An Inventory Model-based Spectrum Pooling for Wireless Service Provider and Unlicensed Users

Sang-Seon Byun, Ilangko Balasingham, and Heung-No Lee

Abstract

In order to comply with more opportunistic and transient service requirements, many research literatures have paid attention to an approach called spectrum pooling. If we imagine a scenario in which wireless service providers (WSPs) manage the spectrum pool, the radio spectrum itself will be traded as done in a market-based scenario, and moreover, every WSP will likely require new strategies in order to make more profits under such dynamic service requirements. Up to now, most researches have concentrated on investigating the inter-WSP strategies such as market equilibrium under the consideration of noncooperative or cooperative WSPs. In this paper, we direct our attentions to an intra-WSP strategy: a WSP coordinates the spectrum order in order to minimize its cost. To this end, we deploy a probabilistic inventory model that helps WSPs to determine economic order quantity and reorder point yielding minimum total expected cost. The total cost is composed of ordering cost, holding cost, and stockout cost. We believe that this model will be the first investigation of intra-WSP strategy that manages and estimates economic aspects of the spectrum pooling.

I. Introduction

In most countries, chunks of spectrum are statically allocated to the wireless service providers (WSPs) [1]; this often leads to spectral overcrowding and low spectrum utilization as reported in [2]. The static allocation also results in large amount of spectrum holes that cannot be allocated to licensed/unlicensed services.

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The *spectrum pooling* [3], [4] scheme can resolve such inflexibilities and inefficiencies of static allocation by opening unassigned or lightly used spectrums to unlicensed users. Spectrum pooling is a spectrum management principle that enables to utilize spectrums not used by licensed users, called *coordinated access band* (CAB) [5]; CABs are gathered and put into a pool from which unlicensed users can rent spectrums. In the scenario of spectrum pooling, the licensed (or primary) user implies the user who has the right to access the spectrums freely; on the other hand, the unlicensed (or secondary) user implies the user that can access the spectrums only when the spectrums are not used by the licensed users.

In the future, it is expected to see more dynamic service offerings and profiles, as users move from long-term or permanent service provider agreements to more opportunistic and transient service models. If we imagine a scenario where wireless service providers (WSPs) manage the spectrum pool, it is feasible that the radio spectrum itself will be traded as done in a market-based scenario where WSP will likely adopt new strategies in order to realize the full potential for profit under uncertain and dynamic spectrum requirements of unlicensed users. Along with this scenario, there have been a plentiful number of research efforts to apply market-based approaches to the spectrum pooling; some of them are [6], [7], [8], [9], and [10]. However, all of them have been concerned with *inter-WSP* or *inter-buyer strategies* - that is, strategic behavior of each WSP or buyer in accordance with behaviors of the other WSPs or buyers - and investigated a market equilibrium that defines a price where the amount of the spectrum demand equals the amount of the supply.

In this paper, we direct our attentions to *intra-WSP strategy*: each WSP minimizes the cost for securing the pool of unused spectrums with fulfilling the demands of unlicensed users instantly. We consider that a WSP leases spectrums from spectrum owner, e.g., Federal Communication Commission in the USA, with a certain price, and then sells the spectrums to the unlicensed users in the form of services (bandwidth) [11]. We also consider the situation where the number of spectrum demands by the unlicensed users is not known deterministically, that is, only statistic parameters such as mean and standard deviation are given. In such a scenario, the goal of WSP is to minimize the maintenance cost of the spectrum pool and maximize the profits while preventing the WSP from ordering excessive quantity of spectrums, which means low spectral efficiency. To this end, we envisage an *inventory model*-based spectrum pooling.

One class of general inventory problems requires that the order quantity decision be made

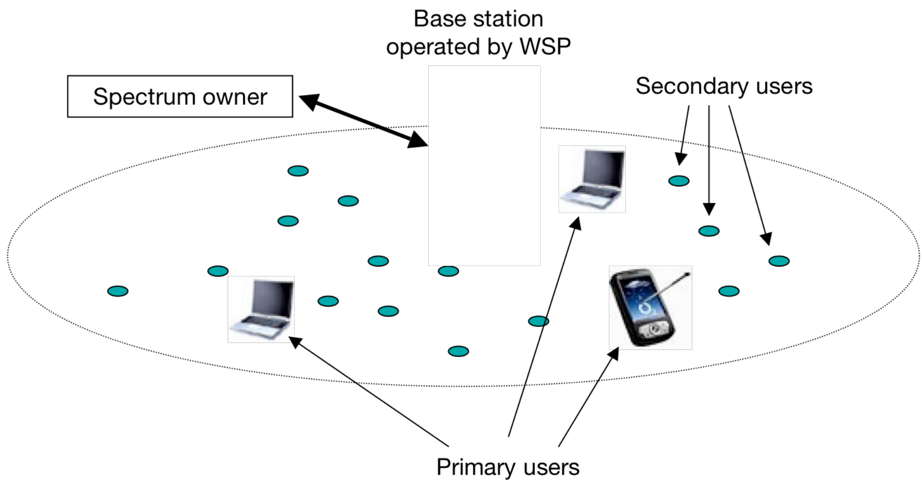

Fig. 1. The system model.

whenever or ideally before a stockout happens. That is, the opportunity to replenish the pool may occur before it becomes depleted. Given that the total spectrum demand over the period is uncertain, the dilemma is to order sufficient so that the full potential for profit can be realized while avoiding or minimizing loss due to excessive ordering. In this paper, we propose a probabilistic inventory model-based approach (Ch. 8 in [12]) that helps WSPs to determine the economic order quantity (EOQ) of spectrums and reorder point towards achieving minimum total expected cost (TEC) under uncertain user spectrum demands.

