Title: Some fundamental properties of speckle.

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Published in: J. Opt. Soc. Am., Vol. 66, No. 11, November 1976.

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Abstract: A probabilistic modeling for speckle pattern is introduced. Ways to suppress the speckle pattern is also presented.

I. Introduction

Speckle

The vast majority of surfaces, synthetic or natural, are extremely rough on the scale of an optical wavelength. Under illumination by coherent light, the wave reflected from such a surface consists of contributions from many independent scattering areas. <u>Interference of these de-phased but coherent wavelets</u> results in the granular pattern we know as speckle.

Note that if the observation point is moved, the **path length**s traveled by the scattered components change, and a **new and independent valued of intensity** may results from the interference process.

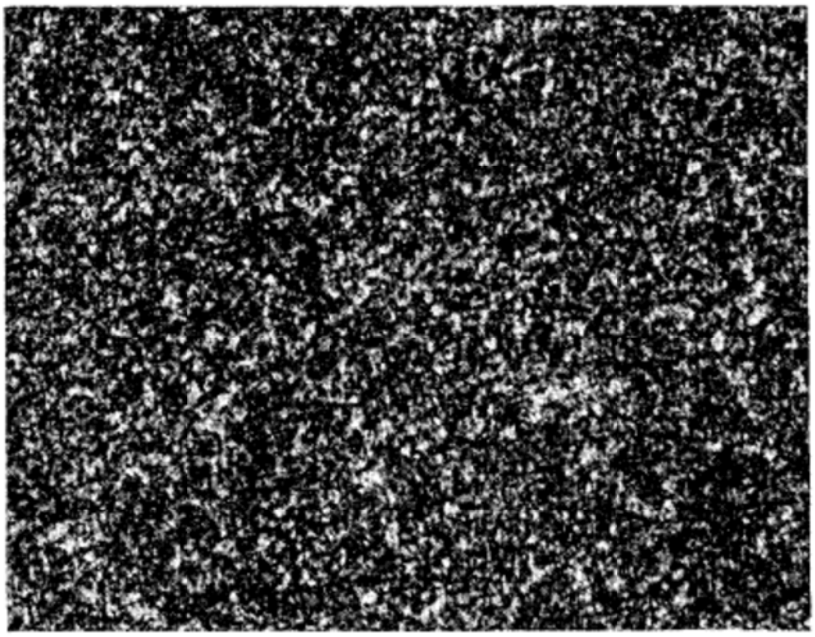

FIG. 1. Typical speckle pattern.

II. Speckle as a random-walk phenomenon

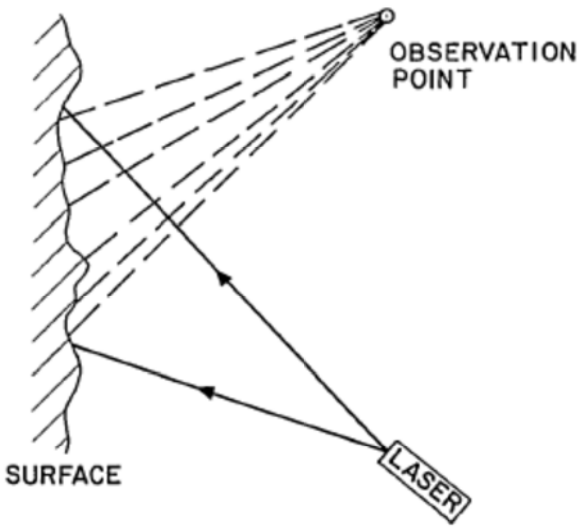

FIG. 2. Speckle formation in the free-space geometry.

The signal:

$$u(x, y, z; t) = A(x, y, z)\exp(i2\pi\nu t)$$

where ν is the optical frequency and $A(x, y, z)$ is a complex phasor amplitude

$$A(x, y, z) = A(x, y, z)\exp(i\theta(x, y, z)) .$$

The irradiance:

The directly observable quantity is the irradiance at (*x*,*y*,*z*), which is given by

$$I(x, y, z) = \lim_{T\to\infty}\int_{-T/2}^{T/2}\left|u(x, y, z; t)\right|^2 dt = \left|A(x, y, z)\right|^2 .$$

The complex amplitude of the field at (*x*,*y*,*z*) may be regarded as resulting from the sum of contributions from many elementary scattering areas on the rough surface. Thus the phasor amplitude of the field can be represented by

$$A(x,y,z)=\sum_{k=1}^{N}a_k=\sum_{k=1}^{N}|a_k|\exp(i\phi_k)$$

where $|a_k|$ and ϕ_k represent the amplitude and phase of the contribution form the *k*th scattering area and *N* is the total number of such contributions.

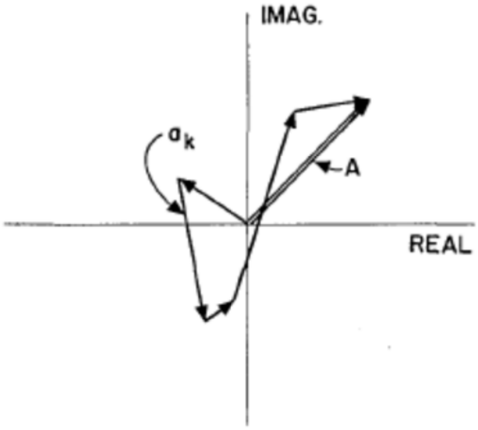

FIG. 4. Random walk in the complex plane.

