A Study on Mixing Sequences in Modulated Wideband Converters

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***Abstract*—In this paper, we study mixing sequences of modulated wideband converters (MWC). The MWC is a sub-Nyquist sampling system which mixes an input analog signal by multiple numbers of fast mixing sequences in parallel. When the mixing sequences are random cyclic shifts of a base sequence for a memory efficiency, the system turns into random partial Fourier structured MWC (RPFMWC). In RPFMWC, the spectrum distribution of a base sequence is important in reconstruction of the input signal. We show that a reconstruction of the input is guaranteed if and only if all the discrete Fourier transform (DFT) elements of a base sequence are nonzero. The sufficient and necessary conditions allow more flexible lengths for bipolar base sequences compared to existing results. In addition, we propose a measure for evaluating a base sequence in order to predict noisy performance of the RPFMWC.**

I. INTRODUCTION

A continuous-time signal, if sampled at the Nyquist rate, can be perfectly reconstructed from the discrete-time samples. However, sampling wideband analog signals at the Nyquist rate would be a severe burden for practical implementation. If the spectrum of a signal sparsely resides across a wideband with unknown spectral constitution, lossless sub-Nyquist sampling can be achievable by exploiting compressed sensing (CS) theory [1], [2]. Researchers have developed concrete sub-Nyquist sampling schemes such as modulated wideband converters (MWC) [3], [4], multi-coset samplers [5], random demodulators [6], compressive multiplexers [7], and multirate-samplers [8]. In this paper, we restrict our attention to MWC.

In the MWC, the input signal is mixed with a multiple number of periodic mixing sequences in parallel. The mixing sequences play a significant role in recovering the input from the sub-Nyquist samples by CS theory. In [3], independently drawn random Bernoulli sequences were chosen as the mixing sequences to exploit a theoretical result of CS. For a memory efficient generation of mixing signals, a well designed base sequence has been employed to generate all the mixing sequences by its random cyclic shifts [9]-[11]. In the literatures, the CS recovery was guaranteed if the discrete Fourier transform (DFT) elements of a base sequence have flat magnitudes. In [9], deterministic bipolar sequences having flat spectrum except at zero-frequency such as maximal length sequences (m-sequences) or Legendre sequences [12] have been chosen as the base sequence. The deterministic sequences can be treated as the spectrally flat sequences when the input signal does not use the frequency near zero, which is practical in wireless communications. In [10] and [11], real- and complex-valued sequences with flat spectra have been respectively chosen as the base sequence.

We scope the MWC using random cyclic shifts of a base mixing sequence, which is referred to as random partial Fourier structured MWC (RPFMWC). The prior works of [9]-[11] restricted their focuses on the base sequences having flat spectra. Although it is well known to construct non-bipolar sequences with flat spectra [13], using arbitrary-valued sequences requires high complexity in implementation. Meanwhile, it is conjectured that a bipolar sequence with flat spectrum exists only for the length 4. Instead, m-sequences and Legendre sequences with the nearly-flat spectra can be considered [9], but their lengths are inflexible. For example, m-sequences exist in lengths $M = 2^n - 1$ for a positive integer n, and Legendre sequences exist in prime lengths M such that $M \equiv 3 \pmod 4$. Since a length of mixing sequence is a major parameter in deciding a sampling rate for the lossless sampling, the inflexible lengths of the spectrally nearly-flat sequences can increase the sampling rate unnecessarily. In the perspective of flexible sequence length, using a bipolar sequence having non-flat spectrum as the base sequence would be a reasonable choice. Therefore, it is needed to investigate the CS recovery performance of the RPFMWC with the bipolar base sequence having non-flat spectrum.

Our main contribution is sufficient and necessary conditions for the base sequence of RPFMWC. We show that the CS recovery of the RPFMWC is guaranteed if and only if all the DFT elements of a base sequence are nonzero. Unlike [9]-[11], our conditions provide an opportunity for the RPFMWC to use the bipolar base sequence having non-flat spectrum that results in more flexibility in the length of base sequence. Our second contribution is to present a measure for evaluating a bipolar base sequence of the RPFMWC in terms of the spectral distribution. The proposed measure can be easily calculated and predicts the noisy recovery performance in the sense of signal-to-noise ratio (SNR) required for successful recovery.