The remainder of this paper is organized as follows: in Section II, the basic system model and probabilistic inventory theory for the spectrum pooling are given. In Section III, we present an iterative method that can determine the best order quantity and reorder point. We evaluate the proposal by numerical experiments in Section IV. Finally, we conclude the paper in the last section.

II. System Model

Prior to giving the system model, the following assumptions are made:

1) For the simplicity, the CAB is composed of noncontinuous multiple homogeneous subcarriers that will be the basic resolution used for transmission with regard to the bandwidth and not with regard to the frequency.

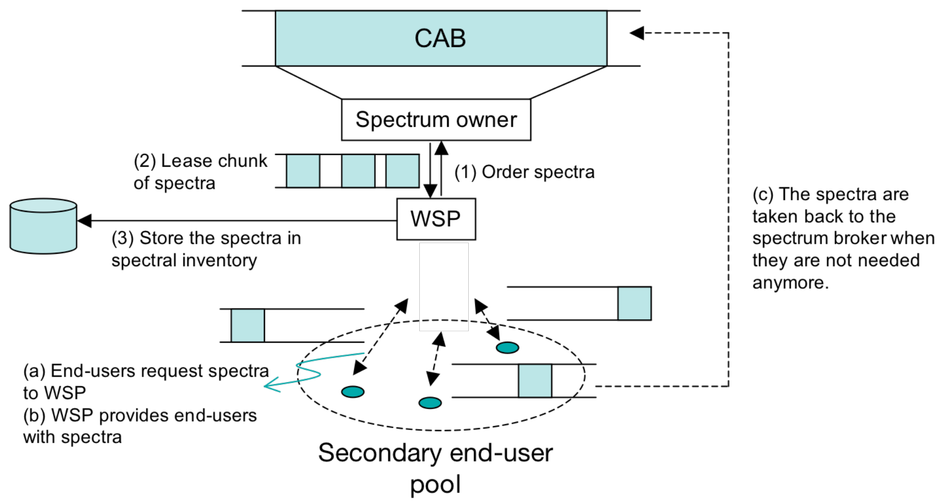

Fig. 2. Asynchronous scenario of the inventory model-based spectrum pooling: the procedure (1) $\sim$ (2) occurs in asynchronous with the procedure (a) $\sim$ (c).

2) We consider single-cell multiuser wireless networks with a single WSP as illustrated in Fig.1.
3) The subcarriers in the CAB are not used by the licensed users.
4) Asynchronous model is considered as described in Fig.2 and its caption: whenever the WSP needs spectrum resources, it makes a request to the spectrum owner, and then the spectrum owner leases a chunk of subcarriers within the CAB to the WSP.
5) The WSP stores the leased subcarriers at its spectrum inventory.
6) The WSP should pay the ordering and holding cost per subcarrier to the spectrum owner, and the holding cost is proportional to the lease period.
7) The WSP makes a profit on selling subcarriers to unlicensed users with higher prices than the holding cost plus the ordering cost. In this paper we concentrate on minimizing the cost (i.e., TEC).
8) Whenever user demands arrive, the WSP assigns subcarriers exclusively from its inventory to users.
9) The subcarriers assigned to unlicensed users are withdrawn and restored to the spectrum owner when the users do not need them anymore.

10) The CAB may be composed of white space in time domain [13] or frequency bands [14]. In this paper, we consider only the CAB in the frequency domain.
11) We assume continuous review[1] and fixed lead-time[2].
12) Both back-orders case and lost-sales case are treated.

Based on the above assumptions, the probabilistic inventory model for the spectrum pooling is given as follows:

There is a fixed and predefined lead-time for spectrum replenishment, denoted by L. Although there is no uncertainty in L, the spectrum demand during L can be unpredictable[3]. This means that the ordering for spectrum replenishment should be initiated in anticipation with the possibility of running out of subcarriers. If spectrums are ordered too early, the holding cost increases. On the other hand, if spectrums are ordered too late, it will incur either back-orders or lost-sales. Besides, there will be a hindrance to real-time market when back-orders or lost-sales occur. Therefore the policy assumed at this model is based on that the replenishment order is initiated when the spectral inventory level, that is, the current number of subcarriers stored in the spectral inventory, falls below a certain value called reorder point (denoted r) in order to minimize any profit loss due to either back-orders or lost-sales and extra holding cost. The value r is one of the quantities to be determined in this inventory model. The other decision variable is Q, the EOQ. That is, Q subcarriers are ordered whenever the inventory level falls to r. Since orders always occur when the inventory level is r, there is no reason to vary the order quantity from one time to next.

In either back-orders case or lost-sales case, p is used as the penalty cost per subcarrier[4], i.e., if the spectrum inventory is exhausted, each subcarrier demanded up until the stock is replenished costs p dollars. Note that this penalty is independent of the stockout duration.

