Evolutionary Channel Sharing Algorithm for Heterogeneous Unlicensed Networks

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***Abstract*—Channel sharing in TV whitespace (TVWS) is challenging because of signal propagation characteristics and diversity in network technologies employed by secondary networks coexisting in TVWS. In this paper, the TVWS sharing problem is modeled as a multiobjective optimization problem, where each objective function tackles an important coexisting requirement, such as interference and disparity in network technologies. We propose an evolutionary algorithm that shares the TVWS among coexisting networks taking care of their channel occupancy requirements. In this paper, the channel occupancy is defined as the time duration; a network desires to radiate on a channel to achieve its desired duty cycle. Simulation results show that the proposed algorithm outperforms existing TVWS sharing algorithms regarding allocation fairness and a fraction of channel occupancy requirements of the coexisting networks.**

***Index Terms*—Coexistence set, evolutionary algorithm, indicator function, pareto optimal, pareto dominance, TV whitespace, whitespace object.**

I. Introduction

TV WHITESPACE (TVWS) refers to the TV spectrum not in use by licensed operators in a spatiotemporal region. Worldwide efforts have been initiated to permit unlicensed devices to operate in TVWS. Therefore, several standards such as IEEE 802.22-2011 [1], 802.11af [2], 802.15.4m [3], and ECMA-392 [4] have been developed to regulate access to TVWS. The MAC/PHY layer technologies in these standards are incompatible. A collocated deployment of secondary devices operating on these standards may create coexistence issues, such as unresolved interference due to a disparity in MAC/PHY layer technologies, spectrum congestion due to indiscriminate spectrum usage, and spectrum scarcity in congested areas [5]–[7]. Such issues, if left unresolved, may result in inefficient use of TVWS. Therefore, IEEE has developed a standard namely 802.19.1 to provide coexistence among secondary devices, namely whitespace objects (WSO), operating on heterogeneous network technologies [8]. The collocated WSOs operating on heterogeneous network technologies are referred to as *hetero*-WSO throughout this paper.

A set of tasks to achieve peaceful coexistence among *hetero*-WSOs sharing the common spectrum is referred to as coexistence decision making (CDM) procedure. A system implementing CDM procedure is referred to as a CDM system [9]. Some literature work exists that implements CDM procedure in the TVWS domain. Most of such work like in [9]–[12] implements a CDM procedure to fully satisfy the channel demands of *hetero*-WSOs. However, such channel allocation policy may cause some of the WSOs to get the channel while rest of them do not. This situation is intensified in a highly-congested area where a limited TV spectrum is available for secondary user activities due to the active presence of licensed operators. Considering the free-to-use status of the TVWS, we aim to define a CDM procedure that accommodates as many as *hetero*-WSOs on the available TVWS by relaxing their channel occupancy demands.

In this paper, we propose an Evolutionary Coexistence decision making (EvCo) algorithm for an 802.19.1-complaint CDM system. The algorithm addresses the critical coexistence issues like allocation fairness, system throughput maximization, and WSO satisfaction, each of which is modeled as an objective function in the TVWS multiobjective optimization problem (MOP), as will be defined in Section IV. The main contributions of the proposed work are summarized as follows.

1) A CDM procedure is implemented as a process of sharing a set of TV channels of predetermined bandwidth among a set of *hetero*-WSOs. Unlike existing CDM formulations in the TVWS sharing domain [9]–[11], the proposed formulation accommodates as many as *hetero*-WSOs on the available TVWS by relaxing their channel demand satisfaction.
2) The proposed CDM system transforms the nonconvex, nonlinear multiobjective function in the TVWS sharing MOP (Section IV-B) into a max-min optimization formulation, using a binary epsilon indicator function (Section IV-D). Such formulation enables the CDM system to achieve a true multiobjective optimization as it does not require a priori articulation of preferences of the decision maker nor does it need to scalarize the multiobjective function in the TVWS sharing MOP. Consequently, a better approximation of global minima of the TVWS sharing MOP is achieved as compared to the existing CDM systems in [9] and [10].
3) An evolutionary algorithm, called EvCo is proposed to obtain a feasible Pareto-optimal solution for the

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TVWS sharing MOP. Our evaluation studies show the superiority of the EvCo over existing TVWS sharing algorithms in [9] and [10] regarding scalability, fairness and WSOs' satisfaction from the allocation.