II. THE MODULATED WIDEBAND CONVERTER

We consider sampling a real signal $x(t)$ whose spectrum consists of N disjoint bands with unknown frequency supports. Bandwidth of each band does not exceed B, and the positive

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spectrum of $x(t)$ is bandlimited by a frequency interval $[f_{\min}, f_{\max})$, where imposing the lower limit is practical in wireless communications due to modulation. The sampling scheme of MWC consists of m channels as depicted in Fig. 1. At each channel, $x(t)$ is mixed by a $T_p = 1/f_p$-periodic mixing sequence $p_i(t)$ of odd length M. A chip rate Mf_p should not be less than the Nyquist rate $2f_{\max}$ to preserve the spectrum of $x(t)$, and we fix $f_p = 2M^{-1}f_{\max}$. Then, the mixture is filtered by an anti-aliasing filter of cut-off frequency $1/2T_s$ and sampled at the rate $f_s = 1/T_s$. In [3], choosing f_s to be an odd times of f_p suffices the lossless sampling, and we set $f_s = f_p$.

Conceptually, the MWC splits the input spectrum by an uniform grid of interval f_p and takes time-samples of the spectral pieces occupied by the disjoint bands of input spectrum. This implies that the sampling rate required for the lossless sampling would be proportional not to the actual bandwidth B, but to the grid interval $f_p \geq B$. In specific, a necessary total sampling rate of the MWC is $mf_s \geq 2Nf_p$, which is minimized to $mf_s \geq 2NB$ when $f_p = B$ [3]. Hence, the gap $f_p - B$ can be considered as the amount of waste in the sampling rate, so it is desired to be $f_p = B$.

In [3], every pattern of $p_i(t)$ was chosen at random to ensure channel independence in a sensing model. When the input of MWC is corrupted by an additive noise $n(t)$, a time-domain representation of the sensing model in [3] had a matrix form of

$$\mathbf{Y} = \mathbf{SFD}\left(\mathbf{Z} + \mathbf{N}\right) \quad (1)$$

where the i-th row of $\mathbf{Y} \in \mathbb{R}^{m\times l}$ contains the output samples $y_i[n]$ for discrete-time indices $n = 1, \cdots, l$, and the i-th row of $\mathbf{S} \in \mathbb{R}^{m\times M}$ is sequence patterns of $p_i(t)$ denoted by $[\alpha_{i,0}\ \alpha_{i,1}\ \cdots\ \alpha_{i,M-1}]$. Deterministic matrices $\mathbf{F}$ and $\mathbf{D}$ are the DFT matrix of length M and an invertible diagonal matrix, respectively [3]. Finally, the k-th row of $\mathbf{Z} \in \mathbb{C}^{M\times l}$ consists of $z_k[n] = \left[\mathrm{LPF}\left\{x(t)e^{j2\pi f_k t}\right\}\right]_{t=nT_s}$, where $\mathrm{LPF}\{\cdot\}$ denotes the low-pass filtering operation of the cut-off frequency f_s and

$$f_k := \begin{cases} (k-1)f_p & \text{if } k \leq (M+1)/2 \\ (k-1-L)f_p & \text{if } k > (M+1)/2 \end{cases}$$

Likewise, the k-th row of $\mathbf{N} \in \mathbb{C}^{L\times l}$ consists of $n_k[n] = \left[\mathrm{LPF}\left\{n(t)e^{j2\pi f_k t}\right\}\right]_{t=nT_s}$. We assume that the spectral constitution of $x(t)$ does not change for the observation time lT_s, and thus $\mathbf{Z}$ becomes a row sparse matrix with sparsity $K = 2N$ as long as $f_p \geq B$ [3]. Finally, the Nyquist sample of $x(t)$ can be obtained by recovering the signal $\mathbf{Z}$ from the measurements $\mathbf{Y}$.

For memory efficiency, we consider the result of [9], where every mixing pattern $[\alpha_{i,0}\ \alpha_{i,1}\ \cdots\ \alpha_{i,M-1}]$ is a random cyclic shifts of a base sequence $\mathbf{b} = [\beta_0,\ \beta_1,\ \cdots,\ \beta_{M-1}]$ satisfying a strict condition that the DFT elements of $\mathbf{b}$ have flat magnitudes. The matrix $\mathbf{S}$ can be rewritten to random row-selections from a row-wise circulant matrix, whose rows are cyclic shifts of $\mathbf{b}$. From the well-known DFT factorization of the circulant matrix, the sensing model of (1) turns into

$$\mathbf{Y} = \mathbf{R_\Omega F \Sigma}\left(\mathbf{Z}' + \mathbf{W}\right) \quad (2)$$

