Predicting the Pruning Potential in Sphere Decoding for Multiple-Input Multiple-Output Detection

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***Abstract*—In this work, a method for predicting the pruning potential of a sphere constraint (SC) for sphere decoding (SD) is developed. Because the direct prediction of the pruning potential is not easy, the orthotope constraint (OC), an approximation of SC, is used instead of SC. This pruning potential prediction makes it possible to increase pruning at the root of the search tree in SD, considering it is the most desirable location for pruning.**

Keywords- multiple input multiple output (MIMO); maximum-likelihood (ML) detection; sphere decoding (SD)

I. INTRODUCTION

The popular tree search algorithm for MIMO detection is sphere decoding (SD) [1],[2], which searches only over vector points, **s**, whose Euclidean distance of **Hs**, from the received signal, **r**, is less than a certain value, $\sqrt{C}$. In the tree search process, a metric is accumulated up to a certain level of the tree. This metric is called partial Euclidean distance (PED). If the PED of a specific path from the top to a node is found to be greater than $\sqrt{C}$, the sub-tree leading from that node is pruned from the tree without having to continue further. This pruning condition is called the sphere constraint (SC).

For the structure of the tree, the pruning at the root is desirable because it prunes the exponentially expanding sub-tree. If we can determine which symbol prunes the most at the root, the tree can be reorganized such that the symbol with the largest pruning capacity is located at the root. Once the pruning potential of each level of the tree is known, the best symbol can be identified as the one with the largest pruning potential. However, predicting the pruning potential (PP) before the tree search starts is very difficult.

In this work, we develop a method for predicting the PP. For PP prediction, the orthotope constraint (OC) [3] is utilized instead of SC. There are two reasons that OC is used: (i) its simplicity in PP calculation and (ii) the fact that the amount of pruning by OC is similar to that done by SC at the root.

The remainder of this paper is organized as follows. In section II, the system model is described. In section III, the development of the method for predicting the pruning potential is presented. In section IV, the simulation results are presented. Section V concludes the paper.

II. SYSTEM MODEL

We consider a complex-valued baseband MIMO channel model with M receive and N transmit antennas ($M \geq N$). The channel is assumed to be known. Consider the system model[1],

$$\mathbf{r} = \mathbf{Hs} + \mathbf{n} \tag{1}$$

Here, $\mathbf{r} = \left[r_1, ..., r_M\right]^T$ denotes the received symbol vector. **H** denotes the $M \times N$ block Rayleigh fading channel matrix. Each entry of the **H** matrix is an independently and identically distributed (IID) complex zero-mean Gaussian random variable, with unity variance. $\mathbf{s} = \left[s_1, ..., s_N\right]^T$ is the transmitted symbol vector, $\mathbf{s} \in \mathcal{O}^N \subset \mathbb{C}^N$, where $\mathcal{O}$ is the finite signal constellation. $\mathbf{n} = \left[n_1, ..., n_M\right]^T$ is the unknown additive white Gaussian noise (AWGN) vector, which comprises the entries of mean zero and variance σ^2 .

III. PREDICTION OF THE PRUNING POTENTIAL

For maximum pruning at the root of the tree, we need to estimate the best symbol by calculating the number of constellation points that SC can prune for each symbol in **s** when it is placed at the root. We do so by identifying the number of constellation points that SC cannot prune and then calculating the number of constellation points it can prune from it. For this purpose, the smallest possible search space, that is, the smallest sphere, needs to be identified. This requires knowledge of the Euclidean distance of the ML solution from the received signal, **r**, which is not easy to obtain before the tree search has started.

We can use other constraints instead of SC. Because SC is not strict at the root of the tree, there may be many constraints

[1] Variables that denote vectors and matrices are set, respectively, in lowercase and uppercase boldface. $\mathbf{H}(k,:)$ denotes the k th row of matrix $\mathbf{H}$, and $\mathbf{H}^\dagger$ is the pseudo inverse of $\mathbf{H}$. Each element of a vector is denoted by a subscript. For example, s_k is the k th element of $\mathbf{s}$. $|\mathcal{O}|$ denotes the cardinality of set $\mathcal{O}$. $\|\mathbf{s}\|_2$ denotes the 2nd norm of vector $\mathbf{s}$.

that are as almost as strict as SC is at the root. One example is the orthotope constraint (OC) [3].