The remainder of this paper is organized as follows. Related work is discussed in Section II. Section III summarizes some of the technical background required to establish the baseline for the techniques used in the paper. The TVWS sharing MOP formulation is described in Section IV. The proposed algorithm is presented in Section V and is compared to existing algorithms in Section VI. Finally, Section VII concludes the paper.

II. Previous Work

In this section, we summarize some standards and algorithms developed for TVWS sharing among WSOs. In some wireless standards, for example, IEEE 802.11af and 802.22.1 [13], PHY and MAC layer extensions for TVWS have adopted new cognitive radio features to protect incumbents from harmful interference from unlicensed devices. However, these standards define self-coexistence in TVWS operations. Other standards like IEEE 802.15.2 [14] and IEEE 802.15.4 [3] have partially addressed the coexistence issue among devices operating in industrial, scientific, and medical bands. Perceiving the need for cross-platform coexistence mechanisms, IEEE has defined the 802.19.1 standard. The standard specifies coexistence protocols and policies for effective utilization of TVWS across platforms. A coexisting system architecture, defined in 802.19.1 [8], has been summarized in Section III.

On algorithmic perspective, Bahrak and Park modeled the spectrum-sharing problem as a MOP, which was then scalarized using a weighted-sum approach and formulated using a modified Boltzmann machine [9]. A CDM algorithm called FACT [9] is then designed to solve MOP [9]. However, the main issue with the weighted-sum approach is its inability to find Pareto-optimal solution points in the non-convex region of the solution space boundary [15]. Another issue with the FACT is its discrepancy in allocation. It allocates the available spectrum to WSOs until a WSO's channel demand is satisfied. However, in highly congested areas, the available spectrum may be insufficient to accommodate the channel demands of all the collocated WSOs.

Bansal *et al.* [10] define the TV channel sharing problem as a vector of lexicographic ordering of throughputs of an access point (AP) which is then transformed into a graph coloring problem. An algorithm called Share and its localized version lShare are then defined to tackle the graph coloring problem. However, Share does not consider the channel allocation under the scenario when interference among neighboring APs is relatively high. This situation is quite possible in highly congested areas where multiple of collocated WSOs are deployed. Hessar and Roy [11] have discussed the TVWS sharing problem, but in the secondary cellular networks. They have used two different formulations. One is to maximize the number of allocated channels; the other is to maximize the total network throughput. Heuristic approaches are adopted, and greedy algorithms are designed for each of these formulations.

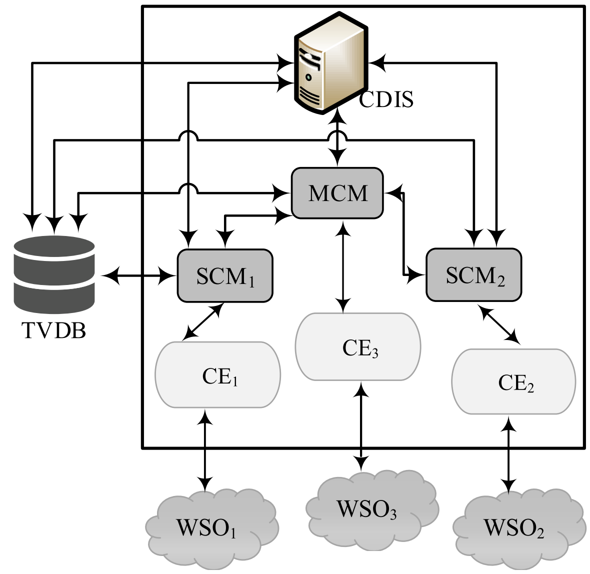

Fig. 1. IEEE 802.19.1-compliant coexistence system with centralized topology.

Within these greedy algorithms, brute force search is applied to find the solution that maximizes the throughput under the minimum fairness in allocation. However, search over the space of a possibly very large number of network and channel collocation combinations leads to a high runtime complexity to find an optimal solution.

III. Technical Background

In IEEE 802.19.1 [8], three types of decision-making topologies namely, autonomous, distributed and centralized are defined. In autonomous decision making the coexistence decision making entity in [8], namely coexistence manager (CM) makes its decisions independently from another CM. In distributed decision making one CM makes its decisions in coordination with another CM. In centralized decision making, the neighboring CMs select one CM as master CM (MCM) which relegates its coexistence decisions to neighboring CMs. The CMs serving the coexisting WSOs are called neighboring CMs in 802.19.1 [8].