[1]The status of the spectral inventory is known at all time

[2]If an order for replenishment must be placed some fixed time in advance, that is, if there is a delay between placement of the order and receipt of the subcarriers, then it is only necessary to anticipate sufficiently far in advance when the spectral inventory will be exhausted and to place the order at the time such that the subcarriers will arrive exactly when the spectral inventory runs out.

[3]In general, lead time can be unpredictable as well due to several factors such as changes in policies and fluctuation in processing time for handling spectrum requests. Therefore, it is more practical to address random lead time. However we assume fixed lead time which is consistent and known to service provider in advance.

[4]We assume identical penalty costs in both cases only for the simplicity. However different penalty costs also can be applied.

The fixed cost of placing an order represented by a, which is used in order to penalize frequent replenishments. The holding cost per subcarrier per unit period is denoted by h, and the cost per subcarrier by c. The unit period can be any time unit that is consistent, e.g., a minute or an hour. The demand rate is denoted by d that is an expected value of a random variable. The units of d are the number of subcarriers per unit period. The probability distribution of spectrum demand - that is, the number of demanding spectrums - during a lead-time L is given by the density function $f(x)$. We assume $f(x)$ follows Gaussian normal distribution; this assumption is justified by central limit theorem that is discussed in Section III. Then the mean value of this probability distribution is represented by μ. The first step in the formulation of the TEC function is to identify the separate components of cost. Accordingly, let

$$\mathrm{TEC}(Q, r) = \mathrm{OC} + \mathrm{SC} + \mathrm{HC} \tag{1}$$

where $\mathrm{TEC}(Q, r)$ is a function of the decision variables Q and r, OC stands for ordering cost, SC for stock cost, and HC for holding cost.

A. Ordering Cost

The ordering cost can be expressed as

cost per order $\times$ *expected number of cycles per unit period.*

Since d is the expected demand per unit period, and Q is the amount sold per cycle, the expected number of cycles per unit period is d/Q, provided that all spectrum demands are met. So, for both the back-orders and lost-sales case,

$$\mathrm{OC} = (a + cQ) \times \frac{d}{Q} = a\frac{d}{Q} + cd. \tag{2}$$

If we assume MQAM (M-ary Quadrature Amplitude Modulation), then d is determined as follows:

Let V and S be the set of all end-users and the set of all subcarriers, respectively, and r_i be the rate achieved by end user $i \in V$. Then

$$r_i = W \sum_{s \in S} \log_2 \left(1 + \frac{P_{is} G_{is} c_3}{\eta} \right) \tag{3}$$

where W is the bandwidth of each subcarrier, P is the is transmission power for the end-user i in the subcarrier s, G_{is} is channel gain, η is the thermal noise, and $c_3 = 1.5/\ln(0.2/\mathrm{BER})$ [14].

In addition, it is assumed that each end-user i wants to meet the rate and power constraints as shown in the following:

$$r_i \geq R_i^{\min}, \tag{4}$$

and

$$\sum_{s \in S} P_{is} \leq P_i^{\max} \tag{5}$$

where R_{min} is the minimum rate required by user i, and P_i^{max} is the maximum transmission power of user i.

If each end-user i has all the information about W, G_{is}, and η, it can decide the number of subcarriers n_i needed in order to be subject to the rate and power constraints. Thus

$$d = \sum_{i \in V} n_i. \tag{6}$$

B. Stockout Cost

The expected stockout cost in either the back-orders or lost-sales case is expressed as

cost per back order or lost sale

× *expected number of back ordered subcarriers or lost sales per cycle*

× *average number of cycle per unit period.*

If random variable x represents the number of spectrums demand during a lead-time, then the number of back-orders or lost-sales is

$$\begin{cases} 0 & \text{if } x \leq r \\ x - r & \text{otherwise} \end{cases} \tag{7}$$

Let $B(r)$ denote the expected number of lost-sales or back-orders per cycle. Then

$$\mathrm{SC} = p\frac{d}{Q}B\left(r\right) \tag{8}$$

where

$$B\left(r\right) = \int_0^r 0f\left(x\right)dx + \int_r^\infty \left(x - r\right)f\left(x\right)dx = \int_r^\infty \left(x - r\right)f\left(x\right)dx. \tag{9}$$

C. Holding Cost

The expected holding cost per unit period is defined as

$$h \times \textit{average spectral inventory over a unit period}.$$

We assume that the average spectral inventory over a unit period is the same as the average inventory over a typical cycle, whereby the behavior of a typical cycle is identical to the expected behavior of all cycles.

Based on the above assumption, we consider the back-orders case first. Since the inventory level is r when the order is initiated, and the expected number of spectrum demands during L is μ, the expected inventory level just before the order arrives is $r - \mu$, and the expected spectral inventory level at the start of a cycle is $Q + r - \mu$. The spectrum demand process then depletes the spectral inventory, another replenishment order is placed, and the cycle ends with an expected inventory level of $r - \mu$. We assume the average spectral inventory over this cycle is

$$\frac{Q}{2} + r - \mu. \tag{10}$$

By this argument, the expression for the expected holding cost per unit period for back-orders case is

$$\mathrm{HC} = h\left(\frac{Q}{2} + r - \mu\right). \tag{11}$$

In lost-sales case, the expected spectral inventory level just before the order arrives is not quite $r - \mu$ since now the spectral inventory level is not allowed to go under zero. Let x be the number of spectrum demands during the lead-time L. Then the spectral inventory level is

$$\begin{cases} r - x & \text{if } x \leq r \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

