A Calibration for the Modulated Wideband Converter Using Sinusoids with Unknown Phases

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Abstract**—** Based on the compressed sensing theory, the sub-Nyquist samplers are proposed to acquire the wideband signals. Among them, although the modulated wideband converter (MWC) is theoretically well designed, a calibration is needed for the implementation. In this paper, we propose a calibration algorithm to estimate the practical system transfer sequentially injecting sinusoids with unknown phases. For the successful calibration, the unknown phases of the sinusoids are estimated in the proposed algorithm, which affect the calibration performance tremendously.

Keywords— calibration, modulated wideband converter (MWC), phase estimation, sub-Nyquist sampling

I. Introduction

The analog-to digital converter (ADC) is essential for the spectrum sensing in cognitive radio. However, the ADC encounters inefficiency when it deals with wideband signals whose carrier frequencies reach up to scores of Giga-Hertz, whereas the bandwidths of the signals are narrow compared to their Nyquist frequencies. For an efficient sampling of the wideband signals, based on the compressed sensing theory (CS) [1, 2], the channelized sub-Nyquist samplers are proposed as troubleshooters, such as multi-coset sampler [3, 4], random modulation pre-integration [5], modulated wideband converter (MWC) [6, 7], random demodulator [8], compressive multiplexer [9], and multirate sampler [10].

The modulated wideband converter (MWC) [6, 7] is one of the realizable sub-Nyquist samplers, which samples spectrally sparse wideband signals. The analog part of MWC receives the priory unknown multiband signal [6] and the signal goes through multiple channels. For each channel, the series of analog components consist of mixer, low-pass filter (LPF), and ADC. The signal is multiplied with pseudo random (PR) sequence at the mixer, and the mixed signal is then filtered by LPF. The ADC samples the signal at a sub-Nyquist rate. After the analog part, the sub-Nyquist samples then are digitally processed to reconstruct the Nyquist samples using sparse recovery algorithms such as orthogonal matching pursuit (OMP) [11], l_1-minimization [12], least absolute shrinkage and selection operator (LASSO) [13].

The theoretical studies on the MWC usually assume that the analog components are ideal and delays do not exist amongst analog components. However, when the MWC is implemented in real world, the assumptions are not valid. This invalid assumption causes model-mismatch problems leading to failure in the reconstruction of Nyquist samples. Thus, calibration techniques for remedying the problems are required for the successful reconstruction.

For the calibration of MWC, researchers have proposed schemes sequentially injecting sinusoids [14 - 17]. In [14], the authors have considered non-linearity of mixer and non-ideal response of filters. In [15], the authors have considered perturbations of ADC. From an assumption the perfect synchronization between the mixer and the ADC, they derive useful information and exploit it to the calibration. However, the perfect synchronization requires additional electronic parts, which might be complex and highly costs to be implemented for fast ADC. In [16], they have considered the delays in analog paths and difference among channel gains. In [17], the authors have considered a problem of non-ideal LPF. Consequently, all the literatures of [14-17] solve their own problems by injecting arbitrarily generated sinusoids with assuming that the phases of the sinusoids are perfectly controllable. However, the phases may be unknown in practice due to the unknown transient between the sinusoid generator and the front-end of the MWC, and the unknown phases again result in the failure in the reconstruction. For the more precise calibration, we need to estimate the phases of sinusoids injected for the calibration.

In this paper, we propose a calibration algorithm for the MWC sequentially injecting sinusoids with unknown phases. In addition to considering the implementation problems from the non-ideality of the analog parts, we consider that the phases of the sinusoids injected during the calibration are unknown and not consistent for every calibration. In the proposed algorithm, the unknown phases and the calibrated transfer function are estimated at the same time. The estimation of the unknown phases is fulfilled by simple multiplications and divisions, and it is easy to be implemented.

II. Background and Problem Formulation

A. The modulated wideband converter

In this section, we briefly introduce about the MWC [6]. The analog part of MWC consists of m channels as shown in Fig. 1. Series of mixer, LPF and ADC compose each channel. For the