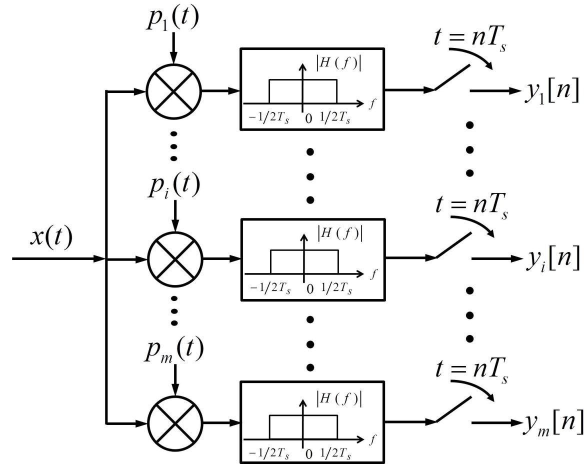


Fig. 1: Sampling scheme of MWC [3]

where $\mathbf{R_\Omega}$ is the $m \times M$ row selection matrix that selects rows of indices specified by a set $\Omega \subset \{1, \cdots, M\}$, whose elements are chosen at random without coincidences. A diagonal part of the DFT factorization remains to $\mathbf{\Sigma} = \mathrm{diag}\left(\boldsymbol{\sigma}\right)$, where $\boldsymbol{\sigma} = \mathbf{bF^H}$ is the inverse-DFT of $\mathbf{b}$. The signal and noise absorb $\mathbf{D}$ and are denoted by $\mathbf{Z}' = \frac{1}{M}\mathbf{DZ}$ and $\mathbf{W} = \frac{1}{M}\mathbf{DN}$ since $\mathbf{D}$ is a deterministic diagonal [3]. Throughout this paper, we refer the MWC of sensing model (2) to as random partial Fourier structured MWC (RPFMWC) for convenience.

In CS theory, the restricted isometry property (RIP) [2] measures near orthogonality of a sensing matrix $\mathbf{\Phi}$ in a sensing model of $\mathbf{y} = \mathbf{\Phi x}$ for every sparse vector $\mathbf{x}$. In [9] and [10], based on the RIP analysis, it was shown that if $\mathbf{\Sigma}$ is unitary then recovering the input is guaranteed. Unfortunately, the sufficient condition is quiet strict for binary sequences since existence of such sequences is known only for length 4. As alternatives, the nearly-flat sequences such as m-sequences and Legendre sequences can be exploited. However, their inflexible lengths can lead to the waste in sampling rate due to the grid gap $f_p - B$.

III. Requirements for base sequences

A. Sufficient and necessary conditions

Theorem 1. *Consider the sensing model of (2) with $f_{\min} = 0$. When $m \geq O\left(K \ln^4 M\right)$, recovering $\mathbf{Z}'$ is guaranteed if and only if $\boldsymbol{\sigma}$ has nonzero elements, i.e., $\mathbf{\Sigma}$ is invertible.*

Proof. Let $\mathbf{X} = \mathbf{\Sigma Z}'$. In (2), if $\mathbf{\Sigma}$ is diagonal and invertible, the sparsity of $\mathbf{X}$ is kept in K, so recovering $\mathbf{X}$ from $\mathbf{Y}$ is guaranteed by the RIP of $\mathbf{\Phi} = \mathbf{R_\Omega F}$ with high probability when $m \geq O\left(K \ln^4 M\right)$ [14]. Once $\mathbf{X}$ is recovered, it is obvious that $\mathbf{Z}'$ is uniquely determined by $\mathbf{X}$ if $\mathbf{\Sigma}$ is invertible. Conversely, if at least one diagonal entry of $\mathbf{\Sigma}$ is zero, $\mathbf{Z}'$ cannot be uniquely determined by $\mathbf{X}$, regardless of the recovery of $\mathbf{X}$. □

Unlike [9] and [10], we show that using spectrally non-flat sequences in the RPFMWC theoretically guarantees recovering the input from the measurements. Theorem 1 allows the

TABLE I: List of Mixing Sequences and Spectral Instabilities for $M = 127$ and $\varphi = 1$

