SPARS 2013

Signal Processing with Adaptive Sparse Structured Representations.
July 8-11, 2013. EPFL, Lausanne.

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The Signal Processing with Adaptive Sparse Structured Representations SPARS workshop seeks novel ideas and results, both experimental and theoretical, for adaptive sparse representations, sampling and computational methods for high-dimensional data that feature structured combinatorial and geometric foundations.

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- Big data applications including but not limited to geophysics, neuroscience, biomedical imaging, array processing, genetics, optics and radar, and feedback control

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Noisy Behavior of MAP-based Sparse Support Detection

Jaewook Kang, Heung-No Lee and Kiseon Kim
Gwangju Institute of Science and Technology (GIST), Republic of Korea. Email: {jwkkang,heungno,kskim}@gist.ac.kr

***Index Terms*—Sparse support detection, belief propagation, phase transition**

I. Introduction and Contribution

In the sparse recovery problems, most recovery algorithms have been developed by focusing on signal estimation, rather than the sparse support detection (SSD). Therefore, those algorithms may not be optimal in terms of SSD. Recently, a few theoretical studies have also indicated that existing estimation-based algorithms have a potentially large gap from the theoretical limit of the support recovery [1],[2]. This paper introduces a detection-oriented algorithm named BHT-BP [3]. The algorithm is designed under the MAP-detection criterion, and it is noteworthy as a low-computational approach using belief propagation algorithm and sparse binary measurement matrices.

The main contribution of this paper is to investigate noisy behavior of the BHT-BP recovery using a phase transition (PT) framework. The PT-framework provides precise guidance on noise level and the signal strength, needed to detect the sparse support set. For a comparison purpose, we consider an estimation-based algorithm, CS-BP [4], which also uses belief propagation algorithm and sparse binary measurement matrices. Our analytical result shows that in the PT diagram the detection success region of BHT-BP is larger than that of CS-BP.

II. Signal Model

We consider a signed sparse signal $\mathbf{X} \in \mathbb{R}^N$ whose elements are *i.i.d.* and have fixed magnitude $|x_{0,i}|$. The support knowledge of $\mathbf{X}$ is represented by a state vector $\mathbf{S} \in \{0,1\}^N$ where each element S_i indicates the supportive state of a signal element $X_i \in \mathbf{X}$,*i.e.*,

$$S_i = \begin{cases} 1, & \text{if } X_i \neq 0 \\ 0, & \text{else} \end{cases} \quad \forall\, i \in \{1, ..., N\}. \tag{1}$$

Then, the support detector observes a noisy measurement vector, given as $\mathbf{Y} = \mathbf{\Phi}\mathbf{X} + \mathbf{W} \in \mathbb{R}^M$, where $\mathbf{\Phi} \in \{0,1\}^{M \times N}$ is a sparse binary matrix and $\mathbf{W} \in \mathbb{R}^M$ is a vector for zero-mean Gaussian noise drawn from $\mathcal{N}(0, \sigma_W^2 \mathbf{I})$.

III. Problem Statement and Main Result

Support detection of BHT-BP and CS-BP is performed in an elementwise manner based on the *decoupling principle* [5]. Therefore, the marginal posterior density $f_{X_i}(x|\mathbf{Y}, \mathbf{\Phi})$ of the signal can be obtained from belief propagation iteration. The CS-BP algorithm is an estimation-based algorithm which obtains a scalar estimate $\widehat{X}_i$ directly from the marginal posterior using the MAP or the MMSE estimator [4]. Therefore, in CS-BP, the supportive state is determined by the value of $\widehat{X}_i$. Accordingly, if CS-BP uses the MAP-estimator ,*i.e.*, $\widehat{X}_{\text{MAP},i} := \arg\max_{x \neq 0} f_{X_i}(x|\mathbf{Y}, \mathbf{\Phi})$, its support detection function can be described by

$$h_{\text{CS-BP},i} := \log \frac{f_{X_i}(x = \widehat{x}_{\text{MAP},i}|\mathbf{Y}, \mathbf{\Phi})}{f_{X_i}(x = 0|\mathbf{Y}, \mathbf{\Phi})} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} 0, \tag{2}$$

where $\mathcal{H}_0 := \{S_i = 0\}$ and $\mathcal{H}_1 := \{S_i = 1\}$ denote two possible hypotheses.

In contrast, for a detection-oriented algorithm BHT-BP, finding the sparse support set is an end in itself. Therefore, the detection function of BHT-BP is designed from the binary MAP-detection for the supportive state S_i, given by

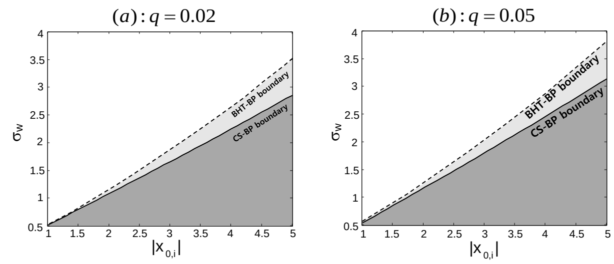

Fig. 1. PT diagram for elementwise support detection. The dashed curve and solid curve indicate BHT-BP and CS-BP, respectively. In these figures, the region above the curves corresponds to the SSD-failure and the region below corresponds to the SSD-success.

$$h_{\text{MAP},i} := \log \frac{\Pr\{S_i = 1|\mathbf{Y}, \mathbf{\Phi}\}}{\Pr\{S_i = 0|\mathbf{Y}, \mathbf{\Phi}\}} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} 0. \tag{3}$$

Using the Bayesian rule, from (3), the detection function of BHT-BP is defined as [3]

$$h_{\text{BHT-BP},i} := \log \frac{q}{1-q} + \log \frac{\int \frac{f_{X_i}(x|S=1)}{f_{X_i}(x)} f_{X_i}(x|\mathbf{Y}, \mathbf{\Phi}) dx}{\int \frac{f_{X_i}(x|S=0)}{f_{X_i}(x)} f_{X_i}(x|\mathbf{Y}, \mathbf{\Phi}) dx} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} 0. \tag{4}$$

where $f_{X_i}(x) = q f_{X_i}(x|S = 1) + (1-q) f_{X_i}(x|S = 0)$ is a sparsifying prior with a rate $q \in (0, 1]$ which is the probability that an element belongs to the signal support.

We draw a phase transition diagram on the plane of noise level σ_W and signal magnitude $|x_{0,i}|$ using the detection function of CS-BP (2) and BHT-BP (4), as shown in Fig.1. For the sake of this, we first obtain the marginal posterior expression as a function of σ_W and $|x_{0,i}|$. Then, using the result, we analyze the failure event for each detection function and compare the failure condition of CS-BP and BHT-BP. For the detail explanation, please see our regular paper [6].

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