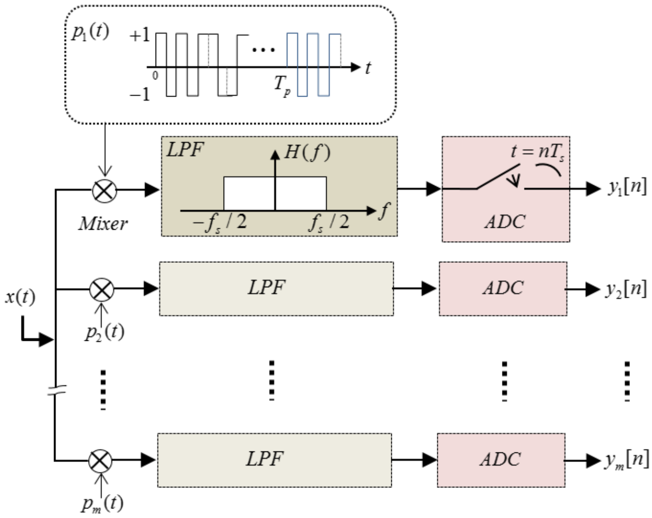

Fig. 1. The m channelized modulated wideband converter [7].

i -th channel, the multiband signal $x(t)$ which has several disjoint bands in $[-f_{\max}, f_{\max}]$, is mixed with PR sequence whose cycle is $T_p = 1/f_p$. For a cycle, the PR sequence $p_i(t)$ has $M = 2M_0 + 1$ chips for $M_0 > 0$, where each chip value $a_{i,v}$ is one of ± 1 for equal duration T_p / M. Thereafter, the mixed signal goes through the LPF whose bandwidth is $f_s/2 = qf_p/2$, where $q = 2q_0 + 1$ is a positive odd integer. After that, the ADC samples them at rate of $f_s = 1/T_s$.

By representing $p_i(t)$ as a form of the Fourier series expansion of bases $e^{-j2\pi f_p lt}$ for $l = -\infty, \cdots, \infty$, the output of i -th channel $y_i[n]$ is represented as

$$\begin{aligned} y_i[n] &= \sum_{l=-\infty}^{\infty} c_{i,l} \cdot LPF\{x(t) \cdot e^{-j2\pi f_p lt}\}\Big|_{t=nT_s} \\ &= \sum_{l=-L_0}^{L_0} c_{i,l} \cdot LPF\{x(t) \cdot e^{-j2\pi f_p lt}\}\Big|_{t=nT_s} \end{aligned} \tag{1}$$

where $LPF\{\cdot\}$ is the low-pass filtering operator, and the infinite order of summation is reduced to a finite order of $L = 2L_0 + 1$ since $x(t)$ is bandlimited and the bandwidth of LPF out of the maximum frequency outputs zero. The reduced order is calculated by $L_0 = M_0 + q_0$ [6]. Fourier series coefficients $c_{i,l}$ of $p_i(t)$ is defined by

$$c_{i,l} \triangleq \frac{1}{T_p} \int_0^{T_p} p_i(t) e^{j\frac{2\pi}{T_p} lt} \tag{2}$$

For an observation time T_o, the matrix form of (1) is

$$\mathbf{Y} = \mathbf{C}\mathbf{Z} \tag{3}$$

where $y_i[n]$ and $LPF\left\{x(t) e^{-j2\pi f_p lt}\right\}\Big|_{t=nT_s}$ respectively exist in the i -th row of $\mathbf{Y} \in \mathbb{R}^{m \times l_d}$ and the l -th row of $\mathbf{Z} \in \mathbb{C}^{L \times l_d}$ for $n = 0, \cdots, l_d - 1$, and $l_d = T_o / T_s$. The transfer matrix $\mathbf{C} \in \mathbb{C}^{m \times L}$ is

$$\mathbf{C} = \begin{bmatrix} c_{1,-L_0} & \cdots & c_{1,0} & \cdots & c_{1,L_0} \\ \vdots & & \vdots & & \vdots \\ c_{m,-L_0} & \cdots & c_{m,0} & \cdots & c_{m,L_0} \end{bmatrix} \tag{4}$$

where $c_{i,-l} = c_{i,l}^*$ and $(*)$ is a complex conjugate operator. Thereafter, additional digital signal processing (DSP) and a CS recovery algorithm reconstruct the Nyquist sample $x[\tilde{n}]$ of the input $x(t)$.

B. Problem formulation

The MWC can reconstruct the Nyquist sample $x[\tilde{n}]$ from the compressed samples $y_i[n]$ by using the theoretical model of (3) when every analog component is ideally designed. However, the transfer matrix $\mathbf{C}$ of (4) does not contain the practical analog characteristics including failure of clock synchronization between the PR sequence and the ADC, irregular channel gains, and the non-ideal responses of the LPF. Since the model mismatch by the non-ideal responses of the LPF can be easily eliminated by digital equalizing filters [17], we focus on the other non-idealities ensuring the linearity in (3) and the calibration using single tones with unknown phase information..

