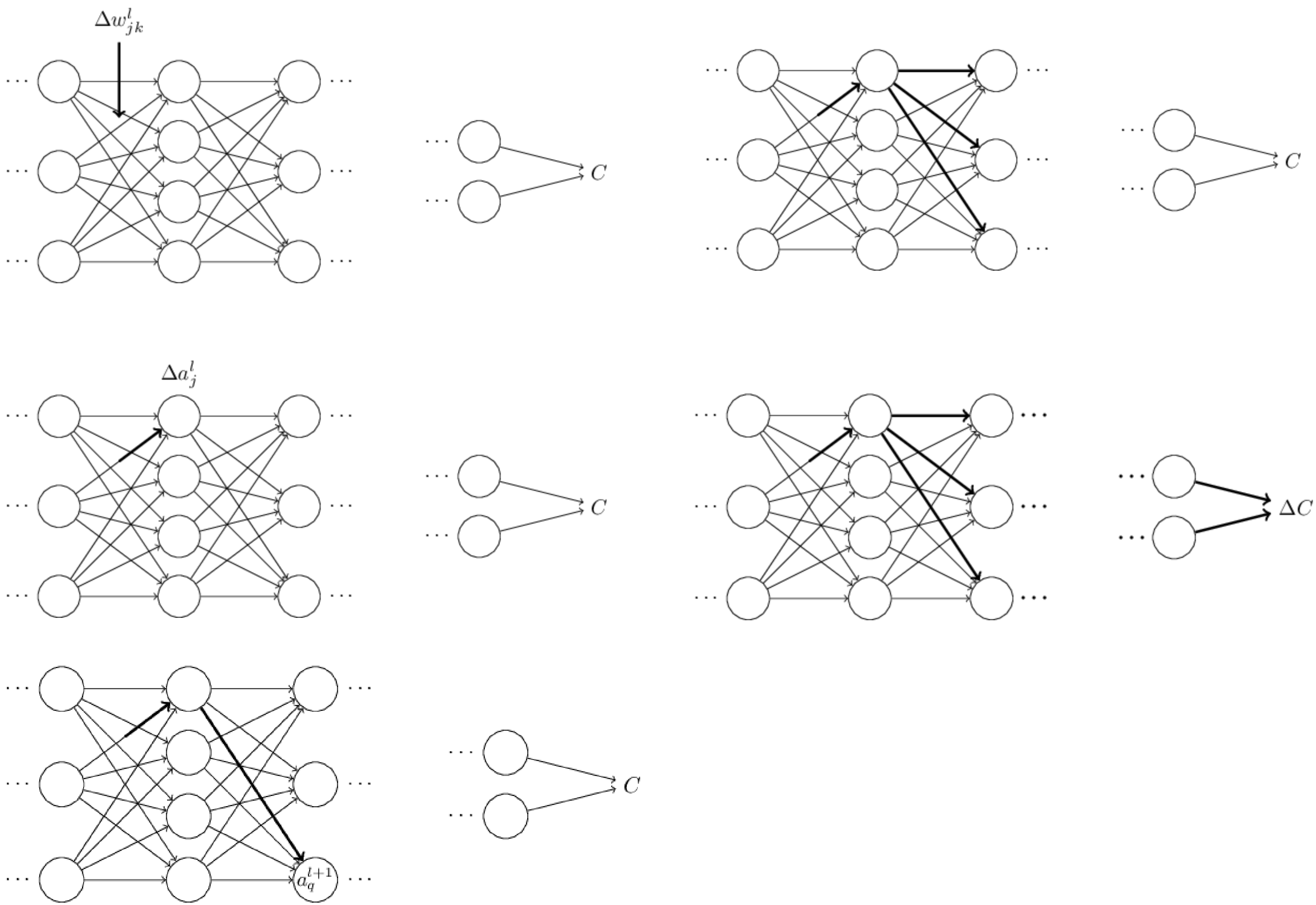
$$\delta_j^l \equiv \frac{\partial C}{\partial z_j^l}$$

- Error in the j^{th} neuron in the l^{th} layer

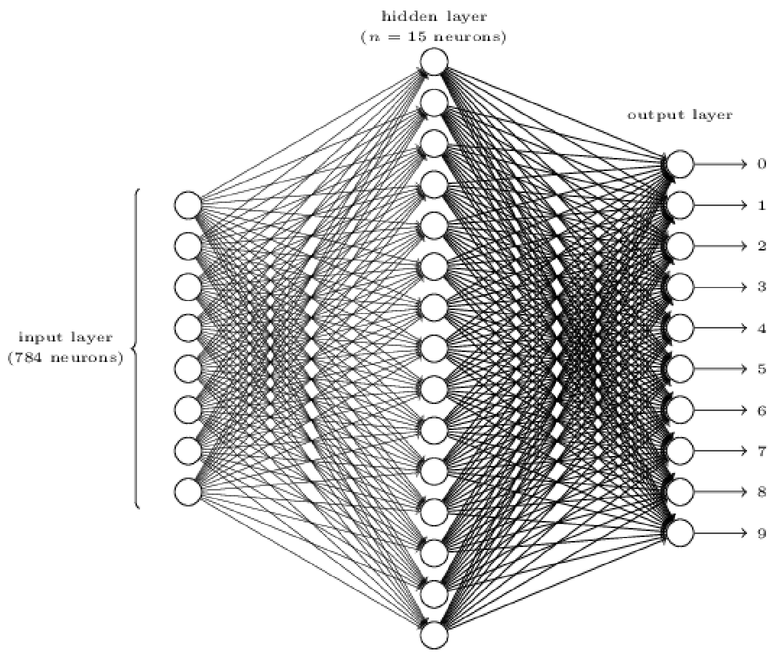
Backpropagation algorithm (4)