Taking the expected value of this function of x yields the expected inventory level,

$$\begin{aligned} & \int_0^r (r-x) f(x)\,dx + \int_r^\infty 0 f(x)\,dx \\ = \; & \int_0^\infty (r-x) f(x)\,dx - \int_r^\infty (r-x) f(x)\,dx \\ = \; & r\int_0^\infty f(x)\,dx - \int_0^\infty x f(x)\,dx + \int_r^\infty (x-r) f(x)\,dx \\ = \; & r - \mu + B(r). \end{aligned} \tag{13}$$

In words, the expected spectral inventory level just before replenishment is greater by an amount that is equal to the expected number of lost-sales. Thus the expected holding cost per unit period for lost-sales case is given by

$$\mathrm{HC} = h\left(\frac{Q}{2} + r - \mu + B\left(r\right)\right). \tag{14}$$

III. Determination of Q and r for Minimum TEC

The total expected cost per unit period is therefore

$$\mathrm{TEC}\left(Q, r\right) = \frac{ad}{Q} + cd + \frac{pd}{Q}B\left(r\right) + h\left(\frac{Q}{2} + r - \mu\right) \tag{15}$$

in the back-orders case, or

$$\mathrm{TEC}\left(Q, r\right) = \frac{ad}{Q} + cd + \frac{pd}{Q}B\left(r\right) + h\left(\frac{Q}{2} + r - \mu + B\left(r\right)\right) \tag{16}$$

in the lost-sales case. This is the function of Q and r that is to be minimized; this means we find the order quantity and reorder point that yield the smallest expected total cost per unit period.

Now we show that the TEC function is convex:

By Leibniz rule, we get

$$\frac{dB\left(r\right)}{dr} = -\int_r^\infty f\left(x\right)dx, \tag{17}$$

and

$$\frac{\partial^2}{\partial r^2}B\left(r\right) = -\left(-f\left(x\right) + \underbrace{\int_r^\infty \frac{\partial}{\partial r} f\left(x\right)dx}_{=0}\right) = f\left(r\right). \tag{18}$$

Then the Hessian matrix of TEC function is given by

$$\begin{bmatrix} \frac{ad}{Q^3} + cd + \frac{pd}{Q^3}B\left(r\right) & \frac{pd}{Q^2}\int_r^\infty f\left(x\right)dx \\ \frac{pd}{Q^2}\int_r^\infty f\left(x\right)dx & \frac{pd}{Q}f\left(r\right) \end{bmatrix} \tag{19}$$

in the back-orders case, or give by

$$\begin{bmatrix} \frac{ad}{Q^3} + cd + \frac{pd}{Q^3}B\left(r\right) & \frac{pd}{Q^2}\int_r^\infty f\left(x\right)dx \\ \frac{pd}{Q^2}\int_r^\infty f\left(x\right)dx & \left(\frac{pd}{Q} + \frac{h}{2}\right)f\left(r\right) \end{bmatrix} \tag{20}$$

in the lost-sales case. Assuming $h > 0$ and $Q > 0$ for all cycles, $\displaystyle\int_r^\infty f\left(x\right)dx > 0$ by (25) and (26). Moreover, it is natural to assume $d > 0$ and $f(r) > 0$ for all $r \in R+$. Therefore, we see that the Hessian matrix of TEC function is positive definite in both cases, and which proves the strict convexity of the TEC function [15].

By the strict convexity, the TEC function has a unique minimum that can be found at the point where the partial derivatives with respect to both Q and r vanish.

Taking the partial derivative with respect to Q,

$$\frac{\partial \mathrm{TEC}\,(Q,r)}{\partial Q} = -\frac{ad}{Q^2} - \frac{pd}{Q^2}B\,(r) + \frac{h}{2} \tag{21}$$

in both of the cases.

Setting it equal to zero and solving for Q, we find

$$Q = \sqrt{\frac{2d\,[a + pB\,(r)]}{h}}. \tag{22}$$

Taking the partial derivative with respect to r, we get

$$\frac{\partial \mathrm{TEC}\,(Q,r)}{\partial r} = h + \frac{pd}{Q}\frac{dB\,(r)}{dr} \tag{23}$$

in the back-orders case, or

$$\frac{\partial \mathrm{TEC}\,(Q,r)}{\partial r} = h + \left(\frac{pd}{Q} + h\right)\frac{dB\,(r)}{dr} \tag{24}$$

in the lost-sale case.

Substituting (17) back into the partial derivative - that is, equations (23) and (24) -, and setting them equal to zero, we get

$$\int_r^\infty f\,(x)\,dx = \frac{hQ}{pd} \tag{25}$$

in the back-orders case, or

$$\int_r^\infty f\,(x)\,dx = \frac{hQ}{pd + hQ} \tag{26}$$

in the lost-sales case.

The right-hand sides of equations (25) and (26) are called critical ratio. The left-hand side is the probability that the demand during L exceeds r, i.e., the reorder point r should be set high enough so that the probability of running out is just equal to the critical ratio. Although we now have expressions in terms of both Q and r, the evaluation of each of the expressions requires the value of the other. Thus we need an iterative approach called Gauss-Seidal algorithm (Ch. 3 in [16]) that guarantees the convergence to the minimum of the TEC function with a certain precision bound whenever the function is convex (Prop. 3. 9 in [16]). It proceeds as follows:

STEP 1: Let $k = 1$.

