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Super-resolution and reconstruction of sparse sub-wavelength images

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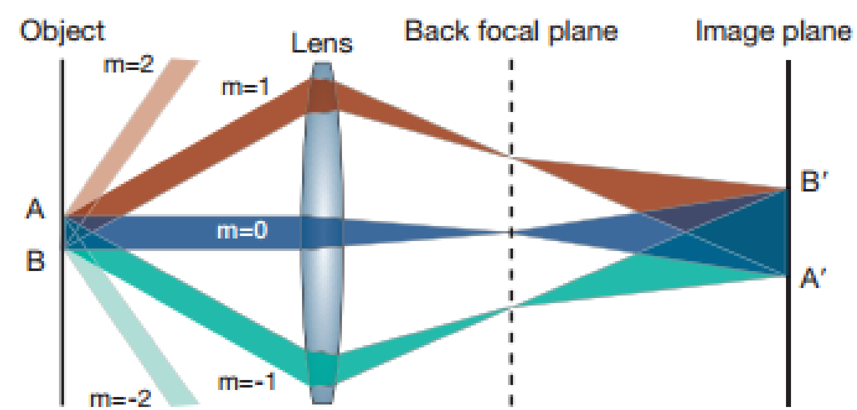
Background

- Observations of sub-wavelength structures with microscopes are difficult because of diffraction limit defined by Ernst Abbe (1873).
- $d = \dfrac{\lambda}{2(nSin\theta)}$, where (n Sin theta) is numerical aperture.
- Light with wavelength lambda travels in a medium with reflective index n and converging to a spot with angle theta will make a spot with diameter d.

- But the true limit of imaging arises from the optical wavelength λ and the best recoverable resolution is λ/2. This is because propagation of EM waves in bulk media behaves as low-pass filter, for distances larger then the wavelength, rendering spatial frequencies larger then 1/lambda evanescent.

[Abbe's theory of image formation]



The object AB (which acts as a diffraction grating) is illuminated with coherent light. The light is diffracted into orders, shown with different colors (some orders are collected by the lens). In the back focal plane, the orders are separated; in the image plane, diffracted orders interfere to form the image B′A′.

Background

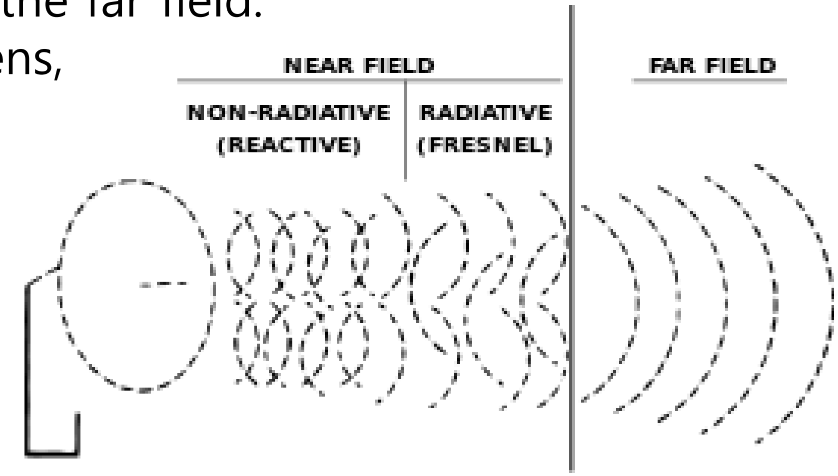
- Sub-wavelength imaging is nothing but capturing this evanescent waves.

- "evanescent waves are formed when waves traveling in a medium undergo total internal reflection at its boundary because they strike it at an angle greater than the so-called critical angle." We can compare this waves by analogy in acoustics as pressure gradients.

- These are waves that travel from a light source or an object to a lens, or the human eye.

- This can alternatively be studied as the far field.

In contrast, the superlens, or perfect lens, captures propagating light waves and waves that stay on top of the surface of an object, which, alternatively, can be studied as both the far field and the near field.