The non-idealities are usually posed when the MWC is implemented. For examples, there are asynchronization between the PR sequence and the ADC and irregularity among channel gains. In theory, the transfer model of (4) assumes perfect synchronization between the initial starting points of the PR sequence and the ADC. However, the synchronization requires additional circuits, and unknown path delays on the channels may disturb the synchronization. We model the asynchronization as a time difference of the initial points between the ADC and the PR sequence, which is equivalent to giving an unknown delay τ_i to $p_i(t)$ in Fig. 1. Likewise, for the irregular channel gains, we give an unknown channel gain w_i to $p_i(t)$. By the definition, the elements of the transfer matrix $\breve{\mathbf{C}}$ distorted by the asynchronization and the irregular channel gains are derived as

$$\begin{aligned} \breve{c}_{i,l} &= \frac{1}{T_p} \int_{\tau_i}^{T_p + \tau_i} w_i \cdot p_i(t - \tau_i) e^{-j2\pi f_p lt} \cdot dt \\ &= w_i e^{-j2\pi f_p l \tau_i} \frac{1}{T_p} \int_0^{T_p} p_i(t) e^{-j2\pi f_p lt} \cdot dt \\ &= w_i e^{-j2\pi f_p l \tau_i} \cdot c_{i,l} \end{aligned} \tag{5}$$

The asynchronization and irregularity among channel gains respectively multiplies complex exponentials and magnitudes to every column and row of (4). Consequently, some non-idealities of analog components distort the transfer matrix $\mathbf{C}$ while keeping the linearity.

By noting that the columns of unknown $\breve{\mathbf{C}}$ correspond to frequency response of the sampling system, we can estimate every frequency response by sequentially injecting single tone signals owing to the linearity of the distorted system. However, when the phases of the input single tone signals are unknown, which results in another complex exponentials in outputs, the estimated response is still untrustworthy. Such a case is impractical since the unknown path delays in analog channels make it hard to predict the phase of outputs even if the phase of input tone is elaborately controlled. Thus, it is needed to develop a calibration algorithm using tone signals with unknown phases for a practical implementation of the MWC.

III. CALIBRATION

We provide a calibration algorithm using single tone signals with unknown phase information for estimation of linear system transfer of the MWC. In specific, we suppose that the system is distorted while keeping the linearity, and our goal is to calibrate a distorted system transfer $\tilde{\mathbf{C}}$. The calibration algorithm consists of two steps; *linear estimation* and *phase equalization*. In the linear estimation step, we input sinusoids with unknown phases to the distorted MWC. Based on a mathematical relationship between the input tone signals and the output samples, we estimate the frequency responses of the system transfer. However, due to the unknown phases of the tone signals, the estimate still contains uncertain components. In the phase equalization step, we equalize the effect of the unknown phases by exploiting a structural characteristic in a matrix representation of the input-output relationship. The phase equalization step outputs the distorted system transfer of a single analog channel of the MWC.

In the linear estimation step, for the i-th channel, we sequentially input tone signals of

$$s(t) = e^{j2\pi f_p k(t-\tau_{i,k})} \tag{6}$$

for $k = 0,1,\cdots,M_0$, where $\tau_{i,k}$ are the priorly unknown phases, which are different among input frequencies and channels. From (1), the output $\tilde{y}_{i,k}[n]$ is derived as

$$\begin{aligned}\tilde{y}_{i,k}[n] &= \sum_{l=-L_0}^{L_0} \tilde{c}_{i.l} \cdot LPF\{e^{j2\pi f_p k(t-\tau_{i,k})} \cdot e^{-j2\pi f_p lt}\}\Big|_{t=nT_s} \\ &= \sum_{l=-L_0}^{L_0} \tilde{c}_{i.l} \cdot \theta_{i,k}^{k} \cdot LPF\{e^{-j2\pi f_p (l-k)t}\}\Big|_{t=nT_s}\end{aligned} \tag{7}$$

where $\tilde{c}_{i.l}$ is elements of the distorted transfer matrix $\tilde{\mathbf{C}}$ and $\theta_{i,k} = e^{-j2\pi f_p \tau_{i,k}}$ is represented as effects of unknown phases. The frequencies $f_p(l-k)$ within the bandwidth of LPF f_s is remained in (7), and the order of summation L_0 is reduced to q_0 when

