A Calibration of the Modulated Wideband Converters with Practical and Unsophisticated Pilot Signals

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Abstract

We propose a calibration algorithm for the modulated wideband converters (MWC). The MWC is a sub-Nyquist sampler, which compresses and samples a wideband signal at the sub-Nyquist rate. For the reconstruction of the wide band signal from the compressed samples in the hardware-implemented MWC, a calibration of the actual system transfer is essential. Our algorithm well calibrates the actual system transfer with the negligible error by using pilot signals. The main advantage is that the algorithm does not require accurate phase of the pilot signals.

Keywords: Modulated wideband converter (MWC), calibration, pilot signals, phase estimation.

1. Introduction

Sub-Nyquist samplers take sample spectrally sparse signals with unknown frequency supports at a low sampling rate below the Nyquist rate. In the last decade, many sub-Nyquist samplers have been developed under theoretical supports of the compressed sensing theory [1]-[5]. Among them, the modulated wideband converter (MWC) [2] is especially efficient for sampling the multi-band signals whose spectra consist of multiple disjoint continuous bands. Moreover, the MWC is one of realizable sub-Nyquist samplers [6].

In the MWC, the reconstruction of the wideband input from the sub-Nyquist samples is done by exploiting system transfer relation. However, when the MWC is implemented, the analog circuits have numerous non-ideal and unexpected characteristics, which lead to the mismatch of the actual system transfer from a theoretical analysis. To trace the actual system transfer, calibration algorithms using pilot signals have been proposed [7]-[10]. In the literatures, a prior assumption was the exact knowledge of the phase of the pilot signals. However, the assumption may be impractical since the signal generator would have errors in adjusting the phase. Moreover, it is hard to exactly measure the phase transitions of the analogue paths. In [11], we recently have proposed a calibration algorithm with unknown phase of the pilots. The algorithm used the direct-current (DC) signal as a reference pilot to equalize the unknown phases of the other pilots. However, it may be not applicable for some analog circuits where the DC component is cut off due to the power consumption.

In this paper, we propose a calibration algorithm using unsophisticatedly generated pilots for the MWC. We do not assume precise knowledge about the phases of the pilots. In our algorithm, we estimate the system transfer and the phase of pilots at the same time by exploiting structural features of the transfer model of the MWC. In spite of the unsophisticated pilots, our algorithm produces the negligible calibration error.

2. Backgrounds and problem formulation

The MWC [2] consists of m analog channels, and each channel has a mixer, a low-pass filter, and a sampler in order (Fig. 1). The mixer at the i -th channel multiplies the input signal $x(t)$ by a pseudo-random (PR) signal in time-domain. The PR signal $p_i(t)$ is T_p - periodic and theoretically defined by

$$p_i(t) = a_{i,l} \text{ for } \frac{lT_p}{M} \le t < \frac{(l+1)T_p}{M} \tag{1}$$

for $l = 0, \cdots, M-1$ within a single period, where M is an odd number and $a_{i,l} \in \mathbb{R}$. Alternatively, due to the periodicity, $p_i(t)$ forms the Fourier series expansion as follow,

$$p_i(t) = \sum_{k=-\infty}^{\infty} c_{i,k} e^{j2\pi k f_p t}, \tag{2}$$

where $f_p = T_p^{-1}$ and $c_{i,k}$ are Fourier series coefficients of $p_i(t)$. The coefficients are computed by [2]

$$c_{i,k} = w_{i,k} \sum_{l=0}^{M-1} a_{i,l} e^{-j2\pi \frac{kl}{M}}, \quad (3)$$

where

$$w_{i,k} \triangleq \frac{1}{T_p} \int_0^{\frac{T_p}{M}} e^{-j\frac{2\pi}{T_p}kt} dt. \quad (4)$$

In the channel, the sampling period is chosen by $T_s = q^{-1}T_p$ for an odd number q, and the cut-off frequency of the low-pass filter is chosen by $f_s/2 = T_s^{-1}/2$ in order to avoid the aliasing. As the result, we can express the output $y_i[n]$ in terms of $x(t)$ by [2]

$$y_i[n] = \sum_{k=-L_0}^{L_0} c_{i,k}\, LPF_{f_s}\left\{ e^{j2\pi k f_p t} x(t) \right\}\Big|_{t=nT_s}, \quad (5)$$

where the low-pass filtering operation with a cut-off frequency $f_c/2$ is denoted by $LPF_{f_c}\{\cdot\}$, $L_0 = \lfloor L/2 \rfloor$, and $L = M + q$ [2]. The operator $\lfloor \cdot \rfloor$ denotes the flooring operation. By solving the equation (5) with respect to $LPF\left\{ e^{j2\pi k f_p t} x(t) \right\}\Big|_{t=nT_s}$, we can reconstruct the Nyquist samples of $x(t)$ from the compressed sub-Nyquist samples $y_i[n]$.