In this paper we implement a CDM system based on the centralized CDM topology defined in [8] as shown in Fig. 1. The system components include coexistence managers (CM), coexistence enablers (CE), and a coexistence discovery and information server (CDIS). The CE registers a whitespace object (WSO) with the system and acts as a communication bridge by translating messages between the WSO and the CM. The CM is responsible for making coexistence decisions related to the reconfiguration of WSOs to solve the coexistence issues. The CDIS maintains a list of WSOs registered to the 802.19.1 system. A TVWS database (TVDB), shown in Fig. 1, contains information about the TV channels available in the geographic region of the WSOs. In a centralized decision-making topology, the neighboring CMs select one of them as master CM (MCM), and rest of them become slave CMs (SCM), as shown in Fig. 1. The MCM performs all coexisting decisions like TVWS sharing among WSOs registered within it and with its slave CMs (SCM). The MCM in the proposed centralized CDM system implements the TVWS sharing process as a MOP, as shown in the following section.

IV. Problem Formulation

In this section, we formulate the TVWS sharing problem as an energy minimization MOP and transform it into a max-min optimization problem using a binary indicator function.

A. Modeling the CDM System

The CDM system, as shown in Fig. 1, is defined as follows,

$$\mathbf{O}^* = TVWS(\mathcal{W}, \mathcal{J}, \mathcal{T}, \mathcal{D}), \tag{1}$$

where $\mathcal{W} = \{1, 2, \ldots, W\}$, $\mathcal{J} = \{1, 2, \ldots, J\}$, and $\mathcal{T} = \{T_1, T_2, \ldots, T_J\}$ represent a set of *hetero*-WSOs, a set of available TV channels and a channel window time set, respectively. The window time is defined as a slot duration of a scheduling repetition period that satisfies the essential system quality of service (QoS) performance. A set of channel-demands of *hetero*-WSOs is defined as, $\mathcal{D} = \left\{ [n_w]_{W\times 1}, [O_w]_{W\times 1}, [p_w]_{W\times 1}, \left[SINR_{w,j}\right]_{W\times J} \right\}$. In 802.19.1 [8], an abstraction is provided that allows WSOs to send their channel demands to their CM. We exploit such information, available at CM, to formulate the channel demands set. The set elements are defined as follows. Let n_w represents the number of TV channels desired by WSO w. The value of n_w depends upon the network technology employed by the WSO, defined as follows. Let $\mathcal{M} = \{1, 2, 3\}$ be a set of network technologies where 1, 2, and 3 refer to the technologies defined in 802.19.1 like IEEE 802.11af, IEEE 802.22, and ECMA392, respectively. The standard definitions of these technologies specify a single channel of regulatory defined bandwidth, e.g., 6 MHz in the US, as a compulsory requirement of TVWS operations. An 802.11af type WSO can operate on 1, 2 or 4 TV channels [2]. However, allocating more than one channels to such a WSO is defined as optional in [2]. The proposed CDM system, thus, supports the channel allocation among WSOs requesting for one TV channel, or multiple, non-contiguous TV channels. Channel allocations which are continuous in frequency slots are also promoted in the proposed system; however, such an allocation is not guaranteed. The $O_w \in \mathcal{D}$ translates to the amount of time that the WSO $w \in \mathcal{W}$ desires to use its desired channel to radiate electromagnetic waves using a pre-allocated transmission power p_w. A WSO's desired bandwidth is defined as, $b_w = n_w b$[MHz] where b represents the bandwidth of a TV channel. The CDM system then solves the following TVWS sharing problem.

Given input parameters in the system Eq. (1), the TV channels must be shared among a set of coexisting WSOs such that the following objectives are satisfied:

- Allocation among WSOs is fair,
- System throughput is maximized,
- WSOs are satisfied regarding their channel demands.

The CDM system achieves the objectives of the TVWS sharing problem by formulating them in the following functions.