$$|l-k| \le q/2 \tag{8}$$

The inequality (8) is yielded from $|f_p(l-k)| \le f_s/2$ and $f_s = qf_p$. From (8), the last of (7) is derived as

$$\begin{aligned}\tilde{y}_{i,k}[n] &= \sum_{l=-L_0}^{L_0} \tilde{c}_{i.l} \cdot \theta_{i,k}^{k} \cdot e^{-j2\pi f_p (l-k)t}\Big|_{t=nT_s}, \quad |l-k| \le \frac{q}{2} \\ &= \sum_{l=-q_0}^{q_0} \tilde{c}_{i.l+k} \cdot \theta_{i,k}^{k} \cdot e^{-j2\pi l\frac{n}{q}}\end{aligned} \tag{9}$$

For $k = 0,1,\cdots,M_0$, the last equation of (9) can be represented as a matrix form of

$$\tilde{\mathbf{Y}}_i = \mathbf{P}_i \tilde{\mathbf{A}}_i \mathbf{F}^{\mathrm{H}} \tag{10}$$

where $\tilde{\mathbf{Y}}_i \in \mathbb{R}^{(M_0+1)\times l_d}$, $\mathbf{P}_i \in \mathbb{C}^{(M_0+1)\times(M_0+1)}$, $\tilde{\mathbf{A}}_i \in \mathbb{C}^{(M_0+1)\times q}$, and $\mathbf{F}^{\mathrm{H}} \in \mathbb{C}^{q\times l_d}$ are readily represented at (10a). The matrices $\tilde{\mathbf{Y}}_i$ and $\mathbf{F}^{\mathrm{H}}$ are already known, but $\mathbf{P}_i$ and $\tilde{\mathbf{A}}_i$ are unknown. Meanwhile, $\tilde{\mathbf{A}}_i$ contains elements of distorted system transfer $\tilde{\mathbf{C}}$. Note that our aim is estimation of distorted system transfer. The matrix $\mathbf{F}^{\mathrm{H}}$ is a form of discrete Fourier transform (DFT) matrix and become full rank matrix when $l_d \ge q$. This condition can be achieved by injecting the signals for an observing time $T_o \ge qT_s$ since $l_d = T_o/T_s$. By multiplying Moore-Penrose pseudoinverse $\mathbf{F}^{\dagger}$ of $\mathbf{F}^{\mathrm{H}}$ to the right side of (10), it turns into

$$\tilde{\mathbf{Y}}_i \mathbf{F}^{\dagger} = \widehat{\mathbf{P}_i \tilde{\mathbf{A}}_i} \tag{11}$$

We define $\mathbf{B}_i \triangleq \tilde{\mathbf{Y}}_i \mathbf{F}^{\dagger}$ whose the (k,l)-th element equals to

$$\begin{aligned}b_{k,l} &= \left(\widehat{\mathbf{P}_i \tilde{\mathbf{A}}_i}\right)_{k,l} \\ &= \theta_{i,k-1}^{k-1} \cdot \tilde{c}_{i,-q_0+(k+l-2)}\end{aligned} \tag{12}$$

As shown in the result of linear estimation (12), the elements $\tilde{c}_{i,l}$ of distorted transfer matrix are multiplied with elements of

$$\underbrace{\begin{bmatrix} \tilde{y}_{i,k=0}[0] & \cdots & \tilde{y}_{i,k=0}[l_d-1] \\ \vdots & & \vdots \\ \tilde{y}_{i,k=M_0}[0] & \cdots & \tilde{y}_{i,k=M_0}[l_d-1] \end{bmatrix}}_{\tilde{\mathbf{Y}}_i} = \underbrace{\begin{bmatrix} \theta_{i,0}^{0} & & \\ & \ddots & \\ & & \theta_{i,M_0}^{M_0} \end{bmatrix}}_{\mathbf{P}_i} \underbrace{\begin{bmatrix} \tilde{c}_{i,-q_0} & \cdots & \tilde{c}_{i,q_0} \\ \vdots & & \vdots \\ \tilde{c}_{i,-q_0+M_0} & \cdots & \tilde{c}_{i,q_0+M_0} \end{bmatrix}}_{\tilde{\mathbf{A}}_i} \underbrace{\begin{bmatrix} e^{-j2\pi(-q_0/q)\cdot 0} & \cdots & e^{-j2\pi(-q_0/q)\cdot(l_d-1)} \\ \vdots & & \vdots \\ e^{-j2\pi(q_0/q)\cdot 0} & \cdots & e^{-j2\pi(q_0/q)\cdot(l_d-1)} \end{bmatrix}}_{\mathbf{F}^{H}} \tag{10a}$$