Two important assumptions

(i) The amplitude and the phase of the *k*th elementary phasor are statistically independent of each other and of the amplitudes and phases of all other elementary phasors.

(ii) The phases of the elementary contributions are equally likely to like anywhere in the primary interval $(-\pi,\pi)$.

With these two assumptions, the similarity of our problem to the classical random walk in a plane becomes complete.

Provided the number *N* of elementary contributions is large, we find (a) the real and imaginary parts of the complex filed at (*x*,*y*,*z*) are independent, zero mean, identically distributed Gaussian random variables, and (b) the irradiance *I* obeys negative exponential statistics, i.e., its pdf is of the form

$$p(I)=\begin{cases}(1/\bar{I})\exp(-I/\bar{I}), & I\geq 0\\ 0, & \text{otherwise}\end{cases}$$

where $\bar{I}$ is the mean irradiance.

A fundamental important characteristic of the negative exponential distribution is that its standard deviation precisely equals its mean. Thus, the contrast of a polarized speckle pattern, as defined by

$$C = \sigma_I / \bar{I}$$

is always unity. Herein lies the reason for the subjective impression that the variations of irradiance in a typical speckle pattern are indeed a significant fraction of the mean.

III. Suppression of speckle

The sum of *M* identically distributed, real-valued, uncorrelated random variables has a mean value which is *M* times the mean of any one component, and a standard deviation which is $\sqrt{M}$ times the standard deviation of one component. Thus, if we add *M* uncorrelated speckle patterns on an irradiance basis, the contrast of the resultant speckle pattern is reduced in accord with the law

$$C = \sigma_I / \bar{I} = 1/\sqrt{M} .$$

Uncorrelated speckle patterns can be obtained from a given object by means of time, space, frequency, or polarization diversity.

Pure spatial diversity occurs, for example, when a reflecting surface is illuminated by several different lasers from different angles. If the **angles of illumination** are sufficiently separated, the path length delays experienced by each of the reflected beams will be different enough to generate uncorrelated speckle patterns.

A second way of changing optical paths (in wave lengths) traveled by a reflected wave is to change the optical **frequency of the illuminations**. If the separation of these frequency components is sufficiently great, *M* uncorrelated speckle patterns will result, with addition on an irradiance basis.

Ex) In a reflection geometry, with angles of incidence and reflection near normal to the surface, the separation required to produce uncorrelated speckle is approximately

$$\Delta \nu \cong c / 2\sigma_z$$

where c is the light velocity and σ_z is the standard deviation of the surface height fluctuations.

Time diversity: If a transparency object is illuminated through a diffuser, then motion of that diffuser results in a continuous changing of the speckle pattern in the image. A **time exposure** in the image plane then results in the addition, on an intensity basis, of a number of uncorrelated speckle patterns, thus suppressing the contrast of the detected speckle pattern.

IV. Discussion

The speckle phenomenon happens in turbid lens imaging (TLI) systems []. On the one hand, it is used as useful information. On the other hand, it is regarded as a noise that we try to suppress.

TLI is one new technique that increases the resolution of an imaging system beyond the physical limitation given by lenses. In TLI, a turbid medium is inserted between a sample and an objective lens. The turbid medium has many small particles in it, and the wavelets in the sample beam experience multiple scattering inside the medium. This scattering is good in a sense that higher angle mode waves, which usually go out of the detector, experience the scattering and may be redirected into the detector. This makes it possible to collect higher mode wave information which tells us the details of the sample. Also, the scattering is bad in a sense that it scrambles the sample image in the detector. But if we know i) the way how the medium scrambles images and ii) the way how to recover it using “i)”, it is no longer a problem. Here we can obtain “i)” because the input and output relationship of the turbid medium can be measured. “ii)” is also known as it is just an inverse problem given linear system model and the system response.

The sample beam is a weighted sum of waves with many spatial frequencies, written as follows,

$$x(t) = \sum_l x_l \exp(i2\pi f_l t)$$

where f_l is a spatial frequency and t is a spatial index.

We measure response of the optical system for a wave $\exp(i2\pi f_l t)$

$$T_l(t) = T\left(\exp(i2\pi f_l t)\right).$$

Assuming TLI system is linear time-invariant (LTI), the response to the input x can be written

$$y(t) = T(x(t)) = T\left(\sum_l x_l \exp(i2\pi f_l t)\right) = x_l \sum_l T_l \ .$$

Here, the index t and l actually have dimension two. Then, the response can be represented by a linear system model as follows

$$\mathbf{y} = \mathbf{T}\mathbf{x} \, .$$

Here, we can estimate **x** with given **y** and **T**, where **y** and **T** are measured in the experiments. Here, note that a column of **T** is a speckle pattern and the columns in **T** are uncorrelated to each other for their path lengths differ enough; if the spatial frequency gap is more than a certain value, it is true. So, we can see that the matrix **T** is well conditioned for recovery.

We also note that the **y** is in fact is a noisy measurement where speckle pattern is included in the noise. For the suppression of the noise, we can use multiple uncorrelated measurement as it is presented in the talk.