Exact Outage Probability and Power Allocation of Two Nodes in Cooperative Networks

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Abstract—**Advanced communication techniques exploiting network coding and cooperative schemes have attracted considerable attention as ways to improve power efficiency in wireless transmission as well as to achieve high throughput and spectral efficiency. In this paper, we consider a cooperative wireless network with two nodes and one base station, and investigate the impact of the outage performance on using non-binary network coding. We derive the exact and general outage probability in our network coding schemes, and obtain the asymptotic expression of the outage probability. We compare outage probabilities between exact and asymptotic results. We discuss the problem of optimal power allocation.**

Keywords—outage probability; non-binary network coding; cooperative networks; power allocation

I. Introduction

Signal fading is one of the underlying causes of performance degradation in wireless networks. One approach to combating the fading is to increase the transmit power. A more advanced way is to exploit diversity techniques such as time, frequency, and space. Cooperative networking is a state-of-the-art technology that utilizes the spatial diversity via user cooperation. Each user takes a part in collaboration and shares the benefit of using a virtual antenna array in transmitting its information to a receiver that is available through another user's antenna [1].

In [2], Ahlswede et al. have proved that the use of simply routing or replicating data in a single-source multicast scenario is not optimal approaches in terms of the flow rate. And they have proposed network coding to enhance the flow rate. Many studies have since been performed to clarify that network coding provides advantages over the existing cooperative network schemes [3]-[6].

There are many studies about analyses of outage probability in cooperative networks in [1], [7]-[11]. In [8], Chen et al. showed that binary network coding (BNC), based on the arithmetic of size 2 of a Galois Field, i.e., GF(2), provides improved diversity gains and bandwidth efficiencies in wireless networks in which each user employs a simple decode-and-forward scheme that assumes a perfect inter-user channel. In practice, there exist channel errors between users, as discussed in [9], where the authors proposed an adaptive decode-and-forward scheme with BNC. It was recently shown in [10] that BNC is not optimal for achieving full diversity in a system of multiple users and relays. Using non-binary network coding (NBNC) with GF(q) for $q > 2$, it was shown that a full diversity order can be achieved in [10] and [11].

In this paper, we consider a cooperative wireless network, where there are two source nodes and one base station (BS) as shown in Figure 1. We investigate the impact of the outage performance on the size of network coding. In [10], although Xiao and Skoglund have showed that the diversity order of NBNC is higher than that of BNC, their outage probability was obtained from a number of approximations: *i*) they did not consider all possible outage scenarios (for a full consideration see [7]), *ii*) all channel outages are treated with the same transmit powers, the same average channel gains, and thus the same average channel SNRs. In this work, however, we derive a general and an exact outage analysis framework with field sizes in network coding, i.e., GF(2) vs. GF(4), transmit powers, transmission rates, and network topologies on the outage performance of the network. We thereby investigate the impact of outage probability on different channel environments. In addition, the exact framework allows us to widely provide an analysis tool. We obtain the asymptotic outage probability for both network coding schemes, and compare outage probabilities between exact and asymptotic results. We show that our asymptotic results are well matched the exact ones in the high SNRs. An analysis of the optimal power allocation (OPA) for both cooperative schemes is useful to determine the power efficiency for various network environments, i.e., according to the variances of the channel gains.

II. System Description

A. Cooperative Schemes

We consider a cooperative scheme for wireless networks as shown in Figure 1. There are two source nodes, nodes 1 (N1) and 2 (N2), and two phases in the cooperative scheme, the broadcasting and the relay phase. In the broadcasting phase, source nodes N1 and N2 transmit their messages, S_1 and S_2 respectively. In the relay phase, when both nodes successfully recover the transmitted messages, the messages are re-encoded and then forwarded to the base station (BS). When a node is unable to successfully perform decoding, it instead repeats its message in the relay phase. When receiving repeated messages, BS as a destination performs maximum ratio combining (MRC) of these messages, and recovers the transmitted messages. In this paper, we assume that the transmission rate is selected to

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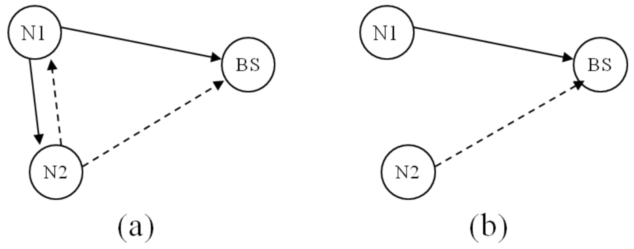

Figure 1. Cooperative scheme: (a) broadcasting phase, (b) relay phase.

be sufficiently lower than the capacity of each channel so that near perfect decoding of messages can be accomplished with the use of a channel code. Thus, for all wireless channels, the received messages are either completely corrupted, and therefore not available at the receiving end, or considered error-free.