#	Sequence type	$\eta(\mathcal{T})$	#	Sequence type	$\eta(\mathcal{T})$
1	m-sequences	1.000	8	Bernoulli 5	13.443
2	Legendre	1.000	9	Gold 2	16.527
3	Bernoulli 1	3.878	10	Bernoulli 6	19.917
4	Bernoulli 2	4.933	11	Gold 3	26.946
5	Gold 1	6.609	12	Bernoulli 7	33.423
6	Bernoulli 3	9.344	13	Bernoulli 8	41.296
7	Bernoulli 4	10.987	14	Bernoulli 9	54.820

RPFMWC to exploit any bipolar sequence of any length as long as the DFT has no zero entry. A more flexible length M of the base sequence makes it easier to reduce the sampling rate by minimizing the grid gap $f_p - B$. While keeping the recovery performance, choosing a longer M subject to $f_p \geq B$ reduces not only f_p but also the sampling rate f_s. We provide an example of the reduction in sampling rate in Section IV.

B. Spectral instability

Although the recovery of the RPFMWC with a base sequence having non-flat spectrum is guaranteed by Theorem 1, the RIP analysis does not explain influence of a base sequence on actual recovery performance of the RPFMWC. Theorem 2 reveals the influence of a base sequence on the recovery by the orthogonal matching persuit (OMP) algorithm [15] in a single measurement vector (SMV) model, where $l = 1$ in (2).

Theorem 2 (Theorem 4.10 in [16])**.** *Consider a sensing model of* $\mathbf{y} = \mathbf{R_\Omega F \Sigma}(\mathbf{x} + \mathbf{w})$ *where the noise vector* $\mathbf{w} \in \mathbb{R}^M$ *is independent of* $\mathbf{R_\Omega}$ *with* $\|\mathbf{w}\|_2 \leq \sqrt{E_n}$. *Fix* β *in* $(0,1)$ *and* δ *in* $(0, 0.5)$. *For a constant* C, *if* $m \geq C\delta^{-2}K\left(K + \ln(M-K) + \ln\beta^{-1}\right)$, *then the OMP algorithm can recover the support of a* K*-sparse vector* $\mathbf{x} \in \mathbb{R}^M$ *in* K *iterations with probability exceeding* $1 - \beta$ *if*

$$\min_{i \in \mathcal{L}} |(\mathbf{\Sigma x})_i| \geq C_1 \|\mathbf{R_\Omega F \Sigma w}\|_2 \tag{3}$$

where $\mathcal{L} := \{1, \cdots, L\}$, $(\mathbf{\Sigma x})_i$ *denotes the* i*-th element of* $\mathbf{\Sigma x}$, *and* $C_1 = 2(1-2\delta)^{-1}$.

Corollary 3. *Consider the sensing model in Theorem 2 and let* $\mathrm{SNR}_{\min} = K \min_{i\in\mathcal{L}} |x_i|^2 / E_n$ *denote minimum SNR. For a sufficiently large* m, *the OMP algorithm can recover the support of a* K*-sparse vector* $\mathbf{x}$ *in* K *iterations with high probability if*

$$\sqrt{\mathrm{SNR}_{\min}} \geq C_2 \frac{\max_{i\in\mathcal{L}} |\sigma_i|}{\min_{i\in\mathcal{L}} |\sigma_i|} \tag{4}$$

where C_2 *is a positive constant.*

Proof. Since $\mathbf{\Sigma} = \mathrm{diag}(\boldsymbol{\sigma})$, the left-hand side of (3) has a lower bound of $\min_{i\in\mathcal{L}} |(\mathbf{\Sigma x})_i| \geq \min_{i\in\mathcal{L}} |\sigma_i| \min_{i\in\mathcal{L}} |x_i|$.

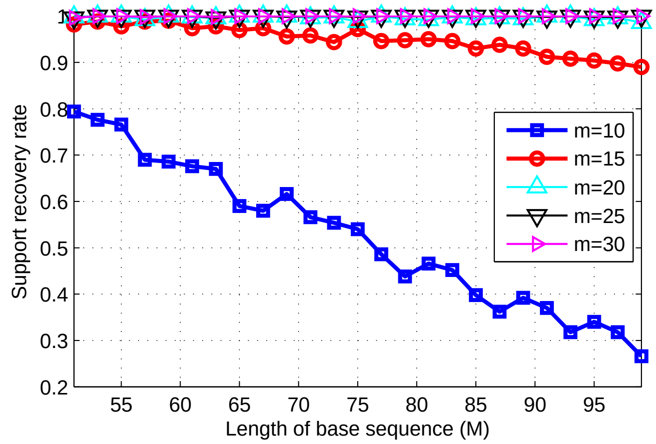


Fig. 2: Successful support recovery rates of the noiseless RPFMWC versus length of the base sequence M with various numbers of channels m. The frequency occupancy of input is fixed to $B = 400$ MHz and $N = 4$. For every M, $K = 2N$.