STEP 2: Let $B(r_k) = 0$, and solve for

$$Q_k = \sqrt{\frac{2d\left[a + pB\left(r_k\right)\right]}{h}} \tag{27}$$

in both cases.

STEP 3: Using Q_k , find r_k from the critical ratio equations:

$$\int_{r_k}^{\infty} f\left(x\right) dx = \frac{hQ_k}{pd} \tag{28}$$

in the back-orders case, or

$$\int_{r_k}^{\infty} f\left(x\right) dx = \frac{hQ_k}{pd + hQ_k} \tag{29}$$

in the lost-sales case.

STEP 4: Using r_k, evaluate

$$B\left(r_k\right) = \sigma f\left(r_k\right) + \left(\mu - r_k\right) G\left(r_k\right) \tag{30}$$

where σ is the standard deviation of the spectrum demand during L, and $G(x)$ is the complementary cumulative distribution function of $f(x)$.

STEP 5: $k = k + 1$, and go to **STEP 2**.

STEP 6: Repeat **STEP 2** $\sim$ **5** until the changes occurred in the values of Q_k and r_k are smaller than ϵ. The final values obtained are the optimal order quantity and reorder point.

If we assume $f(x)$ is Gaussian normal density function, $B(r)$ can be evaluated by equation (30). The consideration that the demand during the lead time is in fact normal is frequently justified by the central limit theorem [17]. Rigorously mentioning, random variable indicating the demand of individual user during the lead time is independent since the minimum rate requirement (R_i^{min}) and power constraint (P_i^{max}) of each user are fully uncorrelated to those of the other users. Then, the total demand can be written as a sum of the demands of individual end users. Therefore the total should approach a normally distributed random variable in the above algorithm if there are enough of them, and even though their distributions are not identical.

In order to determine r_k in **STEP 4**, we first standardize $f(x)$, and then find the value closest to the right-hand side of (28) or (29) in the statistical table of complementary cumulative normal distribution. Then we obtain r_k by reading z-value in the table. Moreover, most mathematical programming languages or libraries provide a function with which the z-value can be computed directly: for instance, the function **gsl_cdf_ gaussian_Qinv()** in GNU scientific library [18].

TABLE I

MAIN EXPERIMENTAL PARAMETERS.

Parameter	Value
Unit period	100 ticks
Lead time	10 ticks
Mean demand	Normally distributed with a mean of 1000 and a standard deviation of 250
Ordering cost (per order)	100
Holding cost (per subcarrier & unit period)	4.0
Cost per subcarrier	2.0
Initial inventory level	200
ϵ	0.00001

IV. NUMERICAL EXPERIMENTS

In this section, we evaluate the inventory model with numerical experiments. At first, we illustrate the convergence of the iterative approach given in Section III. Then we compare the optimal total cost that is yielded by the optimal $\langle Q, r\rangle$ with non-optimal ones. We apply the iterative approach in order to find the best $\langle Q, r\rangle$, and then compute the TEC using the equation (15) in the back-orders case or (16) in the lost-sales case.

We also simulate the inventory model in order to measure the *actual total cost* with various pairs of $\langle Q, r\rangle$. As shown in the equations (15) and (16), the TEC depends on the expected number of demands per unit period, i.e., d, and the expected number of lost-sales or back-orders during lead time, i.e., $B\left(r\right)$. However, the exact values that d and $B\left(r\right)$ have may not be realizable in actual case; therefore we need to illustrate the inventory model with the actual values of d and $B\left(r\right)$ sampled from a given distribution. For these simulations, we divide each unit period into 100 ticks, and let reordered subcarriers arrive at the beginning of a tick if a reorder occurs previously. In addition, we let end-users' purchasing occur at the end of each tick, and generate the spectrum demand following $\mathcal{N} \sim (1000, 250^2)$ on every period - that is the spectrum demand per tick follows $\mathcal{N} \sim (10, 2.5^2)$. The additional parameters for the numerical experiments and simulations are listed in Table I.

Table II and III list $\langle Q, r\rangle$, and $B(r)$ achieved on each iteration by the iterative approach in the back-orders case and lost-sales case, respectively. It is observed that the algorithm converges to the optimal $\langle Q, r\rangle$ after 5th iteration in the back-orders case and 6th iteration in the lost-sales

TABLE II

COMPUTATION RESULTS OF $\langle Q, r\rangle$ AND $B(r)$ ON EACH ITERATION IN THE BACK-ORDERS CASE.

Iteration number	Q	r	$B(r)$
1	632.45532	135.286088	0.893844
2	643.662568	135.049967	0.912676
3	643.896586	135.045070	0.913070
4	643.901585	135.044965	0.913078
5	643.901587	135.044965	0.913078

TABLE III

COMPUTATION RESULTS OF $\langle Q, r\rangle$ AND $B(r)$ ON EACH ITERATION IN THE LOST-SALES CASE.