A. Orthotope Constraint

It has been observed that, at the root of a tree, the amount of pruning by SC is almost the same as that done by OC over wide SNR ranges.

Geometrically, OC can be defined by a collection of squares, each of which is centered at x_k, the k^{th} element of $\mathbf{x} = \mathbf{H}^{\dagger}\mathbf{r}$, the Babai point, and has a width of $\sqrt{C} \cdot \delta_k$ for s_k of $\mathbf{s}$. The constellation points inside the squares satisfy OC, whereas those located outside the squares do not.

OC is expressed as

$$\Delta(s_k) \leq \sqrt{C} \cdot \delta_k \text{ for } k = 1, ..., N \quad (2)$$

Here, $\Delta(s_k) = \max\{|\Re(s_k) - \Re(x_k)|, |\Im(s_k) - \Im(x_k)|\}$ and $\delta_k = \|\mathbf{H}^{\dagger}(k,:)\|_2$.

Note that the widths of the squares have certain ratios to each other.

Definition 1: (Orthotope Square Ratios) The widths of the squares have the ratios $\delta_1, \delta_2, \ldots, \delta_N$ to each other.

B. Pruning Potential

In contrast to the sphere for SC, the smallest search space for OC, or the smallest orthotope, can be identified without any knowledge of the ML solution.

Definition 2: The minimum orthotope (MO) is the smallest orthotope that contains at least one vector point.

Theorem 1. *Any constellation point, s_k, satisfying the following test is in the minimum orthotope.*

$$\Delta(s_k) \leq \sqrt{C_{\min}} \cdot \delta_k,$$

where $C_{\min} = \max_{k=1,2,...,N}\left[\left(\delta_k^{-1}\right)^2 \min_{s_k \in \mathcal{O}}\left(\Delta^2(s_k)\right)\right]$. *Any constellation point not satisfying the test is not in the minimum orthotope.*

Proof: The proof is given in [4]. □

Theorem 2. (Pruning Potential) *Let $S_{k\text{-min}}$ be the set of constellation points for the k^{th} level of the minimum orthotope. Then, the pruning potential of OC for the k^{th} level is $|\mathcal{O}| - |S_{k\text{-min}}|$.*

Proof: The proof is given in [4]. □

Once PP is calculated, the symbol with the largest PP value is selected as the best symbol.

IV. Results

The probability of the failure of the proposed method in estimating the best symbol is obtained by a simulation. When the PPs of the best symbol as well as other symbols are the largest values given by the proposed method, it is not regarded as a failure. As shown in Fig. 1, the failure percentage is lower than 3% over a wide SNR range in an 8×8 system with a 16-QAM constellation. We also performed extensive simulations for other types of constellations as well as antenna numbers and found the results to be consistent.

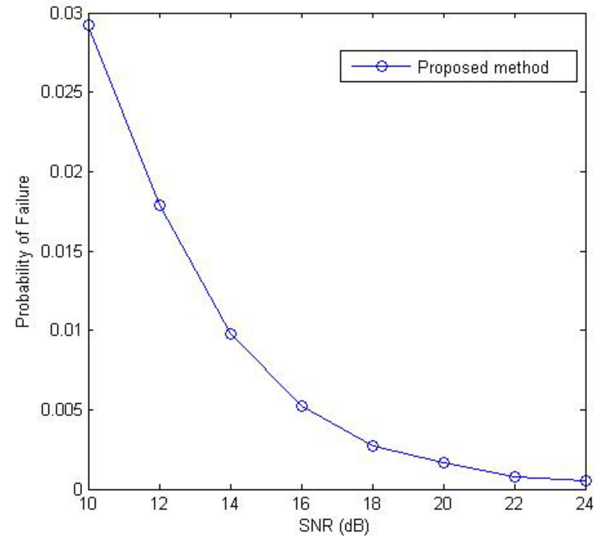

Fig. 1. Probability of failure of best symbol estimation by the proposed method in an 8×8 system with a 16-QAM constellation.

V. Conclusion

A pruning potential (PP) prediction method for sphere decoding (SD) is presented. This method can be used to determine the best symbol to be placed at the root of a tree in SD. The proposed method utilizes the orthotope constraint (OC) instead of the sphere constraint (SC) to calculate PP. Simulation results showed that the failure probability for best symbol estimation was less than 0.03 over a wide SNR range for an 8×8, 16-QAM system.

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