INFONET Seminar Application Group

Frequency Domain Compressive Sensing for Ultrasound Imaging

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Introduction

Conventional Ultrasound imaging systems rely on Shannon-Nyquist theorem. Often US devices use a sampling rate that is at least four times the central frequency.

Consequently large amount of data imply problems in:

- Real-time imaging (especially 3D)
- Data-transfer
- Grows of machinery size

Compressive sensing allows reduce volume of data directly acquiring compressed signal. Recently CS framework was adopted to ultrasound imaging [10-15] (all paper except one are conference papers), Ultrasound Doppler [16-17] or Photoacoustic [18]

Sampling a signal

Sampling can be summarized by measuring linear combination of an analog signal

$$y_k = \langle \varphi_k, f \rangle, \quad \text{for } k = 1, \ldots, m, \quad (1)$$

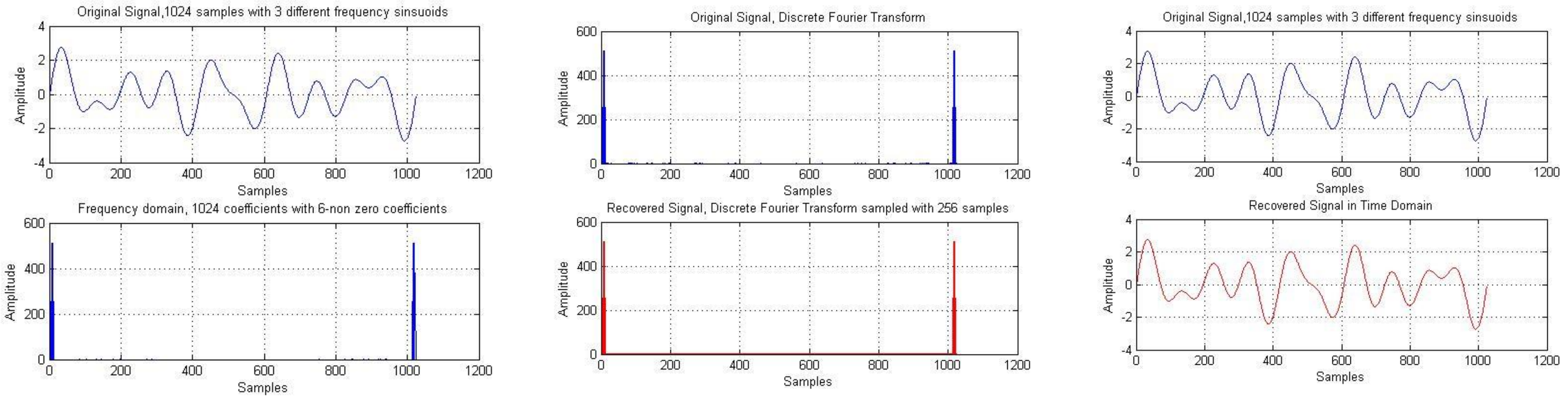
Where y_k are the measurements, φ_k are sampling vectors, and m is the number o f measurements. The most common sampling protocol consist of vectors of Dirac's at equal time. The measurements then simple discretization of $f(t)$.

In CS, the number of measurements is m << n. However, when the number of mea surements is smaller then the signal size, then it is ill-posed inverse problem. If Φ is a matrix of size $m \times n$, concatenating the sampling vectors φ_k, then $y = \Phi f$. T he signal $\hat{f}$ corresponding to the measurements y has infinity many solutions.

CS shows that $\hat{f}$ can be reconstructed if it has sparse representation in a given basi s and that the measurements are incoherent with that basis.

Concept of Sparsity

Sparsity is the idea that signal may have a concise representation in a given basis.