1. **Input** x**:** Set the corresponding activation a^1 for the input layer.
2. **Feedforward:** For each $l = 2, 3, \ldots, L$ compute $z^l = w^l a^{l-1} + b^l$ and $a^l = \sigma(z^l)$.
3. **Output error** δ^L **:** Compute the vector $\delta^L = \nabla_a C \odot \sigma'(z^L)$.
4. **Backpropagate the error:** For each $l = L-1, L-2, \ldots, 2$ compute $\delta^l = ((w^{l+1})^T \delta^{l+1}) \odot \sigma'(z^l)$.
5. **Output:** The gradient of the cost function is given by $\frac{\partial C}{\partial w^l_{jk}} = a^{l-1}_k \delta^l_j$ and $\frac{\partial C}{\partial b^l_j} = \delta^l_j$.

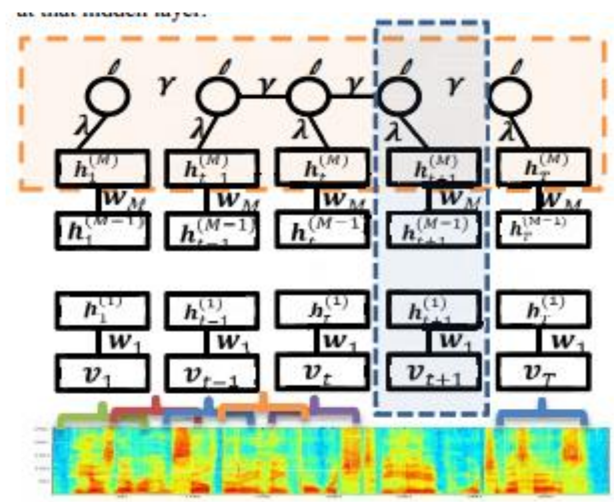
Backpropagation algorithm (5)



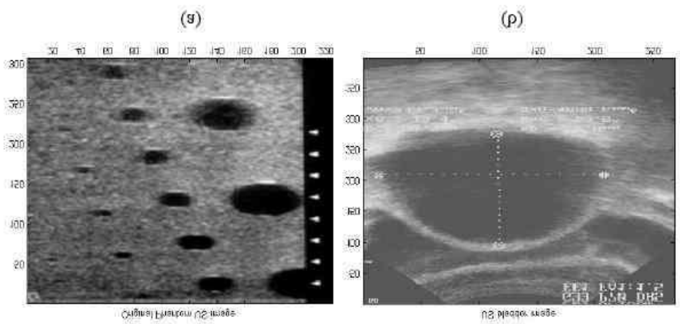
Applications



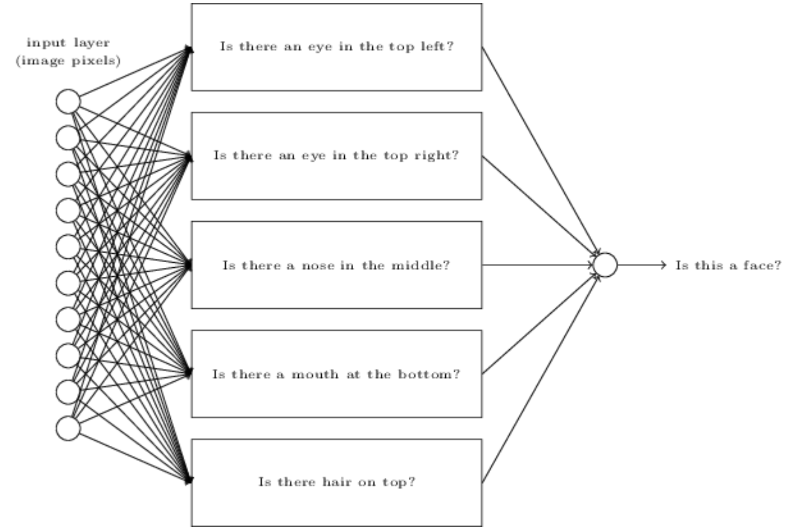
Pic. 10. Digit recognition using NN



Pic. 11. Speech recognition using NN



Pic. 12. ultrasound shape recognition



Pic. 13. Face recognition using NN

Q&A

Thank you