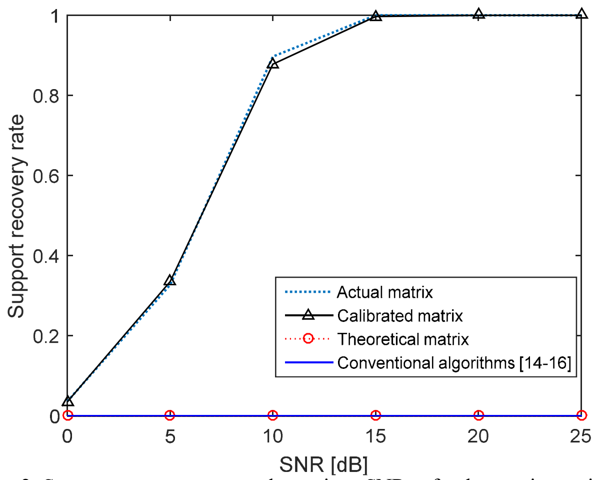

Fig. 2. Support recovery rates under various SNRs of unknown input signals. The system transfer matrix calibrated by the proposed algorithm and the theoretical matrix ignoring practical conditions are used.

$\mathbf{P}_i$. Thus, to acquire the elements $\tilde{c}_{i,l}$ from $\tilde{\mathbf{A}}_i$, we should estimate or equalize the $\mathbf{P}_i$.

In the phase equalization step, we exploit that $\tilde{\mathbf{A}}_i$ is a Hankel matrix, i.e. $(k+1,l-1)$ element equals to (k,l) element for $k \geq 1,\ l > 1$. We generate a matrix $\mathbf{G}_i = diag\{g_1, g_2, ..., g_{M_0+1}\}$ to equalize the unknown phase matrix $\mathbf{P}_i$. All the elements of $\mathbf{G}_i$ are defined as

$$g_k \triangleq \begin{cases} 1 & ,\ k=1 \\ g_{k-1} \cdot \dfrac{b_{k-1,2}}{b_{k,1}} & ,\ otherwise \end{cases} \tag{13}$$

By the definition of (12), the elements g_k are equal to the inverse of the k-th elements of $\mathbf{P}_i$. In specific,

$$\begin{aligned} g_k &= g_1 \prod_{j=1}^{k-1} \frac{b_{j,2}}{b_{j+1,1}} \\ &= g_1 \prod_{j=1}^{k-1} \frac{\theta_{i,j-1}^{j-1} \cdot \tilde{c}_{i,-q_0+j}}{\theta_{i,j}^{j} \cdot \tilde{c}_{i,-q_0+j}} \\ &= g_1 \cdot \frac{\theta_{i,0}^{0} \prod_{j=2}^{k-1} \theta_{i,j-1}^{j-1}}{\theta_{i,k-1}^{k-1} \prod_{j=1}^{k-2} \theta_{i,j}^{j}} \\ &= \left(\theta_{i,k-1}^{k-1}\right)^{-1} \end{aligned} \tag{14}$$

Hence, the multiplication $\mathbf{G}_i \widehat{\mathbf{P}}_i$ becomes identity matrix $\mathbf{I}$. Consequently, we can estimate the $\tilde{\mathbf{A}}_i$ by multiplying $\mathbf{G}_i$ to (12), i.e.

$$\widehat{\tilde{\mathbf{A}}}_i = \mathbf{G}_i \mathbf{B} \tag{15}$$

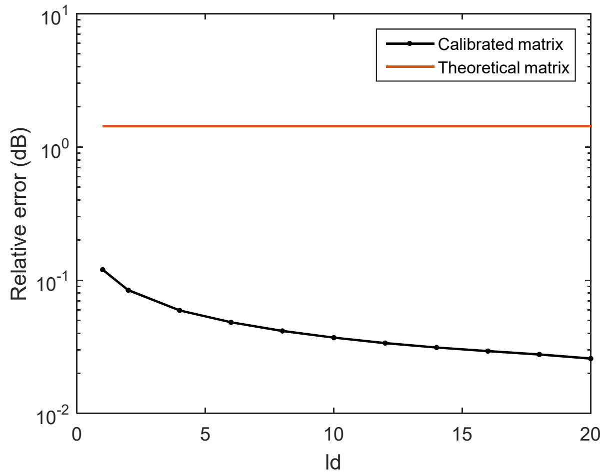

Fig. 3. Relative errors of the calibrated and theoretical matrix compared to the actual system transfer. The parameter l_d means time of injecting calibration signals.