Hence a dense signal in the time domain can be coded with only a few samples. Mathe matically it translates as follows:

$$f(t) = \sum_{i=1}^{n} x_i \psi_i(t) \quad (2)$$

Where $f(t)$ is the original signal, x_i are the coefficients of the signal in sparse basis, and $\psi i(t)$ is an orthonormal basis. The S largest coefficients x_i are noted x_S, and t he corresponding signal $f_S(t)$ If $f(t)$ is sparse in the basis Ψ composed of the vect ors ψ_i, then $f = \Psi x$ and the error $\| f - f_S \|_2$ is small.

Concept of Sparsity

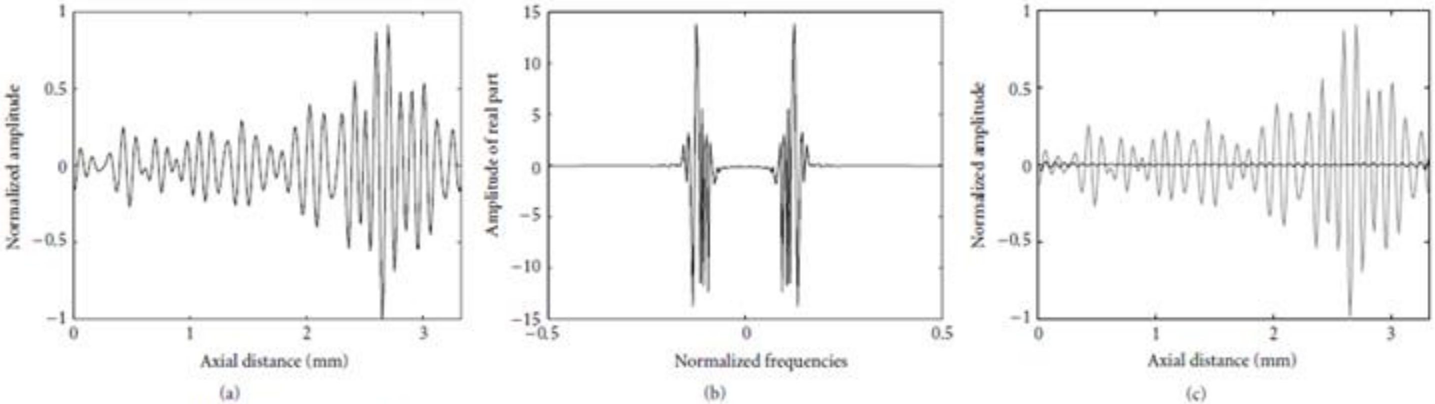


Figure 1: (a) A full US RF signal and (b) its sparse representation via Fourier transform. Most of the coefficients are equal or close to zero. (c) Compressed US RF signal (gray), corresponding to 30% of the largest Fourier coefficients, the rest of them being set to zero. The difference between the full and compressed US RF signal (black) is minimal.

- Sparsity therefore leads to compressive nature of signal: if signal has sparse rep resentation, then the information coding that signal can be compressed on a fe w coefficients.
- However directly acquire only significant coefficients without knowing their po sitions is impossible.
- CS overcomes this issue via an incoherent sampling.

Incoherent Sampling

The term Incoherent sampling conveys the idea that the sampling protocol φ_k in (1) has to be as little correlated as possible with sparse representation ψ_i in (2).

$$y_k = \langle \varphi_k, f \rangle, \quad \text{for } k = 1, \ldots, m, \quad (1) \qquad f(t) = \sum_{i=1}^{n} x_i \psi_i(t) \quad (2)$$

The mathematical definition of incoherence is:

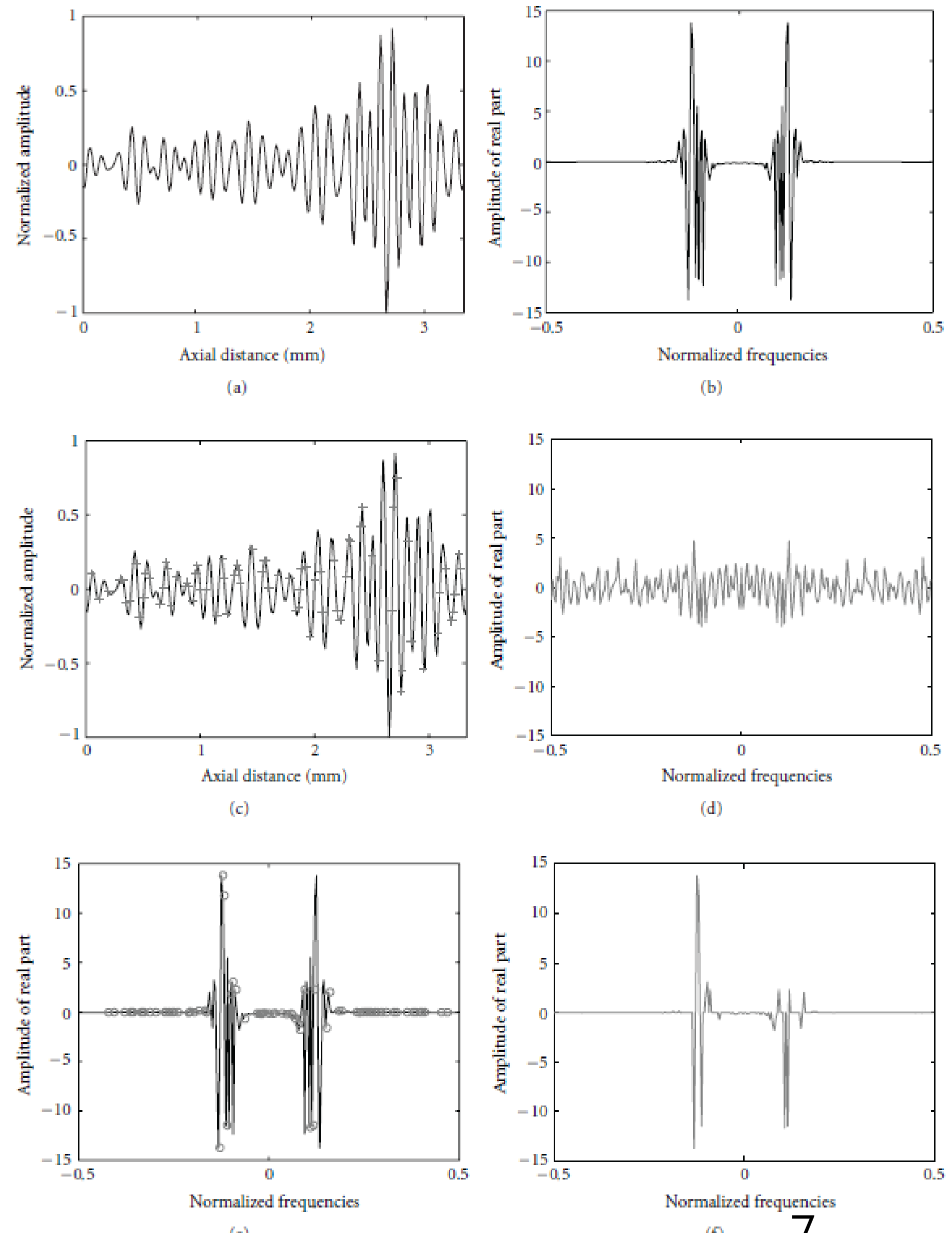
$$\mu(\Phi, \Psi) = \sqrt{n} \max_{1 \le k, j \le n} \left| \langle \phi_k, \psi j \rangle \right|, \quad (3)$$

Where Φ is the sampling basis and Ψ is the sparsifying basis. If the two bases are strongly correlated, then μ will be close to $\sqrt{n}$, and if the are not correlated, then it will be close 1.

- CS requires incoherence .
- If the sampling basis completely random , then it will be maximally incoherent with sparsifying fixed basis.

Incoherent Sampling

- (a) A full US RF signal
- (b) its sparse representation FT
- When sampling is incoherent with the sparsifying basis then the meas urements in that basis are dense (c, d)
- When the sampling basis and the s parsifying basis are coherent (e,f), t he measurements in the sparsifying basis are themselves sparse. There i s significant information missing
- Knowing sparsifying basis and incoh erent measurements in cs make it p ossible to reconstruct original signal using optimization.