Iteration number	Q	r	$B(r)$
1	632.45532	136.297475	0.816844
2	642.704834	136.101002	0.831346
3	642.885325	136.097566	0.831602
4	642.888505	136.097505	0.831606
5	642.888561	136.097504	0.831606
6	642.888562	136.097504	0.831606

TABLE IV

COMPUTATION RESULTS OF TEC AND ACTUAL TOTAL COST PER UNIT PERIOD ACCORDING TO VARIOUS $\langle Q, r\rangle$ IN THE BACK-ORDERS CASE. EACH ACTUAL TOTAL COST IS AVERAGED OVER 1000 DIFFERENT MEASUREMENTS.

$\langle Q, r\rangle$	TEC	Actual total cost
$\langle 644, 135\rangle$ (optimal)	**2339.450797**	**2334.37212**
$\langle 600, 100\rangle$	2383.157047	2354.70207
$\langle 600, 160\rangle$	2347.120074	2347.38654
$\langle 700, 100\rangle$	2374.848897	2353.373735
$\langle 700, 160\rangle$	2348.245778	2348.57604
$\langle 500, 50\rangle$	2701.698141	2739.00922
$\langle 800, 200\rangle$	2375.000893	2376.93254
$\langle 300, 20\rangle$	3435.061749	3500.909220

TABLE V

COMPUTATION RESULTS OF TEC AND ACTUAL TOTAL COST PER UNIT PERIOD ACCORDING TO VARIOUS $\langle Q, r\rangle$ IN THE LOST-SALES CASE. EACH ACTUAL TOTAL COST IS AVERAGED OVER 1000 DIFFERENT MEASUREMENTS.

$\langle Q, r\rangle$	TEC	Actual total cost
$\langle 644, 136\rangle$ (optimal)	**2339.860089**	**2335.591585**
$\langle 600, 100\rangle$	2388.143825	2335.710555
$\langle 600, 160\rangle$	2347.154080	2347.38654
$\langle 700, 100\rangle$	2379.835676	2335.29428
$\langle 700, 160\rangle$	2348.279783	2348.57604
$\langle 500, 50\rangle$	2726.804274	2491.91151
$\langle 800, 200\rangle$	2375.000982	2376.93254
$\langle 300, 20\rangle$	3475.0604065	2761.396525

case. Since there is no change in r between the 4th iteration and 5th iteration in the back-order case, and the 5th iteration and 6th iteration in the lost-sales case, it is sure that $B(r)$ will not alter at the next iteration. From a practical standpoint, the optimal Q and r should be rounded up and rounded down, respectively, since the quantity should be an integer. Besides, we notice that the computation time on each iteration is quite negligible.

Table IV and V show the computation results of TEC with the optimal $< Q, r >$, and compares with TECs yielded by some other pairs of $< Q, r >$. We measure the actual total costs, and list their averages over 1000 simulations. As shown in these tables, we see that better actual total cost - better in terms of average - as well as TEC are obtained in both cases when the optimal pairs of $< Q, r >$ are applied. It is true that better actual cost is not yielded necessarily even after being averaged. However we can argue that it is more probable to get better actual total cost as we increase the number of measurements.

Next we measure TECs and actual total costs with varying only one variable and fixing the other one to the optimum. Fig.3 and Fig.4 plot the measured results in the back-orders case, and Fig.5 and Fig.6 plot the measured results in the lost-sales case. Fig.3 and Fig.6 plot the results with optimal Q and varying r, and Fig.4 and Fig.6 plot the results with optimal r and varying Q. As shown in these graphs, there are differences between the transitions of TECs and those of actual total costs, and the points that yield the minimum total cost are also different. These differences are resulted in by the differences between actual values (of d and $B\left(r\right)$) and

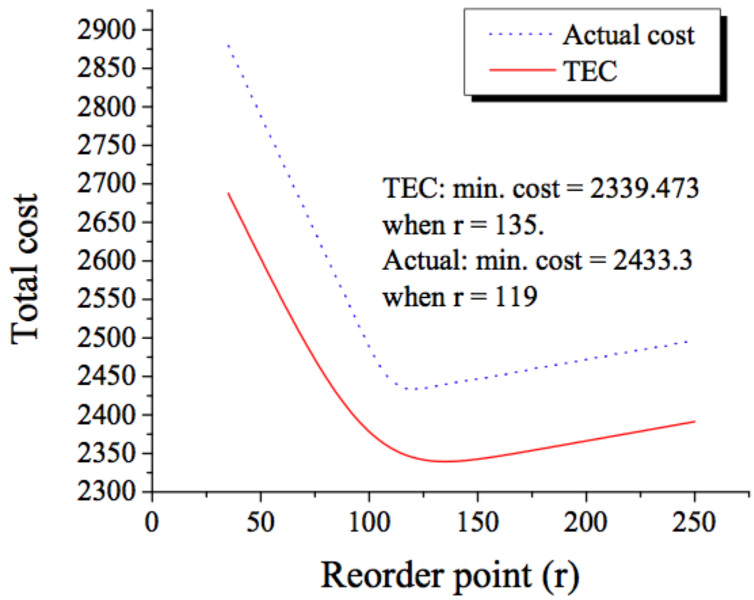

Fig. 3. The transitions of TEC and actual total cost in accordance with r in the back-orders case. Q is fixed to its optimal.