Introduction

- Many attempts to bypass the λ/2 limit on imaging
- "Hardware attempts" ex: superlens, hyperlens, Near-Field Optical Scanning Microscope
  - Not real-time imaging, scanning required
  - Requirement to fabricate on nanometer precision

- "Theoretical approaches" ex: bandwidth extrapolation
- Taylor expansion
- Image is described by spatially defined function, if analytical function known in an arbitrary small region of the far field, then entire function can be found by means of analytic continuation. It allows us theoretically recover sub-wavelength information.
  - Extremely sensitive to noise in the measured data

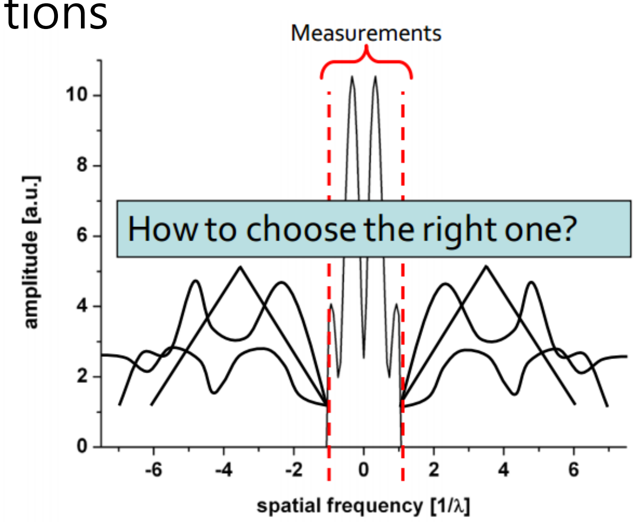
Introduction

- Concept presented how sub-wavelength information could be recovered from far-field of an optical image, overcoming loss of information embedded in decaying evanescent waves
- The purpose is to recover information in spatial frequencies that were cut off by diffraction limit which acts as a low-pass filter
- Requirement is that the image is known to be sparse
- Sub-wavelength imaging reformulated as sparse sampling problem
- Extended version of basis-pursuit used

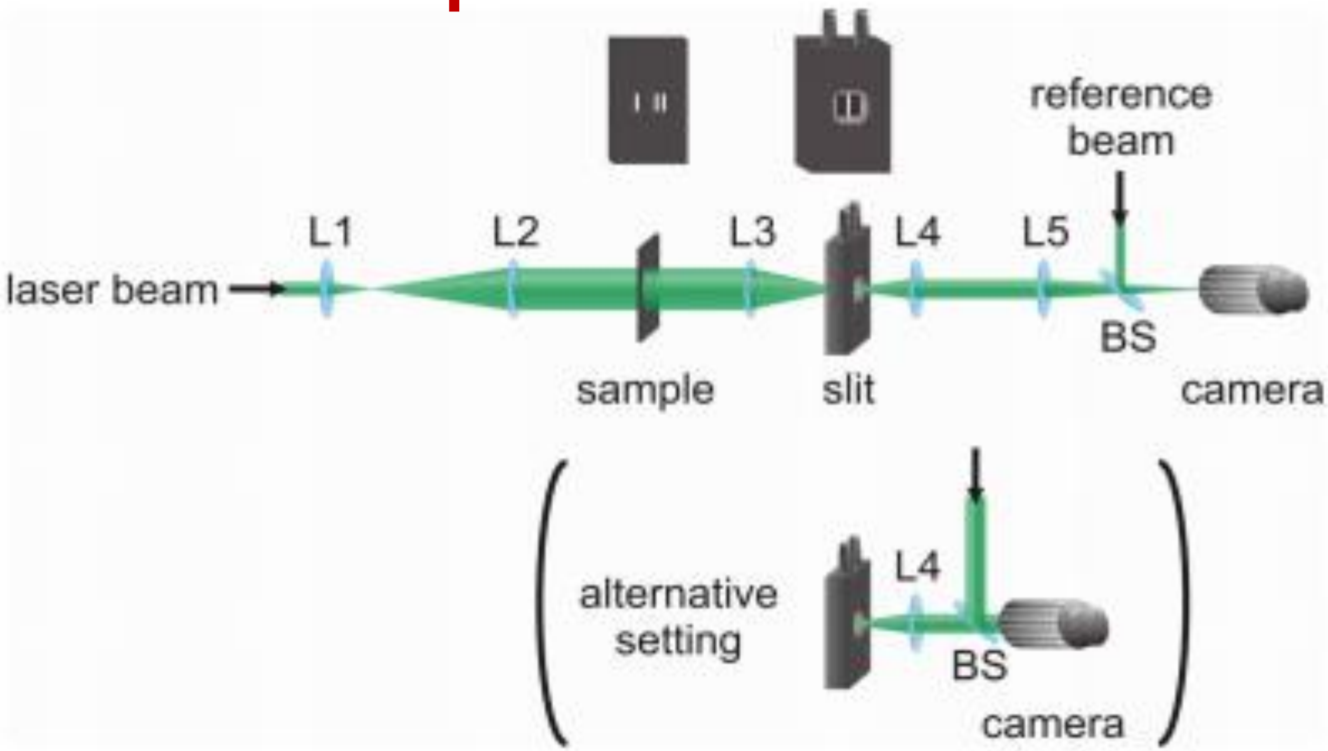
- So why extrapolation methods fail?
- They are not robust to noise in measured data.
- The noise is extremely uncorrelated, hence it is uniformly distributed on the basis functions. All extrapolation methods fail when projection on basis function comparable to noise, which introduces large errors.