At BS, the set of all possible received messages is { S_1 , S_2 , Z_1 , Z_2 }, where the subscript denotes the index of the transmit node. The first two messages are received in the first phase, and the latter two are linearly combined and sent from the sources in the relay phase. The alphabet of the combined message, Z_1 and Z_2, is selected to be a finite field. The two finite fields considered in this paper are GF(2) and GF(4). In this paper, the combined messages Z_1 and Z_2 are generated for each network scheme as follows: *i*) for GF(2) as called *BNC*, $Z_1 = S_1 + S_2$ ad $Z_2 = S_1 + S_2$, *ii*) for GF(4) as called *NBNC-4*, $Z_1 = S_1 + S_2$ and $Z_2 = S_1 + 2S_2$, respectively. All arithmetic operations are performed over finite fields.

B. Channel Model

Our system consists of a multiple access channel network. In the broadcasting and relay phases, all source nodes transmit signals through orthogonal channels using time division multiple access (TDMA) or frequency division multiple access (FDMA). The channels used in this paper are assumed to be spatially independent, flat faded, and perturbed by additive white Gaussian noise (AWGN). We further assume that the channel gains in both the broadcasting and relay phases are mutually independent. The received signal at the *j*-th node is thus.

$$y_{i,j,k} = \sqrt{P_i} h_{i,j,k} x_{i,j,k} + n_{i,j,k} , \tag{1}$$

where k denotes the transmission phase, such as the broadcasting and relay phase, $k \in \{1,2\}$, i denotes the transmitted node, i.e. $i \in \{1,2\}$, namely N1 and N2. Let j denote the received node for $j \in \{1,2,0\}$, where "0" denotes BS. The transmitted and received signals are given as $x_{i,j,k}$ and $y_{i,j,k}$ with $i \neq j$. P_i denotes the transmit power at the *i*-th node. The channel gain is represented by $h_{i,j,k}$, which consists of the fading term $p_{i,j,k}$ and the path-loss coefficient $q_{i,j,k}$; i.e., $h_{i,j,k} = p_{i,j,k} q_{i,j,k}$. Here, we assume that the fading term $p_{i,j,k}$ is random and the path-loss coefficient $q_{i,j,k}$ depends on the distance between nodes *i* and *j*. Noise $n_{i,j,k}$ is AWGN with a normal distribution $\mathcal{N}(0, N_0)$ with a zero mean and power spectral density N_0 . The path-loss coefficient is modeled as $q_{i,j,k} = \left(d_0 / d_{i,j}\right)^{\alpha/2}$, where $2 \le \alpha \le 6$ is the path-loss exponent, $d_{i,j}$ is the distance between nodes i and j, and d_0 is the reference distance. In this paper, we use $d_0 = 1$ and $\alpha = 3$, and $\left|h_{i,j,k}\right|$ is assumed to be Rayleigh distributed such that the channel energy of power $\left|h_{i,j,k}\right|^2$ is exponentially distributed. We assume that the fading term $p_{i,j,k}$ is a complex-valued, independent and identically distributed Gaussian in each dimension with a zero mean and 1/2 variance. The average power of $h_{i,j,k}$ is then represented by the average power of $q_{i,j,k}$, which depends on the distance between the transmitter and the receiver. All channel gains are assumed to be reciprocal, i.e.,; $h_{i,j,k} = h_{j,i,k}$. The instantaneous SNR (signal-to-noise ratio) of each channel is denoted as $\gamma_{i,j,k} := \left|h_{i,j,k}\right|^2 P_i / N_0$, where P_i / N_0 is the transmit SNR at the source node *i*.

C. Outage Probability

The channel capacity as a function of the received SNR at node *j* is given by

$$C_{i,j,k} = \log_2\left(1 + \gamma_{i,j,k}\right), \tag{2}$$

where $C_{i,j,k}$ denotes the channel capacity from nodes *i* to *j* at the *k*-th transmission phase. In this paper, we use the single channel capacity as $C_{i,j,k} = \frac{1}{2}\log\left(1 + \gamma_{i,j,k}\right)$ for each transmission phase because a factor of 2 represents the bandwidth expansion for each node in the cooperative scheme. Channel outage occurs if the capacity is less than the transmission rate R , where R is the desired spectral efficiency in bits/s/Hz. For the Rayleigh fading channel, the outage probability is given and approximated at a high SNR in the following manner:

$$\begin{aligned} P_{out}\left(\gamma_{i,j,k}, R\right) &= \Pr\left\{\gamma_{i,j,k} < \left(2^R - 1\right)\right\} \\ &= 1 - \exp\left(-\frac{2^R - 1}{\Gamma_{i,j}}\right) \\ &\approx \frac{2^R - 1}{\Gamma_{i,j}}, \end{aligned} \tag{3}$$

where $\Gamma_{i,j} = \sigma_{i,j}^2 P_i / N_0$, is the average SNR at the receiver *j*, $\sigma_{i,j}^2$ is the variance of the channel gain $h_{i,j,k}$ which only depends on the distance such that $\sigma_{i,j}^2 = \sigma_{i,j,1}^2 = \sigma_{i,j,2}^2$ The outage probability $P_{out}\left(\gamma_{i,j,k}, R\right)$ is a function of the average SNR and the transmission rate.

We assume that MRC is used at BS for combining identical transmissions. For the case of MRC, the probability of an outage event is a function of two exponentially distributed random variables, which denote the instantaneous SNR for each channel. Thus, the outage probability for MRC at BS is represented as $\Pr\left\{\gamma_{s,0,k} + \gamma_{r,0,k} < 2^{2R} - 1\right\}$, for $s, r \in \{1,2\}$. The outage probability with two random variables is obtained from the following cumulative distribution function (CDF). Let $w := u + v$, where u and v are independent exponential random variables with parameters μ_u and μ_v . The CDF of the random variable w is given by

$$P_w(w) = \begin{cases} 1 - \left[\left(\dfrac{\mu_v}{\mu_v - \mu_u} \right) e^{-\mu_u w} + \left(\dfrac{\mu_u}{\mu_u - \mu_v} \right) e^{-\mu_v w} \right] & \mu_u \neq \mu_v, \\ 1 - (1 + \mu w) e^{-\mu w} & \mu_u = \mu_v = \mu. \end{cases} \quad (4)$$

III. DERIVATION ON OUTAGE PROBABILITY

In this section, we aim to derive the exact and general outage probability that allows us to investigate the effect of different outage events, transmit powers, channel gains, and field sizes in network coding, i.e., BNC vs.NBNC-4.

A. Outage Events in the Cooperative Network

In the broadcasting phase, both source nodes transmit their messages to BS in an orthogonally multiplexed manner, and they can listen to each other's message. In the relay phase, the two source nodes conduct independently with no knowledge of whether their own broadcasted message was successfully decoded or not by their neighbor node. No feedback channel is assumed between the two nodes. As such, there are four possible cooperation scenarios depending on whether the decoding of messages was successful or not in the broadcasting phase. These four outage events are shown in Figure 2, and the four cooperative scenarios for each of the four outage events are denoted as Cases 1, 2, 3, and 4.

In Case 1, both nodes successfully decode the partner's message. In the relay phase, each node linearly combines the neighbor's message with a network coding, and forwards the encoded message to BS, resulting in a fully cooperative scenario. In Case 2, N1 successfully decodes the message from N2, but N2 does not successfully decode the message from N1. Hence, N1 combines N2's message and forwards the re-encoded message to BS in the relay phase in the same manner as in Case 1. However, N2 repeats its message in the relay phase. At the BS, the repeated messages are decoded using the MRC strategy. Case 3 is similar to Case 2 except that the role of N1 is switched with that of N2. In Case 4, no node successfully decodes its neighbor's message in the broadcasting phase, and hence each node uses the available channel in the relay phase to just repeat its own message made

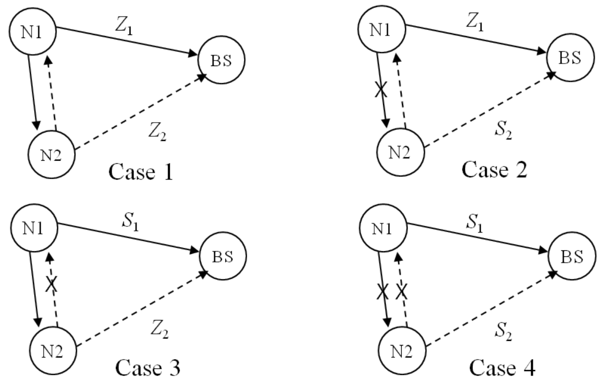

Figure 2. Four cooperative scenarios for relay phase transmission based on the decoding results in the broadcasting phase.

in the broadcast phase. Thus, in this case, the system automatically reverts to a non-cooperative case. In our cooperative schemes, we assume that the BS knows which case out of the four cases has occurred. Next, we derive and evaluate the outage probability for the NBNC-4 scheme for each scenario.