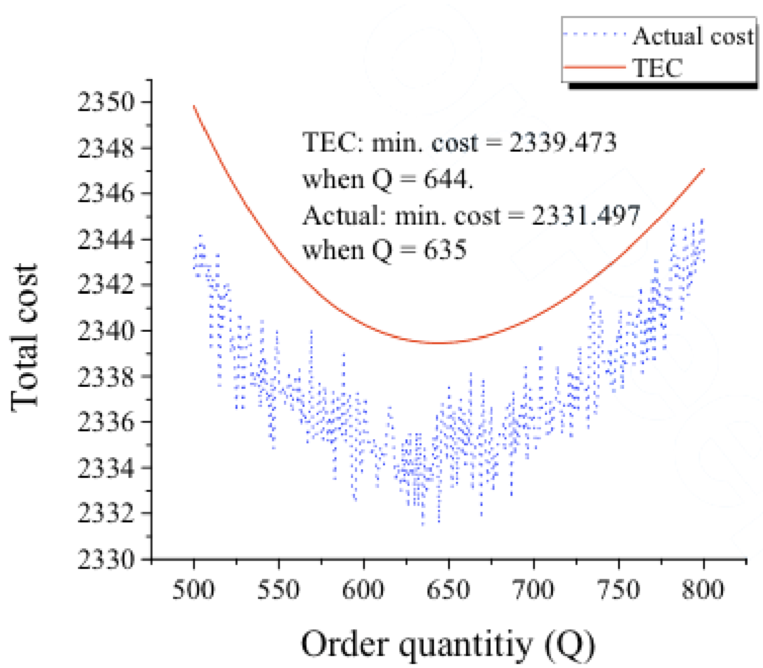

Fig. 4. The transitions of TEC and actual total cost in accordance with Q in the back-orders case. r is fixed to its optimal.

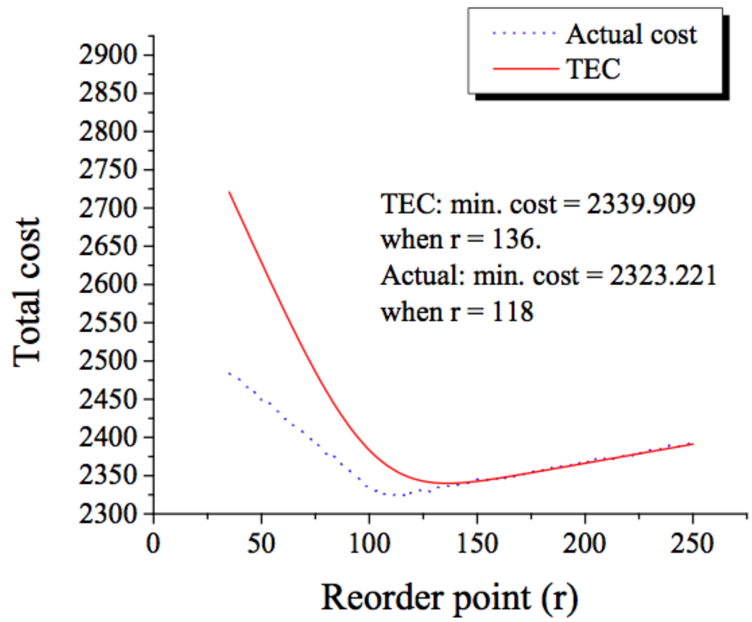

Fig. 5. The transitions of TEC and actual total cost in 52 accordance with r in the lost-sales case. Q is fixed to its optimal.

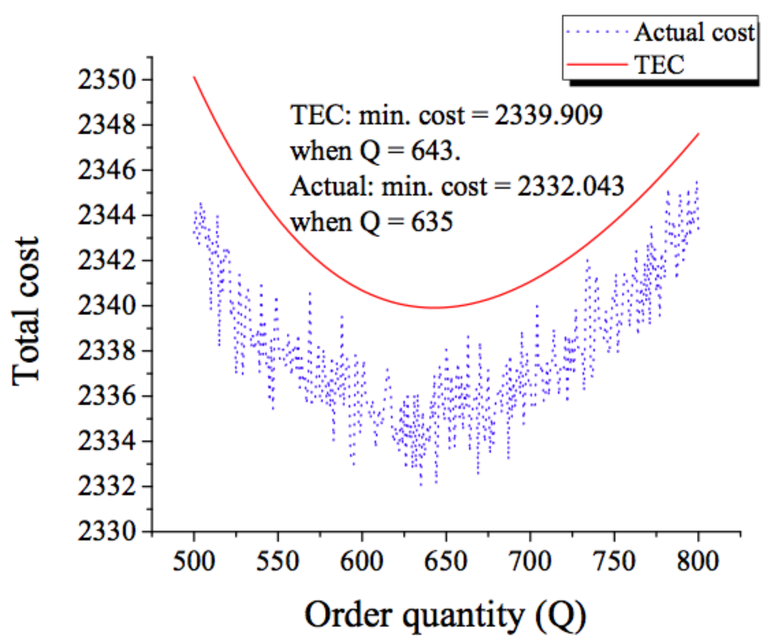

Fig. 6. The transitions of TEC and actual total cost in 52 accordance with Q in the lost-sales case. r is fixed to its optimal.