Introduction

- If information is sparse in some domain then it is possible to find some a proper basis where we could separate further into two subspaces:
- where projections of the measured data much larger then the noise
- and second where the projection are very small and can be set to zero without losing information.
- CS identifies subspace where projection are large therefore CS method does not suffer from noise.
- To do that CS uses knowledge that signal is sparse, which implies that information could be represented in a compact way in some basis spanning only a subspace of possible solutions

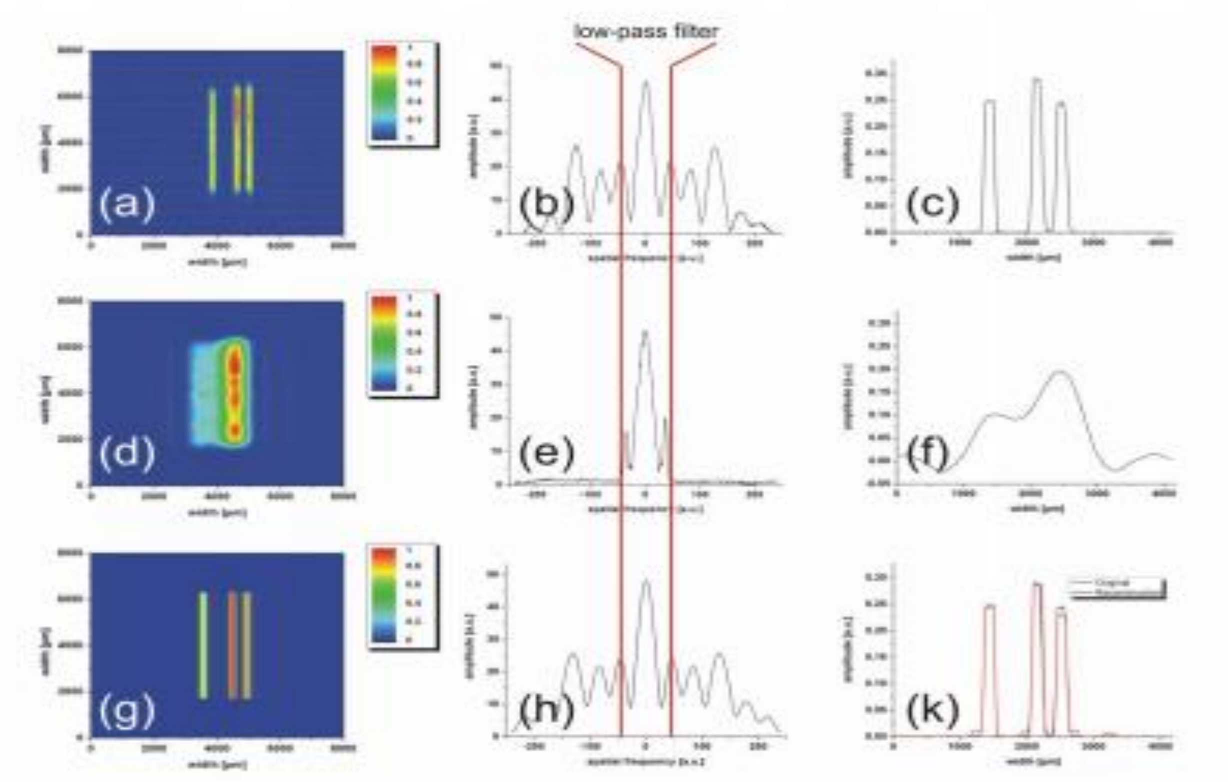


Experimental setup



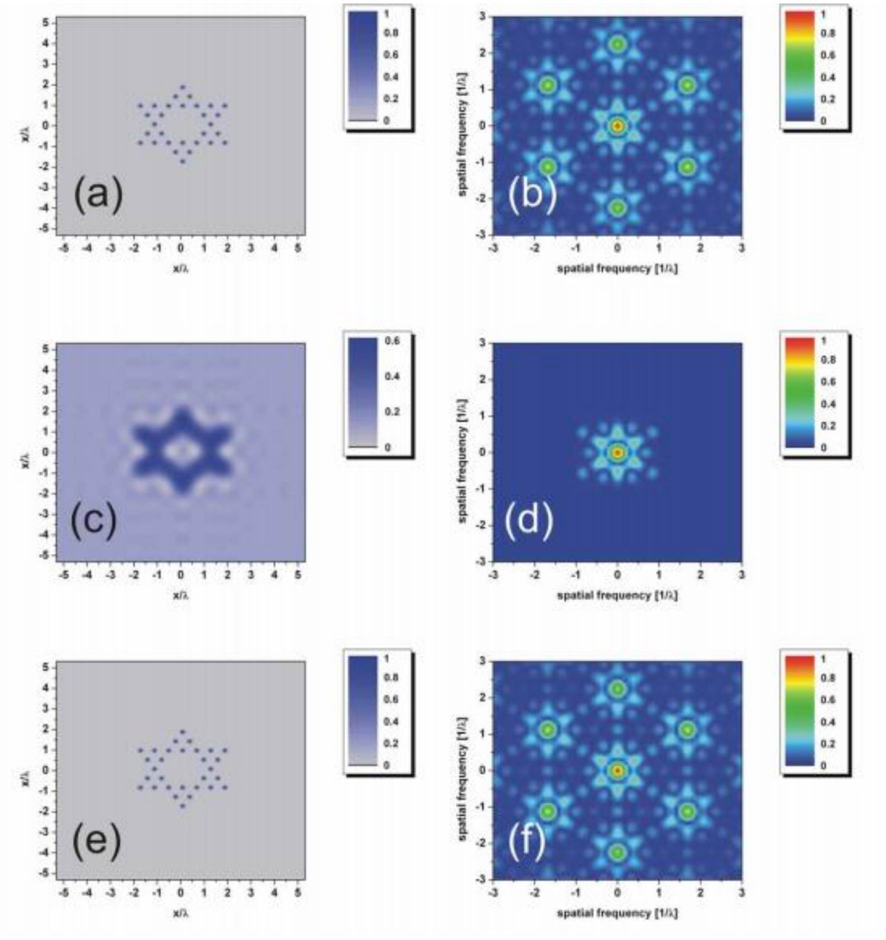
The laser beam is collimated using lenses L1 and L2, before the sample is illuminated. The signal then Fourier transformed using lens L3, low-pass filtered by the slit and again Fourier transformed into the real plane by L4. additional Fourier transform lens L5 and then image superimposed using beam splitter.

Results



Experimental proof reconstruction of amplitude information
(a) original information of three vertical stripes, (b) Fourier spectrum,
(c) horizontal cross-section of the amplitude,