In (5), the unknowns are not spectrally orthogonal since the bandwidth f_s is q -times greater than the shifting intervals f_p. In [2], a digital processing is provided to keep the spectral orthogonality of the unknowns and to expand the number of equations for the reconstruction at the same time. We call it *channel expander*, which produces $z_{i,u}[n]$ by following procedure

$$z_{i,u}\left[\tilde{n}\right] = \left(y_i[n] e^{-j2\pi \frac{u}{q} n} * h[n] \right)\Big|_{n=q\tilde{n}} \quad (6)$$

for $u = -q_0, \cdots q_0$, where $h[n]$ is a digital low-pass filter with the cut-off frequency of $f_p/2$ and $q_0 = \lfloor q/2 \rfloor$. Then, the discrete-time Fourier transform (DTFT) of the consequent input-output relation is given by [2]

$$Z_{i,u}\left(e^{j2\pi f_p t}\right) = \sum_{k=-M_0}^{M_0} c_{i,k+u} X\left(f - kf_p\right) \quad (7)$$

for $-f_p/2 \le f < f_p/2$, where $X(f)$ is the Fourier transform of $x(t)$ and $M_0 = \lfloor M/2 \rfloor$.

In the perspective of implementations, we model that the PR signals are distorted. In specific, the distorted amplitudes $\tilde{a}_{i,l}$ are unknown and the actual system transfer coefficients $\tilde{c}_{i,k}$ are incomputable.

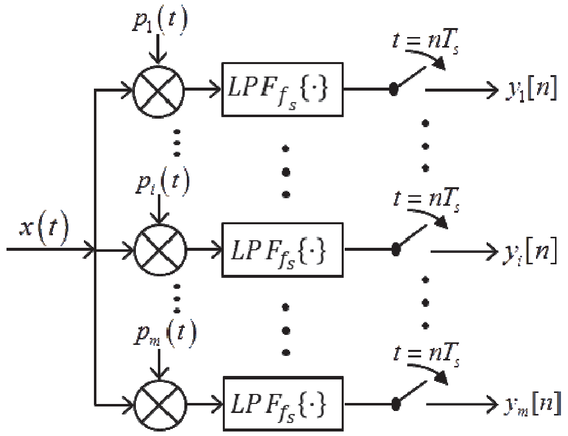

Fig. 1. Block diagram of the analog part of the MWC [2].

For the calibration of $\tilde{c}_{i,k}$, we can inject pilot signals and observe the outputs. However, when the phases of the pilots are unknown, the output does not directly indicate $\tilde{c}_{i,k}$. In [11], a calibration algorithm with equalizing the unknown pilot phases has been suggested. However, the algorithm required the DC pilot signal as reference, which is not practical for radio-frequency (RF) circuits. In conclusion, it is needed to calibrate $\tilde{c}_{i,k}$ by injecting sinusoid pilots with unknown phases.

3. Proposed calibration algorithm

We propose a calibration algorithm for the MWC. The proposed algorithm uses sinusoid pilot signals and does not require prior knowledge of the phases of the pilots or DC input. The proposed algorithm consists of three part; the phase estimator, the actual transfer coefficients estimator, and the final sign decision.

We model the pilot sinusoid signal of frequency vf_p for a positive integer v by

$$\begin{aligned} &x_v(t) \\ &= 2\cos\left(2\pi(v+\Delta_v) f_p t + \theta_v\right) \\ &= e^{j\theta_v} e^{j2\pi(v+\Delta_v) f_p t} + e^{-j\theta_v} e^{-j2\pi(v+\Delta_v) f_p t}, \end{aligned} \quad (8)$$

where θ_v are the unknown phase and an arbitrary real-valued differentiator $|\Delta_v| \in (0, 0.5)$. In other words, the spectrum $X_v(f)$ is modeled by

$$\begin{aligned} X_v(f) = e^{j\theta_v} \delta\left(f - (v+\Delta_v) f_p\right) \\ + e^{-j\theta_v} \delta\left(f + (v+\Delta_v) f_p\right). \end{aligned} \quad (9)$$