Signal Reconstruction through Optimization

Reconstruction performed via convex optimization:

$$\min \|\hat{x}\|_1 \quad \text{subject to } y = \Phi f = \Phi\Psi\hat{x}, \quad (4)$$

$\hat{x}$ is the reconstructed sparse signal.

The optimization searches amongst all the signals that verify the measurements y , the one with the smallest l1 norm, that is the sparsest.
Optimization removes the interferences caused incoherent under sampling from t he sparse representation of the measurements.

Sampling Protocols in Ultrasound

The data acquisition in US is performed in spatial domain. The sampling basis has to be in coherent with sparsifying basis. There are several sampling protocols adapted and incohe rent with sparsifying basis. Basically they all consist in taking samples at random locations (taking samples at specific times on RF signal or taking some RF lines at specific locations)

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- Eight different protocols are proposed:

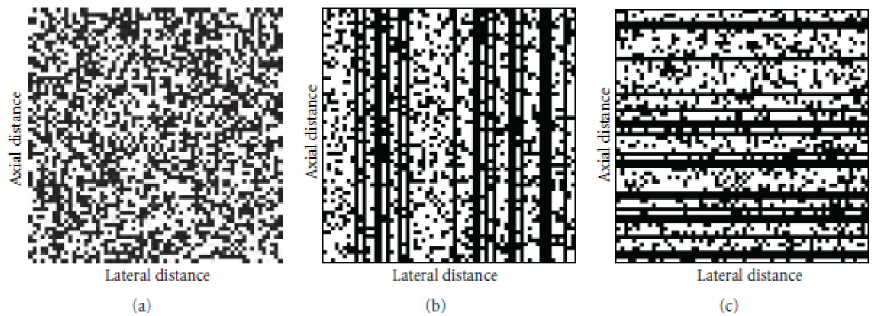
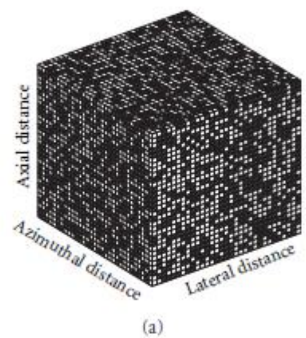


Figure 6: Sampling masks Φ_1 (a), Φ_2 (b), and Φ_3 (c) adapted to a spatial sampling of the US images. The white pixels correspond to the samples used for CS. The proportion of samples here is 50% of the original image.