B. Outage Probabability for NBNC-4 and BNC

In the following, we focus on the derivation of outage probability for the NBNC-4 scheme. First, network coding in the relay phase is performed. Message transmission consists of two phases as described in the previous subsection. We analyze the outage event based on MRC. In this work, we assume that the instantaneous SNRs for the broadcasting and relay phases are mutually independent. We consider the outage probability for N1, which is identical to that for N2 using the symmetric argument. We define the transmission rate for each node as R_1 and R_2 .

Case 1: In this case, both nodes correctly decode each other's messages. Correct decoding events are defined as follows:

$$\left\{C_{1,2,1} > R_1\right\} \cap \left\{C_{2,1,1} > R_2\right\}. \quad (5)$$

We consider the outage events for Case 1. Suppose that transmitted messages in the broadcasting phase from N1 and N2 are not decoded successfully at BS. This amounts to an outage event except when both of the combined messages with rates R_1 and R_2 , respectively, are successfully decoded in the relay phase. In this case, the outage probability can be written as

$$\Pr\left\{\left(C_{1,0,1} < R_1\right) \cap \left(C_{2,0,1} < R_2\right) \cap \left[\left(C_{1,0,2} < R_1\right) \cup \left(C_{2,0,2} < R_2\right)\right]\right\}. \quad (6)$$

In addition, consider the case in which the transmitted message in the broadcasting phase from N1 is not decoded successfully, but the transmitted message in the broadcasting phase from N2 is decoded successfully. An outage occurs only when decoding of both messages in the relay phase fails. This outage probability can be written as

$$\Pr\left\{\left(C_{1,0,1}<R_1\right)\cap\left(C_{2,0,1}>R_2\right)\cap\left(C_{1,0,2}<R_1\right)\cap\left(C_{2,0,2}<R_2\right)\right\}. \quad (7)$$

As a result, the outage probability of N1 for Case 1 can be obtained as

$$\begin{aligned}P^1_{NBNC-4} &= \Pr\{\gamma_{1,2,1}>r_1\}\times\Pr\{\gamma_{2,1,1}>r_2\}\\ &\times\Big[\Pr\{\gamma_{1,0,1}<r_1\}\times\Pr\{\gamma_{2,0,1}<r_2\}\\ &\times\left(1-\Pr\{\gamma_{1,0,2}>r_2\}\times\Pr\{\gamma_{2,0,2}>r_2\}\right)\\ &+\Pr\{\gamma_{1,0,1}<r_1\}\times\Pr\{\gamma_{2,0,1}>r_2\}\\ &\times\Pr\{\gamma_{1,0,2}<r_1\}\times\Pr\{\gamma_{2,0,1}<r_2\}\Big],\end{aligned} \quad (8)$$

where $r_1 = 2^{2R_1}-1$ and $r_2 = 2^{2R_2}-1$.

Case 2: In this case, N1 correctly decodes message S_2 from N2, but N2 does not correctly decode message S_1 from N1. This corresponds to the following events:

$$\left\{C_{1,2,1}<R_1\right\}\cap\left\{C_{2,1,1}>R_2\right\}. \quad (9)$$

According to the transmission protocol, BS receives N2's message S_2 twice, and decoding is performed using MRC. Hence, the outage probability of N2 for MRC is obtained as

$$\begin{aligned}\Pr\{\mathrm{MRC}_2\} &= \Pr\{\gamma_{2,0,1}+\gamma_{2,0,2}<2^{2R_2}-1\}\\ &= 1-\left(1+\frac{r_2}{\Gamma_{2,0}}\right)\exp\left(-\frac{r_2}{2\Gamma_{2,0}}\right).\end{aligned} \quad (10)$$

The outage probability in the conditional case is

$$\begin{aligned}&\Pr\Big\{\left(\{C_{1,0,1}<R_1\}\cap\{C_{1,0,2}<R_2\}\right)\\ &\cup\left(\{C_{1,0,1}<R_1\}\cap\{C_{1,0,2}>R_1\}\cap\{\mathrm{MRC}_2\}\right)\Big\}.\end{aligned} \quad (11)$$

Thus, the overall outage probability for Case 2 is

$$\begin{aligned}P^2_{NBNC-4} &= \Pr\{\gamma_{1,2,1}<r_1\}\times\Pr\{\gamma_{2,1,1}>r_2\}\times\Big[\Pr\{\gamma_{1,0,1}<r_1\}\\ &\times\left(\Pr\{\gamma_{1,0,2}<r_2\}+\Pr\{\gamma_{1,0,2}>r_1\}\times\Pr\{\mathrm{MRC}_2\}\right)\Big].\end{aligned} \quad (12)$$