From the input-output relation (7), the outputs $Z_{i,u}^{(v)}$ of the channel expander from the input $X_v(f)$ are represented as

$$Z_{i,u}^{(v)}\left(e^{j2\pi f_p t}\right)=e^{j\theta_v}\tilde{c}_{k-v}\delta\left(f-\Delta_v f_p\right)+e^{-j\theta_v}\tilde{c}_{k+v}\delta\left(f+\Delta_v f_p\right). \tag{10}$$

Then, we can estimate $\tilde{c}_{i,k+v}$ and $\tilde{c}_{i,k-v}$ contaminated by the unknown phases θ_v from following relations

$$\begin{cases} Z_{i,k}^{+(v)} \triangleq Z_{i,k}^{(v)}\left(e^{j2\pi f T_p}\right)\Big|_{f=\Delta_v f_p} = e^{j\theta_v}\tilde{c}_{i,k-v} \\ Z_{i,k}^{-(v)} \triangleq \left(e^{j2\pi f T_p}\right)\Big|_{f=-\Delta_v f_p} = e^{-j\theta_v}\tilde{c}_{i,k+v}. \end{cases} \tag{11}$$

The first step of the proposed algorithm is to estimate the unknown phases θ_v. Based on the phase estimation, the transfer coefficients $\tilde{c}_{i,v+k}$ for $k=-q_0,\cdots,q_0$ are estimated. First of all, the proposed algorithm estimates θ_{M_0} by injecting $x_{M_0}(t)$. From (3), the transfer coefficients have a property of $\tilde{c}_{i,k}w_{i,k-M}=\tilde{c}_{i,k-M}w_{i,k}$. From this and the relation (11), we have

$$\frac{Z_{i,-1}^{+(M_0)}}{Z_{i,0}^{-(M_0)}}=\frac{e^{j\theta_{M_0}}\tilde{c}_{i,-1-M_0}}{e^{-j\theta_{M_0}}\tilde{c}_{i,M_0}}=\frac{e^{j\theta_{M_0}}\tilde{c}_{i,M_0}w_{i,-1-M_0}}{e^{-j\theta_{M_0}}\tilde{c}_{i,M_0}w_{i,M_0}}=e^{2j\theta_{i,M_0}}\frac{w_{i,-1-M_0}}{w_{i,M_0}}, \tag{12}$$

where $M=2M_0+1$, and this implies the phase estimator $\widehat{e^{j\theta_{M_0}}}$,

$$\widehat{e^{j\theta_{M_0}}}=\pm\sqrt{\frac{Z_{i,-1}^{+(M_0)}}{Z_{i,0}^{-(M_0)}}\cdot\frac{w_{i,M_0}}{w_{i,-1-M_0}}}. \tag{13}$$

Based on $\widehat{e^{j\theta_{M_0}}}$, the proposed algorithm estimates q transfer coefficients $\widehat{c_{i,M_0+k}}$ for $k=-q_0,\cdots,q_0$. For the phase estimation of the other pilots, we exploit another property. From the relation (7), outputs of the channel expander from distinct pilots of different frequencies can share common transfer coefficients. For example, using the relation (11), $\tilde{c}_{M_0-q_0}$ commonly appears in two outputs,

$$\begin{cases} Z_{i,q_0}^{-(M_0-(q-1))}=e^{-j\theta_{M_0-(q-1)}}\tilde{c}_{i,M_0-q_0} \\ Z_{i,-q_0}^{-(M_0)}=e^{-j\theta_{M_0}}\tilde{c}_{i,M_0-q_0}, \end{cases} \tag{14}$$

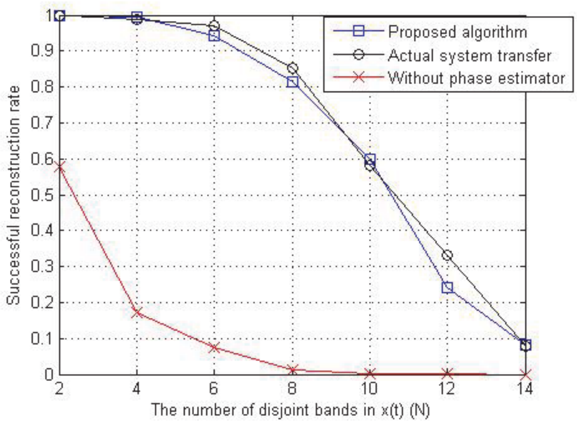

Fig. 2. Success rate of the frequency support reconstruction with varying the number of disjoint continuous bands N of the input multi-band signal.