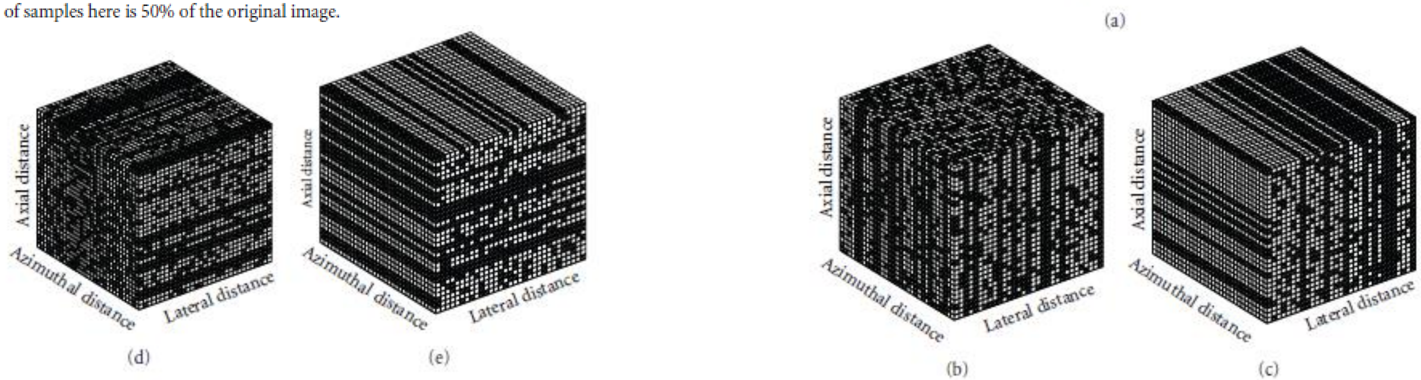


Figure 7: Sampling masks Θ_1 (a), Θ_2 (b), Θ_3 (c), Θ_4 (d), and Θ_5 (e) adapted to a spatial sampling of the 3D US volumes. The white pixels correspond to the samples used for CS. The proportion of samples here is 50% of the original volume.

Reconstruction of Ultrasound Images

The acquisition consist in taking samples of the RF signals. This sampling protocol is simila r to a basis of Diracs. Basis incoherent with Diracs and where US images are sparse is need ed. Fourier basis is maximally incoherent with Diracs and because the US image k-space is sufficiently sparce. The function to minimize is

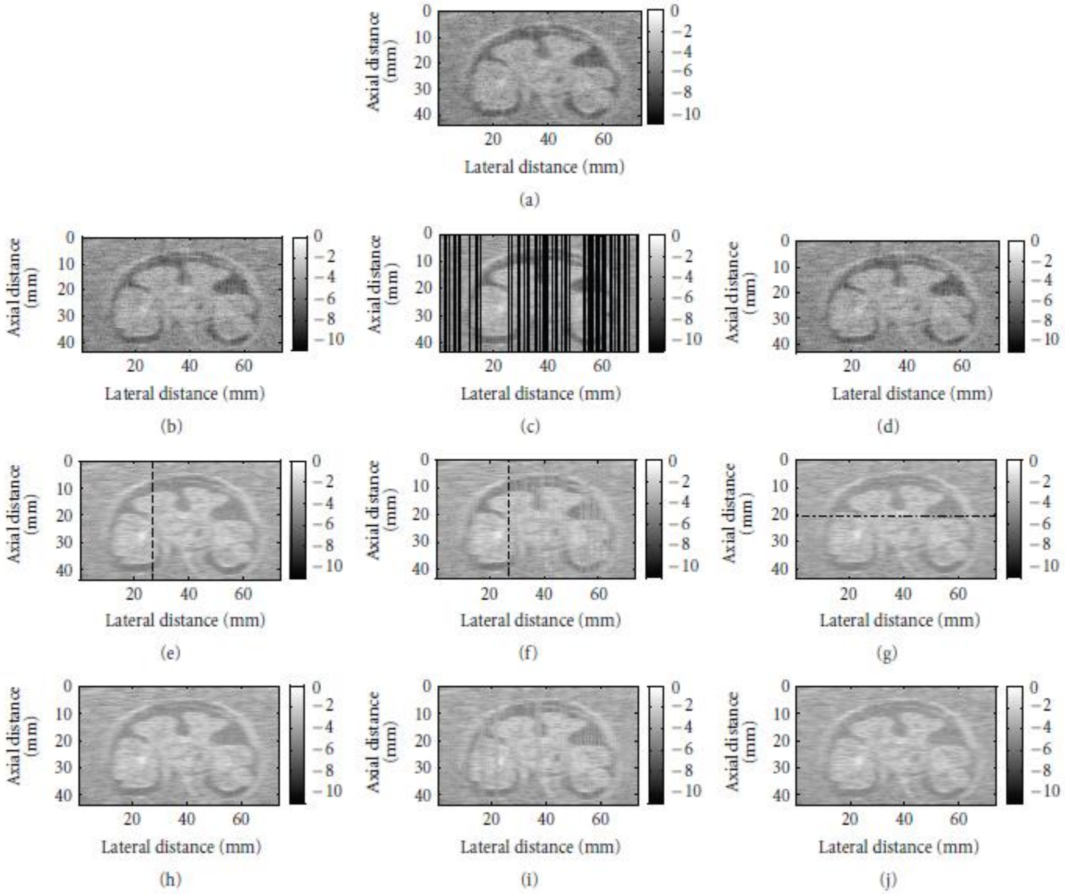
$$\arg\min_{M} \|AM - y\|_2 + \lambda |M|_1 , \quad (8)$$

where M is the k-space of the US RF image m (M= Fm), and A is the sampling scheme (A = ΦF), y are the RF US image measurements and λ is a coefficient weighting for sparsi ty.