1) Fairness in Allocation: Fairness, from a spectrum allocation perspective, is regarded as equity in access to radio resources. It is defined in terms of a fraction of demand serve metric as,

$$R_w := \begin{cases} \dfrac{r_w}{d_w}, & \text{if } r_w < d_w \\ 1, & \text{otherwise.} \end{cases}$$

The $d_w = \sum_{j=1}^{n_w} O_w b_j \log_2\left(1 + SINR_{w,j}\right)$ represents the amount of data that the WSO w desires to transmit and $r_w = \sum_{j=1}^{n_w} O_{w,j} b_j \log_2(1 + SINR_{w,j})$ represents the amount of data the WSO can transmit. Optimizing $\mathbf{R} = (R_1, R_2, \ldots, R_W)'$ by maximally equalizing $R_w \approx R_m, \forall w, m \in \mathcal{W}$ results in fair allocation among *hetero*-WSOs. A fairness function is thus defined as an energy minimization function based on Jain's fairness index [16] as follows,

$$\bar{f}_F(\mathbf{O}) = \left[1 - \frac{\left[\sum_w R_w(\mathbf{O})\right]^2}{W \sum_w R_w(\mathbf{O})^2} \right]. \tag{2}$$

2) System Throughput Maximization: The gain in system throughput depends on multiple factors. Some common factors are formulated as follows.

a) Contiguous Channel Allocation: Contiguous channel allocation allows a network to have adaptive channel widths that can increase system throughput by more than 60% compared to a fixed-width configuration [17]. In this paper, the contiguous channel allocation is promoted as follows. Let A=(0,T_j], and an allocation of a channel j to a WSO w be defined using an indicator function, as follows:

$$\mathbf{1}_A(O_{w,j}) := \begin{cases} 1, & \text{if } O_{w,j} \in A \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

For each block of channels, a monotone increasing cost function is defined as, $(\mathbf{1}_A(O_{w,j}) - \mathbf{1}_A(O_{w,j+1}))$. The function adds a cost of two for each block of channels. The contiguous channel allocation then becomes the energy minimization function, defined as,

$$\bar{f}_C(\mathbf{O}') = \sum_w \left[\sum_j \left(\mathbf{1}_A(O_{w,j}) - \mathbf{1}_A(O_{w,j+1})\right)^2 \right] I_w, \tag{4}$$

where the updated solution metric, $\mathbf{O}'$, is defined by concatenating a zero column on both, the leading and trailing end of the solution matrix $\mathbf{O}$, i.e., $\mathbf{O}' := \left[[0]_{W\times 1} || \mathbf{O} || [0]_{W\times 1}\right]$. The function, I_w, forces the cost of channel allocations to w^{th} WSO to be zero if a single channel or a single block of contiguous channels is allocated, defined as follows,

$$I_w := \begin{cases} 0, & \text{if } \sum_j \left(\mathbf{1}_A(O_{w,j}) - \mathbf{1}_A(O_{w,j+1})\right)^2 \leq 2 \\ 1, & \text{otherwise.} \end{cases}$$

b) WSO Homogeneity: In this subsection, we aim to discuss how a set of WSOs with the same MAC technologies are encouraged to share a TVWS channel, referred to as WSO *homogeneity* in this paper. Homogeneity in MAC technology is a merit to pursue because sharing a channel among WSOs with incompatible MAC technologies results in higher switching delay and error rates due to unresolved synchronization issues [9]. The *homogeneity* in MAC technology is promoted using the control overhead in the technologies in $\mathcal{M}$, defined as follows. Let a variable $C_{w,m(w)}$ be defined as the cost of sharing a channel between two WSOs, $w, m \in \mathcal{W}$, where *m(w)* represents a WSO m sharing a channel with WSO w. Let $\tau_w \in \mathcal{M}$ represent MAC technology of WSO w and β_w represent its control overhead. The control overhead is defined as the amount of time required by a WSO to perform control signaling while operating in the TVWS. This value is fixed and predetermined based on the underlying network technology of the WSO. For example, if a 802.22 WSO employs OFDMA, one OFDM symbol is used for both the frame preamble and the frame header; except for the first frame in the superframe which consumes two additional symbols (1/4 cyclic prefix mode). If we consider two OFDM symbols per frame as a control region then using a symbol duration, T_{Sym}=0.3733 ms [1], the control overhead per frame is computed as, 0.7466 ms. Other settings may generate different overhead. Similarly, if a WSO m operates in a different network technology than that of the WSO w, its control overhead will be different from that of WSO w. The total overhead in a channel varies as the channel is shared among *hetero*-WSOs. The value of the parameter $C_{w,m(w)}$ is then defined simply by adding the control overhead of all WSOs sharing a channel as follows:

$$C_{w,m(w)} := \begin{cases} \beta_w + \beta_m, & \text{if } \tau_w \neq \tau_m \forall (w,m) \in \mathcal{W} \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