Case 3: In this case, N2 correctly decodes N1's message S_1, but N1 cannot decode node 2's message S_2. The corresponding event is

$$\left\{C_{1,2,1}>R_1\right\}\cap\left\{C_{2,1,1}<R_2\right\}. \quad (13)$$

Using the same approach as Case 2, we obtain the overall outage probability as follows

$$\begin{aligned}P^3_{NBNC-4} &= \Pr\{\gamma_{1,2,1}>r_1\}\times\Pr\{\gamma_{2,1,1}<r_2\}\times\Pr\{\mathrm{MRC}_1\}\\ &\times\left(\Pr\{\gamma_{2,0,1}<r_2\}+\Pr\{\gamma_{2,0,1}>r_2\}\times\Pr\{\gamma_{2,0,2}<r_2\}\right).\end{aligned} \quad (14)$$

The outage probability for N1 that uses MRC is

$$\begin{aligned}\Pr\{\mathrm{MRC}_1\} &= \Pr\{\gamma_{1,0,1}+\gamma_{1,0,2}<2^{2R_1}-1\}\\ &= 1-\left(1+\frac{r_1}{\Gamma_{1,0}}\right)\exp\left(-\frac{r_1}{2\Gamma_{1,0}}\right).\end{aligned} \quad (15)$$

Case 4: Neither node decodes the message in the broadcasting phase successfully. The overall outage probability for Case 4 is

$$P^4_{NBNC-4} = \Pr\{\gamma_{1,2,1}<r_1\}\times\Pr\{\gamma_{2,1,1}<r_2\}\times\Pr\{\mathrm{MRC}_1\}. \quad (16)$$

Next, the exact outage probability with NBNC-4 for N1 is obtained by adding the results so far, i.e., (8), (11), (14), and (16), as follows:

$$P_{NBNC-4} = P^1_{NBNC-4}+P^2_{NBNC-4}+P^3_{NBNC-4}+P^4_{NBNC-4}. \quad (17)$$

Using the high SNR approximation given in the last line of (3), we can approximate the outage as follows:

$$P_{NBNC-4} \approx \frac{A_1}{P_1^2P_2}+\frac{A_2}{P_1P_2^2}+\frac{A_3}{P_1^3}, \quad (18)$$

where $A_1 := \frac{2r_1^2r_2N_0^3}{\sigma_{1,0}^4\sigma_{2,0}^2}+\frac{r_1^2r_2N_0^3}{2\sigma_{1,2}^2\sigma_{1,0}^2\sigma_{2,0}^2}+\frac{r_1^2r_2N_0^3}{2\sigma_{1,2}^2\sigma_{2,1}^2\sigma_{1,0}^2}$, $A_2 := \frac{r_1r_2^2N_0^3}{\sigma_{1,0}^2\sigma_{2,0}^4}+\frac{r_1r_2^2N_0^3}{\sigma_{2,1}^2\sigma_{1,0}^2\sigma_{2,0}^2}$, and $A_3 := \frac{r_1^2r_2N_0^3}{\sigma_{1,2}^2\sigma_{1,0}^4}$.

Similar to the analysis done in the NBNC-4 scheme, the outage probability analysis for BNC is done, which shows that the outage probabilities for BNC are identical to those for NBNC-4, except for the first case, i.e., $P^2_{BNC}=P^2_{NBNC-4}$, $P^3_{BNC}=P^3_{NBNC-4}$, and $P^4_{BNC}=P^4_{NBNC-4}$. The reason for this is that the outage events, in each case of Cases 2, 3, and 4, for the BNC scheme are identical to those of NBNC-4. The only difference comes from Case 1.

The outage probability of BNC for Case 1 is given by

$$\begin{aligned}P^1_{BNC} &= \Pr\{\gamma_{1,2,1}>r_1\}\times\Pr\{\gamma_{2,1,1}>r_2\}\times\Pr\{\gamma_{1,0,1}<r_1\}\\ &\times\Big[\Pr\{\gamma_{2,0,1}<r_2\}+\Pr\{\mathrm{MRC}_1\}\times\Pr\{\gamma_{2,0,1}>r_2\}\Big].\end{aligned} \quad (19)$$

The exact outage probability of the BNC scheme is as again obtained by summing the results

$$P_{BNC} = P^1_{BNC}+P^2_{BNC}+P^3_{BNC}+P^4_{BNC}. \quad (20)$$

In the high SNRs, (20) is then given by:

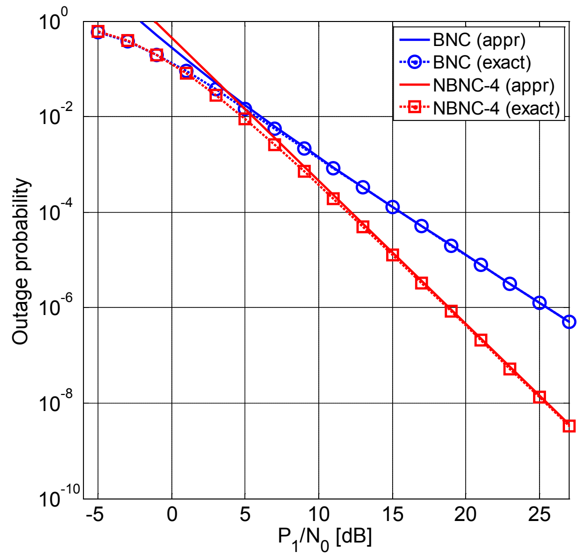

Figure 3. Exact outage probability with BNC and NBNC-4 for N1: $P_1 = P_2$, (a) $\sigma_{1,0}^2 = 1, \sigma_{1,2}^2 = 8, \sigma_{2,0}^2 = 8$

$$P_{BNC} \approx \frac{B_1}{P_1 P_2} + \frac{B_2}{P_1^2 P_2} + \frac{B_3}{P_1 P_2^2} + \frac{B_4}{P_1^3}, \tag{21}$$

where $B_1 := \frac{r_1 r_2 N_0^2}{\sigma_{1,0}^2 \sigma_{2,0}^2}$, $B_2 := \frac{r_1^2 r_2}{2\sigma_{1,2}^2 \sigma_{1,0}^2 \sigma_{2,0}^2} + \frac{r_1^2 r_2}{2\sigma_{1,2}^2 \sigma_{2,1}^2 \sigma_{1,0}^2}$, $B_3 := \frac{r_1 r_2^2 N_0^3}{\sigma_{2,1}^2 \sigma_{1,0}^2 \sigma_{2,0}^2}$, and $B_4 := \frac{r_1^2 r_2 N_0^3}{\sigma_{1,2}^2 \sigma_{1,0}^4}$.

We evaluate the outage probability of N1 for both BNC and NBNC-4 schemes in terms of the average SNRs and the transmission rates R_1 and R_2. We show that using NBNC-4 provides improved outage probabilities compared to BNC for different channel environments. In Figure 3, we show evaluation results for which the benefits of network coding can be obtained at mid to high SNR regions. We compare the asymptotic version of outage probabilities and the exact ones in Figure 3. Both asymptotic results (18) and (21) well match with the exact outage probabilities (17) and (20) in the high SNR region.

In order to investigate the influence of different channel gains, we assume that the transmit powers of the two nodes are equal, i.e., $P_1 = P_2$, and use the same transmission rates $R_1 = R_2 = 1$ b/s/Hz. As shown in Figures 3, we investigate the effect of the variances of channel gains. We can observe that the NBNC-4 scheme achieves a diversity order of 3, whereas a diversity order for the BNC scheme is 2. In Figure 3, we set all variances of the channel gains as $\sigma_{1,0}^2 = 1, \sigma_{1,2}^2 = 8, \sigma_{2,0}^2 = 8$. This means that the link quality between N2 and BS is better than N1-BS. Since the variance of the channel gain depends on the distance, the case of Figure 3 reflects the channel environments in which N2 is the middle of N1 and BS with the equal power allocation (EPA). Note that the diversity order for the three different schemes holds.

IV. Optimal Power Allocation

The power efficiency technique investigated in this study is transmit power allocation. We investigate this problem for the two network codes, using the outage analysis framework developed in Section III.B.

A. Fomulation of optimal power allocation

We assume that each node knows all the channel state information by using an appropriate channel feedback scheme. We investigate the outage performance of optimal transmit power allocation subject to a total power constraint. In other words, the OPA solution is obtained based on minimization of the outage probability under a total power constraint.

We use the outage probabilities (18) and (21), for the BNC and NBNC-4 schemes, to deal with the optimization problem. Note that these are functions of transmit powers, variances of channel gains, and transmission rates. The optimization problem can be written as follows, given the variances of the channel gains and the transmission rate,

$$\begin{aligned} &P_{out}\left(P_1^*, P_2^*, \sigma_{1,2}, \sigma_{1,d}, \sigma_{2,d}, R_1, R_2\right) \\ &= \min_{P_1, P_2} P_{out}\left(P_1, P_2, \sigma_{1,2}, \sigma_{1,d}, \sigma_{2,d}, R_1, R_2\right), \end{aligned} \tag{22}$$

subject to $P_1 + P_2 = P_t$, $P_1 \geq 0$, and $P_2 \geq 0$, where P_t is the total transmit power, and P_1^* and P_2^* denote the optimal transmit powers for two nodes. For the outage probability for the NBNC-4 scheme, the Lagrangian with Lagrange multiplier λ can be written as

$$L(P_1, P_2, \lambda) = \frac{A_1}{P_1^2 P_2} + \frac{A_2}{P_1 P_2^2} + \frac{A_3}{P_1^3} + \lambda\left(P_1 + P_2 - P_t\right) \tag{23}$$