The result of equalization $\widehat{\tilde{\mathbf{A}}}_i$ contains positive frequency elements $\tilde{c}_{i,l}$ for $l = 0,1,...,L_0$. The negative elements are acquired by using the conjugate symmetric property of transfer matrix $\tilde{c}_{i,-l} = \tilde{c}_{i,l}^*$. Thus, by injecting calibration signals to the i-th channel of MWC, we can estimate i-th row of distorted transfer matrix $\tilde{\mathbf{C}}$. For the m channels, the linear estimation and phase equalization are repeated. The calibrated transfer matrix $\tilde{\mathbf{C}}$ is acquired from the m estimated rows. With the calibrated transfer matrix $\tilde{\mathbf{C}}$, the unknown multiband signal can be reconstructed successfully.

IV. Simulations

In this section, the simulations were presented to verify the performance of proposed calibration algorithm. For the simulation, we generated an unknown multiband signal $x(t)$, which consisted of 6 disjoint bands with $f_{\max} = 2GHz$. We used 4-channel MWC to sample $x(t)$. There were $M = 127$ Fourier coefficients of PR sequence at intervals of $f_p = 31.5MHz$, and the sampling rate of ADC was $f_s \simeq 220MHz$, which induced $q = 7$. To construct a distorted environment, we set an asynchronization as a certain time difference of the initial points between the ADC and the PR sequence to $7 \cdot T_p$. To estimate the distorted transfer matrix $\tilde{\mathbf{C}}$, we injected $30dB$ tones as (6). The signal-to noise ratio (SNR) was defined as $SNR = 10 \cdot \log_{10}(\|s\|_2^2 / \|n\|_2^2)$, where s and n were respectively the calibration signal and noise vector. In addition, we also generated unknown input phases $\tau_{i,k} \in [0, T_o]$ randomly. We exploited the OMP [11] to estimate supports of x from the compressed outputs $y_i[n]$. Note that the supports correspond to actual indices of signal spectra among subbands.

We compared the recovery performances using the calibrated transfer matrix and theoretical transfer matrix without calibration. We also generated the actual transfer matrix $\mathbf{C}_a$ from the distorted environment. To verify success of recovery, we measured the supports recovery rate whose flag is denoted as one when the recovered supports were coincided to original supports. As shown in Fig.2, the theoretical transfer matrix failed to recover the original supports due to the distortion by the asynchronization between the ADC and the PR sequence. On the contrary, the calibrated transfer matrix successfully recovered them like the actual matrix. In addition, to verify the importance of the phase equalization step in calibration, we implemented the conventional calibration algorithms of [14-16]. The proposed phase equalizing calibration only succeeded to recover the supports when the unknown input phases exist. The algorithms of [15, 16] cannot cover the distortion of asynchronization. Above all, since the system transfer is distorted by the remained unknown phases again, their failures were unavoidable. Hence, the phase equalization should be executed for the successful calibration.

We demonstrated the effects of the increased injecting time of calibration signals. By increasing the injecting time, the number of compressed samples l_d of (10) was enlarged, which contributed to estimate the actual transfer matrix more precisely. To verify it, we compared relative errors of the calibrated and theoretical matrix to the actual matrix $\mathbf{C}_a$ along the l_d. The relative error was defined as

$$relative\ \ error = \left\| \tilde{\mathbf{C}} - \mathbf{C}_a \right\|_F / \left\| \mathbf{C}_a \right\|_F \tag{16}$$

where $\tilde{\mathbf{C}}$ is calibrated or theoretical matrix, and $\left\| \cdot \right\|_F$ is Frobenius norm. As shown in Fig. 3, the calibrated sensing matrix was closed to the actual matrix unlike the theoretical matrix. The relative errors of theoretical matrix were a certain constant, because the asynchronization and the system transfer are unchanged. Consequently, the result demonstrated that calibration performance was enhanced along the l_d.

V. CONCLUSION

In this paper, we proposed a calibration algorithm for the practical implementation of MWC. By inserting tone signals whose phases were not known but the frequencies were known in advance, the MWC was calibrated successfully. Through the calibration algorithm, the unknown input phases were also estimated for the successful calibration. Under the distorted environment, the simulation showed that only the calibrated transfer matrix successfully recovered the original supports set. In addition, the simulation also showed that the unknown input phases should be estimated for the successful calibration.

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