where $q=2q_0+1$ and $e^{j\theta_{M_0}}$ is estimated by (13). By injecting $x_{M_0-(q-1)}(t)$ and referring to the relations (14) and (13), the proposed algorithm estimates the next phase $\widehat{e^{j\theta_{M_0-(q-1)}}}$,

$$\widehat{e^{j\theta_{M_0-q+1}}}=\frac{\widehat{e^{j\theta_{M_0}}}\cdot Z_{i,-q_0}^{-(M_0)}}{Z_{i,q_0}^{-(M_0-q+1)}}. \tag{15}$$

Again, based on $\widehat{e^{j\theta_{M_0-(q-1)}}}$, the proposed algorithm estimates $q-1$ transfer coefficients $\widehat{\tilde{c}_{i,M_0-q+1+k}}$ for $k=-q_0,\cdots,q_0-1$ and sequentially estimates the phase $\theta_{M_0-2(q-1)}$ of the next pilot $x_{M_0-2(q-1)}(t)$.

In the same manner, the proposed algorithm sequentially estimates the phases and corresponding transfer coefficients by consecutively injecting $x_{M_0-p(q-1)}(t)$ as increasing p. In summary, the phase estimator is defined by

$$\widehat{e^{j\theta_{M_0-p(q-1)}}}=\begin{cases} \pm\sqrt{\dfrac{Z_{i,-1}^{+(M_0)}}{Z_{i,0}^{+(M_0)}}\cdot\dfrac{w_{i,M_0}}{w_{i,-1-M_0}}}, & \text{if } p=0, \\ \dfrac{\widehat{e^{j\theta_{M_0-(p-1)(q-1)}}}\cdot Z_{i,-q_0}^{-(M_0-(p-1)(q-1))}}{Z_{i,q_0}^{-(M_0-p(q-1))}}, & \text{otherwise} \end{cases} \tag{16}$$

for $p=0,\cdots,\lceil(q_0-M_0)/(q-1)\rceil$, and the transfer coefficients estimator is defined by

$$\widehat{\tilde{c}_{i,k+M_0-p(q-1)}}\triangleq Z_{i,k}^{-(M_0-p(q-1))}\widehat{e^{j\theta_{M_0-p(q-1)}}} \tag{17}$$

for $k=-q_0,\cdots,q_0$. In (16), the sign of the estimator $\widehat{e^{j\theta_{M_0}}}$ of the first phase θ_{M_0} are still not decided. Since the estimations are sequentially conducted, the estimator consequently produces two sets of the transfer coefficient estimations having opposite signs. To find the correct estimator set, the proposed algorithm finally injects any non-negative pilot signal. By comparing the sign of reconstruction results by the two estimator sets, the algorithm finally decides the correct sign of the estimations.

4. Empirical results

To demonstrate the calibration performance of the proposed algorithm, we compare performances of reconstructing the frequency supports of a spectrally sparse signal input $x(t)$. We construct a noiseless simulation environment with the system parameters $m=4$, $f_s=221$ [MHz], $q=7$, and $M=127$. We generate random binary PR signals, $x(t)$ with N disjoint continuous bands of random frequency supports, and the pilot signals with random phases and conduct the proposed algorithm and reconstruction procedure over 500 trials. We count the number of successful reconstruction of the frequency supports among the total trials. We vary $N=2,4,6,\cdots,14$ and repeat the tests. Fig. 2 shows the results. The square marker represents the result using the calibrated system transfer by the proposed algorithm. The round marker represents the reconstruction using the actual system transfer for reference. The 'X' shaped marker represents the result using a calibration without the phase estimation, which ignores the unknown phase of the pilot signals. The result demonstrates that the proposed algorithm successfully calibrate the system transfer and estimate the unknown phases of the pilot signals at the same time.

5. Conclusion

In this paper, we have proposed a calibration algorithm for the MWC. Owing to the unexpected features in the electronic circuits, the calibration of the system transfer is essential for the hardware implemented MWC to reconstruct the input signal from the compressed samples. Our proposed algorithm had the advantage of requiring practical pilot signals. First, the phases of the pilot signals were not needed to be known. Predicting the phase of injected sinusoids to complicated electronic circuits is impractical. Without the knowledge of the phases, our algorithm calibrated the system transfer with the small errors. Secondly, real-valued pilot signals were exploited. By using real-valued sinusoids, our algorithm calibrated the complex-valued frequency response of the MWC. Lastly, the DC reference input was not required. Unlike our previous work in [11], the phase estimators did not require the DC reference. This makes the proposed algorithm to be more compatible with RF systems.

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