Results on a 2D Simulation Image

The proposed CS method was used to reconstruct the k-space of an RF image simulated using the Field ii simulation program. Map of kidney used with linear transducer. Sampli ng frequency 20 MHz.

The sampling Φ_1, Φ_2, Φ_3 were studied to compare CS recons truction using a fixed optimal set to 0.005 and the reweight ed λ minimization ℓ_1



Results on a 2D Simulation Image

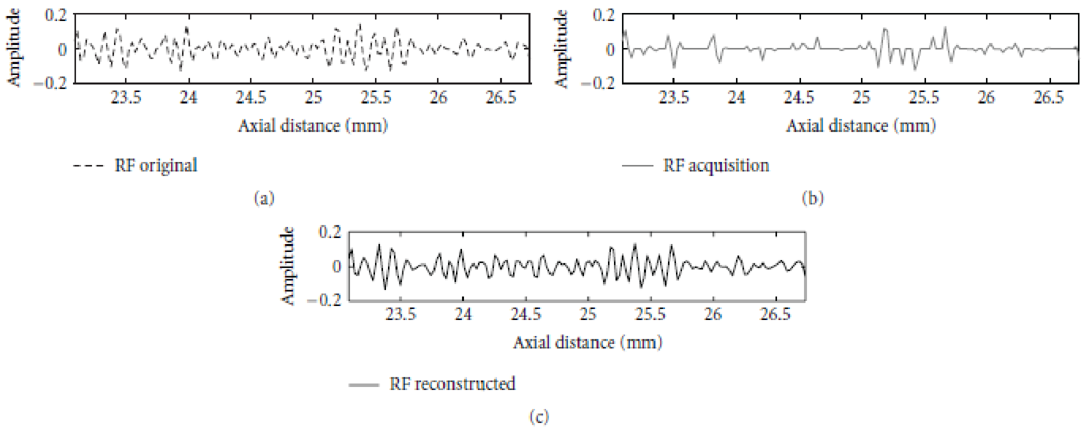


FIGURE 9: An example of a local region of an RF line after CS reconstruction using Φ_1 sampling pattern, corresponding to the dotted line in Figure 8(e).

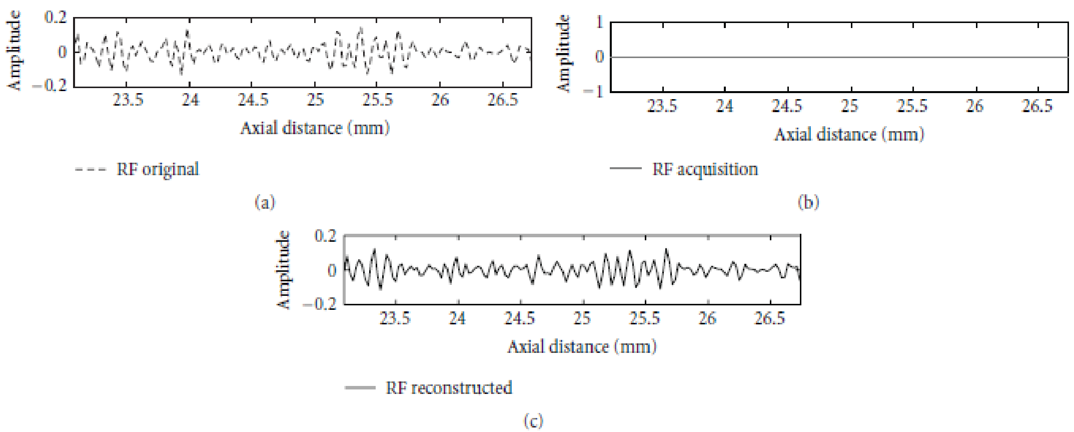


FIGURE 10: An example of a local region of an RF line after CS reconstruction using Φ_2 sampling mask, corresponding to the dotted line in Figure 8(f). This line was not sampled at all.