On the other hand,

$$\begin{aligned}\|\mathbf{R_\Omega F \Sigma w}\|_2 &\leq \|\mathbf{R_\Omega F}\|_F \|\mathbf{\Sigma w}\|_2 \\ &\leq \|\mathbf{R_\Omega F}\|_F \max_{i\in\mathcal{L}} |\sigma_i| \|\mathbf{w}\|_2 \\ &\leq \sqrt{mM} \max_{i\in\mathcal{L}} |\sigma_i| \sqrt{E_n}\end{aligned}$$

By letting $C_2 = \sqrt{mKM}C_1$, the inequality (4) is sufficient for (3). $\square$

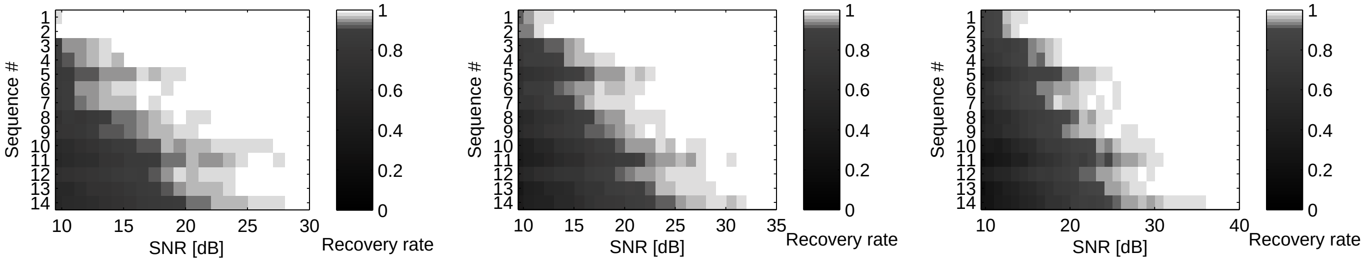
Under the existence of noise, Corollary 3 states that the minimum SNR required for successful recovery depends on the spectral distribution of a base sequence. Corollary 3 is for the real-valued SMV model, which is a special case ($l = 1$) of our sensing model (2). Nevertheless, the sufficient condition (4) supports an intuition that the RPFMWC with the higher ratio of $\frac{\max_{i\in\mathcal{L}}|\sigma_i|}{\min_{i\in\mathcal{L}}|\sigma_i|}$ would require the higher SNR for successful recovery. Also, note that if $E_n \to 0$ then the OMP recovery is always guaranteed as long as $\min_{i\in\mathcal{L}} |\sigma_i|$ is nonzero, which confirms the result of Theorem 1.

In the sensing model of (2), the positive spectrum of $x(t)$ is assumed to have a lower limit $f_{\min}$. This implies that first $\varphi = \lfloor (f_{\min} + 0.5 f_s)/f_p \rfloor$ rows and last $\varphi - 1$ rows of $\mathbf{Z}'$ are zero by the definition of $z_k[n]$, and we let $\mathcal{T} := \{\varphi + 1, \cdots, M - \varphi + 1\}$ denote a set of indices for the active rows of $\mathbf{Z}'$. The set $\mathcal{T}$ indicates two continuous frequency intervals where the input spectrum can exist. We then define a measure for the spectral distribution of a base sequence on the indices of $\mathcal{T}$.

Definition 4. *Consider the RPFMWC system of (2). The spectral instability* $\eta(\mathcal{T})$ *of a base sequence is defined by*

$$\eta(\mathcal{T}) := \frac{\max_{i\in\mathcal{T}} |\sigma_i|}{\min_{i\in\mathcal{T}} |\sigma_i|} \tag{5}$$

As observed in Corollary 3, it is preferred to choose a base sequence $\mathbf{b}$ minimizing $\eta(\mathcal{T})$ when the input signal to



(a) $K = 8$ (b) $K = 12$ (c) $K = 16$

Fig. 3: Successful support recovery rates of the RPFMWC versus SNR for various base sequences of length $M = 127$. The number of channels is set to $m = 50$, and the sequence numbers are assigned in Table I.

the RPFMWC with a sufficiently large m is corrupted by noise. It is obvious that sequences having flat spectrum have the minimal $\eta(\mathcal{T}) = 1$. However, since a bipolar sequence with flat spectrum cannot be achieved, the first alternative is to consider deterministic sequences having the nearly-flat spectrum such as m-sequences and Legendre sequences. The spectra of the deterministic sequences are regarded as flat or $\eta(\mathcal{T}) = 1$ if $\varphi \geq 1$.