Similarly, for the BNC scheme, the Lagrangian is

$$L(P_1, P_2, \lambda) = \frac{B_1}{P_1 P_2} + \frac{B_2}{P_1^2 P_2} + \frac{B_3}{P_1 P_2^2} + \frac{B_4}{P_1^3} + \lambda\left(P_1 + P_2 - P_t\right) \tag{24}$$

for either the BNC or NBNC-4 scheme. Using the first-order derivative condition, the optimal power must satisfy

$$\frac{\partial L(P_1, P_2, \lambda)}{\partial P_1} = \frac{\partial L(P_1, P_2, \lambda)}{\partial P_2} = 0 \tag{25}$$

To find the optimal transmit power P_1 at the source for both cooperative schemes, we use the following equations:

$$\lambda_{N,1} P_1^3 + \lambda_{N,2} P_1^2 + \lambda_{N,3} P_1 + \lambda_{N,4} = 0, \tag{26}$$

where $\lambda_{N,1} := 3A_1 - 3A_2 - 3A_3$, $\lambda_{N,2} := 9A_3 P_t - 5A_1 P_t + A_2 P_t$, $\lambda_{N,3} := 2A_1 P_t^2 - 9A_3 P_t^2$, $\lambda_{N,4} := 3A_3 P_t^3$; and

$$\lambda_{B,1} P_1^4 + \lambda_{B,2} P_1^3 + \lambda_{B,3} P_1^2 + \lambda_{B,4} P_1 + \lambda_{B,5} = 0, \tag{27}$$

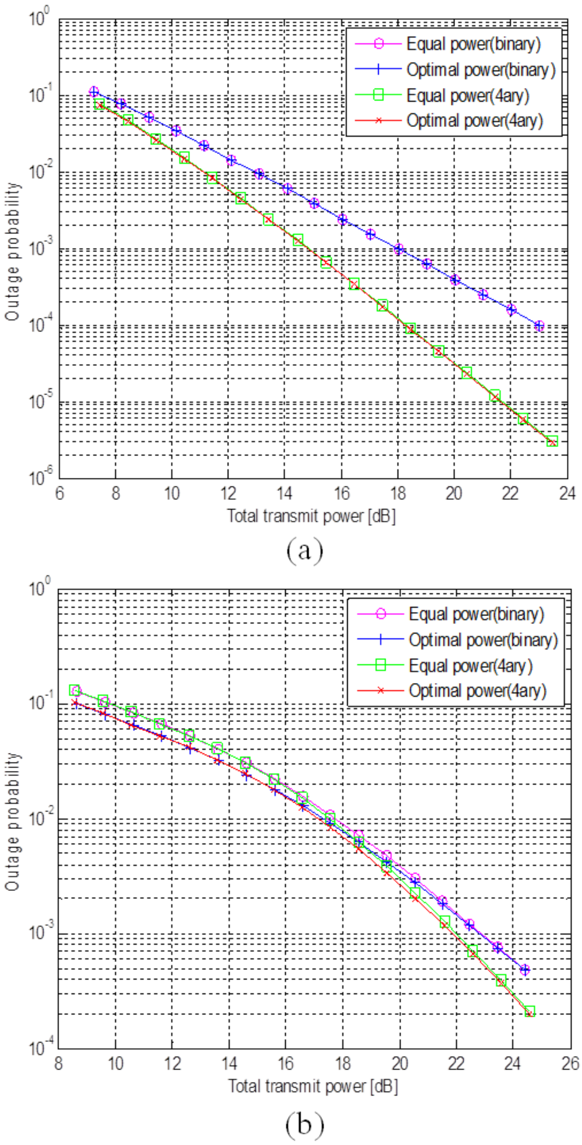

Figure 4. Outage probability as a function of total transmit power: (a) $\sigma_{1,d}^2 = \sigma_{1,2}^2 = \sigma_{2,d}^2 = 1$, (b) $\sigma_{1,d}^2 = 1, \sigma_{1,2}^2 = 0.037, \sigma_{2,d}^2 = 0.125$.

where $\lambda_{B,1} := 2B_1$, $\lambda_{B,2} := 3B_2 - 3B_3 - 3B_4 - 3B_1P_t$, $\lambda_{B,3} := B_1P_t^2 + 9B_4P_t - 5B_2P_t + B_3P_t$, $\lambda_{B,4} := 2B_2P_t^2 - 9B_4P_t^2$, $\lambda_{B,5} := 3B_4P_t^3$. For both cases, (26) and (27) correspond to the NBNC-4 and BNC schemes under the total power constraint. We investigate the effect of varying the variances of channel gains on the optimum ratio of power allocation, while the outage probability is minimized.