Results on a 2D Simulation Image

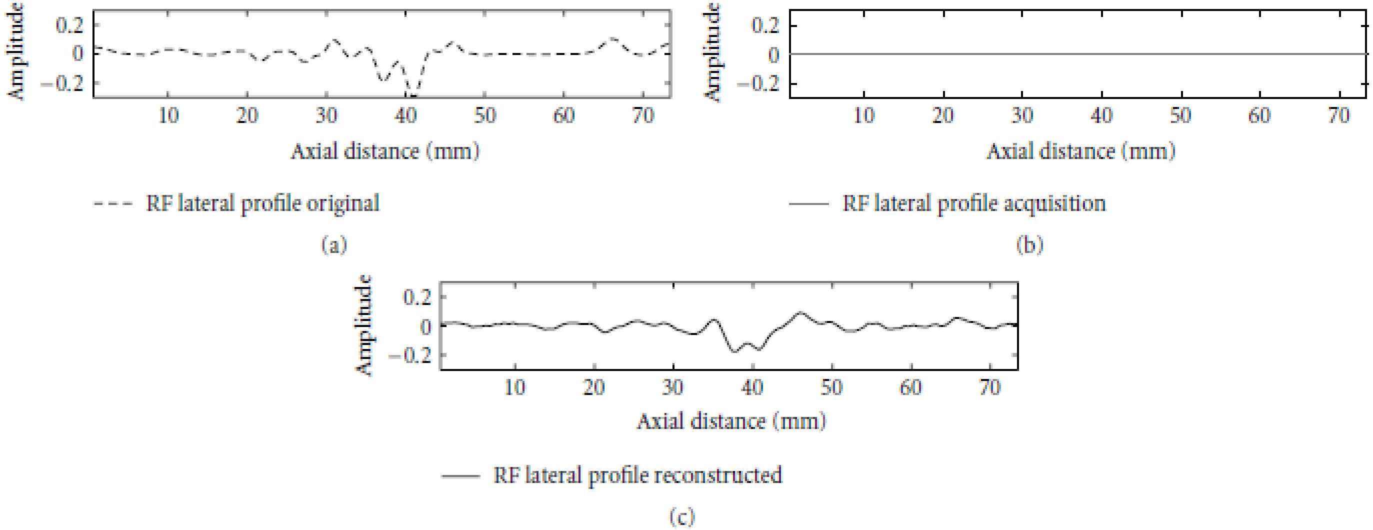


Figure 11: An example of a lateral profile of aN RF image after CS reconstruction using Φ_3 sampling mask, corresponding to the dotted line in Figure 8(g). This lateral profile was not sampled at all.

Table 1: NRMSE between the CS-reconstructed RF US image and the original simulated image of a kidney for different sampling ratios and patterns.

		Φ_1	Φ_2	Φ_3
Classic ℓ_1 minimization ($\lambda = 0.005$)	25%	0.48	0.58	0.6
	33%	0.33	0.45	0.49
	50%	0.15	0.28	0.29
Reweighted ℓ_1 minimization	25%	0.51	0.56	0.64
	33%	0.36	0.47	0.49
	50%	0.17	0.24	0.3

Discussion and Conclusion

Sampling using high sampling rate is neither easy nor cost-effective in high frequency applications.
It was showed potential of CS to reduce data volume and wrap acquisition at the price of a reconstruction using the ℓ_1 norm.

Future work

- Better sparsity basis
- Investigation of several optimization routines
- The aim is to reach fastest and most reliable reconstruction from as little amount of data as possible.

Thank you