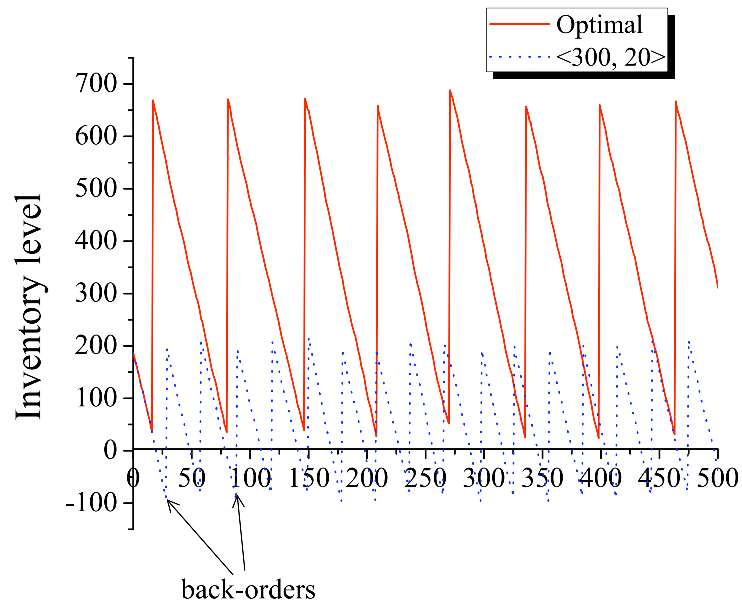

Fig. 7. The transitions of the inventory level over ticks with optimal strategy and $\langle Q, r\rangle = \langle 300, 20\rangle$ in the back-orders case

expected ones. Obviously, if we perform the simulations with more samples of d and $B(r)$, the transitions of the actual total cost will approximate to those of the TEC. Anyway, given d and $B(r)$, there is an optimal pair of $\langle Q, r\rangle$ that yields minimum actual total cost. The pair $\langle Q, r\rangle$ that yields minimum TEC can be found using the iterative algorithm, however, there is no formal way of finding the optimal $\langle Q, r\rangle$ that yields minimum actual cost except testing all the feasible pairs of Q and r. Thus, in order to reduce the search space in actual case, we propose to search for the optimal $\langle Q, r\rangle$ around the $\langle Q, r\rangle$ that yield the minimum TEC.

Fig.7 and Fig.8 illustrate the transitions of the spectrum inventory level over ticks in the back-ordres case and in the lost-sales case, respectively. For these measurements, we applied the same parameters listed in Table I, and set $d = 1000$. As shown in these graphs, we observe that back-orders or lost-sales are yielded when $\langle Q, r\rangle = \langle 300, 20\rangle$. On the other hand, there is neither back-orders nor lost-sales with the optimal strategies, i.e., $\langle Q, r\rangle = \langle 644, 135\rangle$ for back-orders case and $\langle Q, r\rangle = \langle 644, 136\rangle$ for lost-sales case.

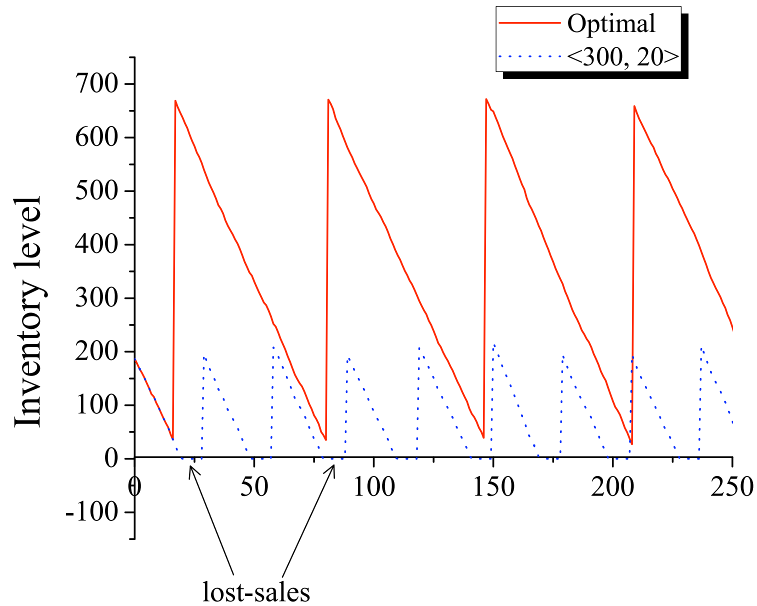

Fig. 8. The transitions of the inventory level over ticks with optimal strategy and $\langle Q, r\rangle = \langle 300, 20\rangle$ in the lost-sales case

V. Conclusion

In this paper, we discuss a scenario of wireless communication environment where users require opportunistic and transient service model. The model considers that each user has a freedom in switching to different WSPs, which forces uncertain and dynamic spectrum usages. Thus each WSP should secure the spectrum pool before user demands arrive with minimizing the maintenance cost of the spectrum pool to enhance its profit.

For this purpose, we deploy the probabilistic inventory model to the scenario in order to determine economic order quantity of subcarrier and reorder point that minimizes total expected cost. By numerical experiments, we show that better total expected cost and actual total cost are achieved when we apply the best order quantity and reorder point obtained by the inventory model. It means that this model may become fairly beneficial to WSP who anticipates minimal cost.

In this paper, we consider enough number of users to exploit the central limit theorem. As one of our future work, we will investigate the inventory model applying other types of distributions of the random variable expressing the demand during the lead time.

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