B. Discussion for various link qualities

In this subsection, we discuss optimal transmit power allocation for various channel environments. Let us consider two cases, *i)* $\sigma_{1,2}^2 = \sigma_{1,d}^2 = \sigma_{2,d}^2 = 1$, *ii)* $\sigma_{1,d}^2 = 1$, $\sigma_{1,2}^2 = 0.037$, $\sigma_{2,d}^2 = 0.125$, respectively. From them, one can find the exact outage probabilities by substituting them into (17) and (20). We fix the total transmit power, i.e., $P_t = P_1 + P_2$ at a particular level, and shows the outage probability as a function of total transmit power. The corresponding results are shown in Figure 4. Note that in the cases the link quality of the wireless channel, i.e., N1-to-N2 and N2-to-BS, is the same and they are good in terms of SNRs. In such cases, as indicated by Figure 4(a), EPA is as good as OPA. In the second case, OPA is obviously better than EPA in general; but this behavior is substantial only in the low-SNR region. From Figure 4(b), we note that as the total transmit power increases, EPA results approach the results of OPA, again.

V. CONCLUSIONS

In this paper, we investigated the impact of the outage performance on the field size of the linear network coding, GF(2) vs. GF(4). To evaluate the benefit of the increase of the field size, we derived the exact outage probability for the considered network coding schemes. We also obtained the asymptotic results of the exact outage probabilities for both BNC and NBNC-4 schemes, showed that our approximation expressions are well matched the exact ones in the high SNR regions. We then analyzed the diversity order for the both network coding schemes. Our results indicate that the diversity order using NBNC-4 is three, but that using BNC is only two. We showed that the optimal power allocation for both schemes are performed. For future work, it will be meaningful to see if the proposed NBNC scheme can be extended to a larger scale network where a more number of nodes are involved in cooperative networks.

REFERENCES

[1] J. L. Laneman, David N. C. Tse and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," IEEE Trans. Info. Theory, vol. 50, no. 12, pp. 3062–3080, 2004.

[2] R. Ahlswede, N. Cai, S.-Y. R. Li and R. W. Yeung, "Network information flow," IEEE Trans. Info. Theory, vol. 46, no. 4, pp. 1204–1216, 2000.

[3] M. Yu, J. Li, R. S. Blum, "User cooperation through network coding," in IEEE Intl. Conf. Commun., Glasgow, Scotland, pp. 4064–4069, 2007.

[4] C. Peng, A. Zhang, M. Zhao, Y. Yao and W. Jia, "On the performance analysis of network-coded cooperation in wireless networks," IEEE Trans. Wireless Commun., vol. 7, no. 8, pp. 3090–3097, 2008.

[5] X. Bao and J. Li, "Adaptive Network Coded Cooperative (ANCC) for Wireless Relay Networks: Matching Code-on-Graph with Network-on-Graph," IEEE Trans. Wireless Comm., vol. 7, no. 2, pp. 574–583, 2008.

[6] L. Xiao, T. E. Fuja, J. Kliewer and D. J. Costello, "A Network Coding Approach to Cooperative Diversity," IEEE Trans. Info. Theory, vol. 53, no. 10, pp. 3714–3722, 2007.

[7] T. E. Hunter, S. Sanayei and A Nosratinia, "Outage Analysis of Coded Cooperation," IEEE Trans. Info. Theory, vol. 52, no. 2, pp. 375–391, 2006.

[8] Y. Chen, S. Kishore and J. Li, "Wireless diversity through network coding," in IEEE Wire. Comm. Net. Conf. (WCNC), Las Vegas, USA, pp. 1681–1686, 2006.

[9] D. H. Woldegebreal and H. Karl, "Network-coding based adaptive decode and forward cooperative transmission in a wireless network: outage analysis," in 13th European Wireless Conference, Paris, France, pp. 1–6, 2007.

[10] M. Xiao and M. Skoglund, "Multiple-User Cooperative Communications Based on Linear Network Coding," IEEE Trans. Commu., vol. 58, no. 12, pp. 3345–3351, 2010.

[11] J. L. Rebelatto, B. F. U-Filho, Y. Li and B. Vucetic, "Multi-User Cooperative Diversity through Network Coding Based on Classical Coding Theory," IEEE Trans. Sig. Proc., vol. 60, no. 2, pp. 916–926. 2012.