Let sharing a channel j between *hetero*-WSOs w and m be expressed using an indicator function as,

$$I_{w,m(w)}(j) := \begin{cases} 1, & \text{if } O_{w,j} O_{m,j} > 0 \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

The homogeneity function then becomes an energy minimization function, defined as follows:

$$\bar{f}_H(\mathbf{O}) = \sum_{w=1}^{W} \sum_{j=1}^{J} I_{w,m(w)}(j) C_{w,m(w)}, \quad \forall m \in \mathcal{W}, m \neq w. \quad (7)$$

c) SINR: Let $\mathcal{S}_j \subseteq \mathcal{W}$ be a set of WSOs with a maximal gain on channel j. The $\mathcal{S}_j$ is selected such that the total occupancy time of WSOs sharing channel j does not exceed the window time T_j as,

$$\mathcal{S}_j = \Big\{ w \in \mathcal{W} | \max \left(\left[SINR_{w,j} \right] \right) : \sum_{w \in \mathcal{S}_j} O_{w,j} \leq T_j \Big\}, \quad \forall j \in \mathcal{J}. \quad (8)$$

Let $T^0 := \sum_{\forall j \in \mathcal{J}} \sum_{\forall w \in \mathcal{S}_j} O_{w,j} b_{w,j} \log_2 \left(1 + SINR_{w,j}\right)$ be the maximum throughput that can be achieved if available TV channels are allocated to WSOs with maximal channel gain. The throughput optimization then becomes an energy minimization function, defined as follows:

$$\bar{f}_T(\mathbf{O}) = \left(T^0 - \sum_{w=1}^{W} r_w \right). \quad (9)$$

To optimize system throughput, the functions in (4), (8), and (9) must be optimized concurrently.

3) WSO Satisfaction From the Allocation : A WSO w is satisfied from the allocation if it achieves its desired data volume d_w. A quantifiable satisfaction can be defined regarding an energy minimization function as follows.

$$\bar{f}_S(\mathbf{O}) = \frac{1}{W} \sum_{w=1}^{W} \left(\frac{d_w - r_w}{d_w} \right)^2. \quad (10)$$

B. TVWS Sharing Problem Formulation

To achieve the TVWS sharing objectives in Section IV-A, the CDM system needs to optimize objective functions in (2), (4), (7), (9), and (10) simultaneously. Let $\mathcal{J}_w := \{ j | O_{w,j} > 0, \forall j \in \mathcal{J} \}$ be a set of channels allocated to WSO w, and let $\mathcal{J}^c := \mathcal{J} \backslash \mathcal{J}_w$. Let $\mathcal{R} = \{\beta_w, \forall w \in \mathcal{W}\}$ be a set of WSOs' control overheads. The TVWS sharing problem then becomes a MOP defined as follows:

$$\underset{\mathbf{O}}{\text{minimize}} \ \bar{\text{F}}(\mathbf{O}) = \left(\bar{f}_F(\mathbf{O}), \bar{f}_T(\mathbf{O}), \bar{f}_S(\mathbf{O}), \bar{f}_C(\mathbf{O}), \bar{f}_H(\mathbf{O}) \right)^T$$

$$\text{subject to} \sum_{w=1}^{W} O_{w,j} \leq T_j, \forall j \in \mathcal{J} \quad (11a)$$

$$\sum_{\forall j \in \mathcal{J}} O_{w,j} \leq n_w O_w, \ \forall w \in \mathcal{W} \quad (11b)$$

$$\sum_{\forall j \in \mathcal{J}} O_{w,j} > \beta_w, \forall w \in \mathcal{W} \quad (11c)$$

$$\beta_w < O_{w,j} \leq O_w, \forall j \in \mathcal{J}_w, \forall w \in \mathcal{W} \quad (11d)$$

$$O_{w,j} = 0, \forall w \in \mathcal{W}, \forall j \in \mathcal{J}^c \quad (11e)$$