(d,e,f) using optical slit the signal is low pass filtered and yielding a highly blurred image.
(g,h,k) Reconstruction using CS techniques

Results



(a,b) the original image of an arrangements of circles and its respective Fourier transform. (c,d) after some propagation distance all spatial frequencies above 1/λ are lost so that actual image is blurred and cannot be resolved.
(e,f) Applying CS algorithm reveals the underlying sub-wavelength structure in real space because Fourier spectrum fully resolved.

Basis Pursuit

Solve the (convex) optimization problem:

$$\min_{x} \|x\|_1 \text{ subject to } \|Ax - y\|_2 < \varepsilon$$

Find the sparsest x that is consistent with measurements

x: unknown image
y: measured image
A: Low-pass filter + sparsity basis
ε: Noise parameter

$$\|x\|_1 = \sum_i |x_i|$$

The requirement on L1 norm is to promote sparsity

Unique sparse solution

$$y = Wd_1 = Wd_2 \rightarrow W(d_1 - d_2) = Wz = 0$$

Triangle inequality

$$\|d_1 - d_2\|_0 \leq \|d_1\|_0 + \|d_2\|_0 \rightarrow \|z\|_0 \leq S_1 + S_2$$

If every S1+S2 columns of W are linearly independent then

$$Wz = 0 \rightarrow z = 0 \forall z \rightarrow d_1 = d_2$$

If $\|d\|0 \leq \frac{1}{2}(1 + \frac{1}{\mu(W)})$, then there is a unique sparse solution

Thank you