Despite the minimal $\eta(\mathcal{T})$, the deterministic sequences are not available for every lengths. For the flexible length of base sequence, bipolar Bernoulli sequences can be considered as the base sequence of RPFMWC. However, no research efforts have been reported to answer which outcomes of the Bernoulli sequences are suitable for the RPFMWC in the aspect of the minimum SNR required for successful recovery. The spectral instability of Definition 4 can be used as a measure to predict the actual recovery performance of the RPFMWC with a bipolar Bernoulli base sequence in the presence of noise.

IV. Simulations

In this section, the validity of Theorem 1 and spectral instability $\eta(\mathcal{T})$ is demonstrated by simulation results. The results show successful recovery rates of supports on the row sparse matrices $\mathbf{Z}$ in (2). For the support recovery, we use the l_2-norm based OMP algorithm for a multiple measurement model (MMV) [17]. A successful recovery is declared if an estimated support includes the true support with a few additional entries. The results are averaged by 500 trials, and the input signal and the noise are redrawn at every trial. The spectrum of $x(t)$ consists of even N conjugate symmetric rectangular bands of bandwidth B with random carriers within an observation time lT_s, where $l = 12$ is the length of samples of each channel. For a sampling rate $f_s = f_p$, the minimum frequency of input is fixed to $f_{\min} = f_s/2$, which implies $\varphi = 1$.

Figure 2 shows successful recovery rates versus sequence length M in noiseless RPFMWC with various numbers of channel m. In the simulation, the input is bandlimited to $f_{\max} = 20$ GHz where the bandwidth of each rectangular band is set to $B = 400$ MHz and $N = 4$. Also, we vary M while satisfying $f_p \geq B$, which implies $K = 2N$. Samples of Bernoulli sequences satisfying the conditions of Theorem 1 are used as the base sequence. As expected by Theorem 1, the Bernoulli sequences having non-flat spectrum can achieve a reliable recovery. In addition, the result shows that the increment of M rarely affects the recovery performance for high m sufficient for a reliable recovery (lines for $m = 20, 25, 30$). By noting that the longest Legendre sequences satisfying $f_p \geq B$ exist in $M = 83$, one can reduce the sampling rate at every channel by 77.89 MHz by employing a Bernoulli sequence of length $M = 99$ satisfying the conditions of Theorem 1.

Figure 3 shows successful recovery rates versus SNR for candidate base sequences with various K. In addition to the Legendre and the m-sequences, we pick the candidate bipolar base sequences of length $M = 127$ satisfying the conditions of Theorem 1 from Bernoulli and Gold [18] sequences of arbitrary initial seeds. The candidate base sequences are listed in Table I in increasing order of the spectral instability $\eta(\mathcal{T})$. In the simulation, the input is bandlimited to $f_{\max} = 1$ GHz. We fix the number of channels $m = 50$ since Corollary 3 is effective for a sufficiently large m. By setting $B = f_p$, the sparsity of $\mathbf{Z}$ is $K = 2N$. As expected, it is shown that the RPFMWC of a base sequence having the lower $\eta(\mathcal{T})$ is likely to require the lower SNR for the reliable recovery.

V. Conclusion

We have studied mixing sequences for the RPFMWC that uses random cyclic shifts of a base sequence. The prior works of [9] and [10] restricted their focuses on base sequences with flat spectra due to their theoretical recovery guarantees. On the contrary, we have considered using a general base sequence having non-flat spectrum in the RPFMWC and have shown that the performance is theoretically guaranteed for the spectrally non-flat base sequences. Employing the spectrally non-flat bipolar sequences can reduce the hardware complexity and the waste in sampling rate. Then, we showed how the spectrum of a base sequence influences on the noisy recovery performance, and defined the spectral instability as a measure to evaluate a base sequence in the noisy RPFMWC system. Empirical results validated that a base sequence having the lower spectral instability is expected to show the better recovery performance in the noisy RPFMWC.

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