The constraint in (11a) ensures that the total occupancy time of all allocated WSOs on channel j does not exceed the channel window time T_j . The constraint (11b) ensures that the total occupancy time of a WSO w on all allocated channels does not exceed its total desired channel occupancy time. The constraint in (11c) ensures that each WSO gets allocation on at least one channel, ensuring a minimum fairness in allocation. The constraint in (11d) ensures that for each allocated channel to WSO w, the occupancy time of the WSO satisfies the minimum and the maximum allocation constraints, $\beta_w \in \mathcal{R}$, and O_w, respectively. The constraint in (11e) sets all the variables $O_{w,j}$ to zero where the WSO w is not scheduled in the TV channels, i.e., $\mathcal{J}^c$. This constraint, in conjunction with (11d), allows the optimization routine to adjust the channel occupancies of *hetero*-WSOs such that the CDM system can accommodate as many as *hetero*-WSOs in the system.

A single solution point $\mathbf{O}$ that could optimize all objectives in $\bar{\mathbf{F}}$ in (11) is not possible as these objectives contradict each other. They need to be balanced by applying Pareto-optimality, defined as follows. Let $\mathcal{P}$ be a feasible solution set defined on the domain of the MOP in (11), $\Omega = [0, 1]$. Then, finding a Pareto-optimal solution requires establishing a preference relation on the solution points in $\mathcal{P}$, i.e., $\mathbf{O}^1 \in \mathcal{P}$ is preferable to $\mathbf{O}^2 \in \mathcal{P}$ if $\bar{\mathbf{F}}(\mathbf{O}^1)$ dominates $\bar{\mathbf{F}}(\mathbf{O}^2)$. The dominance concept for MOP in (11) can be defined as follows. $\bar{\mathbf{F}}(\mathbf{O}^1)$ dominates $\bar{\mathbf{F}}(\mathbf{O}^2)$ if and only if $\bar{f}_m(\mathbf{O}^1) \leq \bar{f}_m(\mathbf{O}^2)$, for every $m \in \{F, T, S, C, H\}$ and $\bar{f}_n(\mathbf{O}^1) < \bar{f}_n(\mathbf{O}^2)$ for at least one index $n \in \{F, T, S, C, H\}$. A Pareto-optimal solution is then defined as follows [18].

Definition 1: A solution point $\mathbf{O}^* \in \mathcal{P}$ is Pareto-optimal to (11) if and only if there is no other solution point $\mathbf{O} \in \mathcal{P}$ such that $\bar{\mathbf{F}}(\mathbf{O})$ dominates $\bar{\mathbf{F}}(\mathbf{O}^*)$.

The MOP in (11) has nonconvex, nonlinear property as discussed in the following section.

C. Non-Convergence Issue

Let $\mathcal{P} \subseteq \mathbb{R}^{W \times J}$ be a closed, nonempty subset of $\mathbb{R}_0^+$ consisting of all feasible solution points defined in the domain on the MOP in (11). If $\mathcal{P}$ is a convex set, we refer to variable $\mathbf{O} \in \mathcal{P}$ a convex variable, if $\mathcal{P}$ is a nonconvex set, the variable $\mathbf{O} \in \mathcal{P}$ is considered as a nonconvex variable.

Theorem 1: For a nonconvex $\mathcal{P}$, the function $\bar{\boldsymbol{F}}$ in (11) is a nonconvex function.

Proof. See Appendix A.

If $\mathcal{P}$ is a nonconvex set, the optimization problem like MOP in (11) can be hard in general [19]. Moreover, step function in (4) makes the MOP in (11) as nonlinear function. In such a case, no algorithm can converge to a global Pareto-optimal solution, at least in a polynomial time [19]. There exist some methods that converge to such a solution. For instance, when $\mathcal{P}$ is a nonconvex finite set, a simple brute force method [19], a branch-and-bound [19] and a branch-and-cut [19] methods all are guaranteed to converge to the global Pareto-optimal solution. However, these and such methods have non-polynomial worst-case runtime [19]. It is often burdensome to use them for optimizing the TVWS sharing problem where a shorter runtime is desirable for a reason mentioned in Section I. Our aim is to give up the accuracy and use a method that can find a good approximate of a global Pareto-optimal point in a shorter runtime. The evolutionary strategy (ES) based heuristic technique can provide such a solution quickly [20], [21]. It is specifically suitable for a nonconvex, non-differentiable optimization problem [20] like MOP in (11). Moreover, the computational costs of optimization techniques in the ES are lower as ES does not require complex gradient or hessian calculations. Therefore, we adopt ES technique to design an algorithm (Section V) to tackle the TVWS sharing MOP in (11).

An evolutionary algorithm, like the EvCo proposed in Section V, requires a fitness function to rank the solution points. Therefore, we define an indicator based optimization function, as defined in the following section, and use it as a fitness function. Before we define the indicator function, the objective functions in (2), (4), (7), (9), and (10) are normalized for following reason. The objective functions' values are defined in different intervals, e.g., $0 \leq \bar{f}_F \leq 1, \bar{f}_T \in \mathbb{R}_0^+, 0 \leq \bar{f}_C \leq W \times J, 0 \leq \bar{f}_S \leq 1, \bar{f}_H \in \mathbb{R}_0^+$. The larger valued functions like $\bar{f}_T$, $\bar{f}_C$, and $\bar{f}_H$ may diminish the effect of small valued functions like $\bar{f}_F$ and $\bar{f}_S$. To get an equal effect of these objective functions in the indicator function, we normalize them as follows,

$$f_\alpha(\mathbf{O}) = \frac{\bar{f}_\alpha(\mathbf{O}) - \bar{f}_\alpha^{\min}}{\bar{f}_\alpha^{\max} - \bar{f}_\alpha^{\min}}, \forall \alpha \in \{F, T, S, C, H\} \tag{12}$$

where $\bar{f}_\alpha^{\min}$ and $\bar{f}_\alpha^{\max}$ represent the minimum and maximum objective function values over all solution points in $\mathcal{P}$, respectively, defined as follows.

$$\bar{f}_\alpha^{\min} = \text{minimum}\{f_\alpha(\mathbf{O}),\ \forall \mathbf{O} \in \mathcal{P}\},$$
$$\bar{f}_\alpha^{\max} = \text{maximum}\{f_\alpha(\mathbf{O}),\ \forall \mathbf{O} \in \mathcal{P}\}.$$

The TVWS sharing MOP in (11) is then redefined using normalized objective functions as follows.

$$\underset{\mathbf{O}}{\text{minimize}}\, \mathrm{F}(\mathbf{O}) = (f_F(\mathbf{O}), f_T(\mathbf{O}), f_S(\mathbf{O}), f_C(\mathbf{O}), f_H(\mathbf{O}))^T$$
$$\text{subject to constraints in (11a) to (11e).} \tag{13}$$

D. Problem Formulation Using Binary Epsilon Indicator Function

A binary epsilon indicator function measures the quality of two sets of solution points with respect to each other [22]. In our case a set of solution points is called a cluster; the clustering is defined in Section V. The indicator function performs preference ordering on a set of clusters, by establishing Pareto-dominance on the corresponding objective function vector defined as follows.

Let $C_k = \{\mathbf{O} \in \mathcal{P}\}$ be a cluster then, $\forall \mathbf{O} \in \mathcal{P}$ a set of K clusters is defined as, $\mathcal{C} = \{C_1, C_2, \ldots, C_K\}$. Let $k, l \in \{1, 2, \ldots, K\}$ be the indices to the cluster set $\mathcal{C}$ then a binary epsilon indicator function applied to the $\mathbf{F(O)}$ in (13) can be defined as follows [22]:

$$I_{\varepsilon^+}(C_k, C_l) = \min_\varepsilon \Big\{\forall \mathbf{O}^q \in \mathcal{C}_l\ \exists \mathbf{O}^p \in \mathcal{C}_k : f_\alpha(\mathbf{O}^p) - \varepsilon \leq f_\alpha(\mathbf{O}^q),\ \alpha = \{F, T, S, C, H\}\Big\}. \tag{14}$$

According to the definition in (14), $I_{\varepsilon^+}(C_k, C_l)$ denotes the minimum amount, ε, which is required to improve each objective function $f_\alpha(\mathbf{O}^p), \forall \alpha \in \{F, T, S, C, H\}$ for each member of C_k such that C_k is weakly preferable to C_l. The indicator function in (14) is redefined as max-min optimization formulation [23] as follows:

$$I_{\varepsilon^+}(C_k, C_l) = \max_{\mathbf{O}^q \in \mathcal{C}_l} \min_{\mathbf{O}^p \in \mathcal{C}_k} d_\varepsilon(\mathbf{O}^p, \mathbf{O}^q) \tag{15}$$

where a distance function is $d_\varepsilon(\mathbf{O}^p, \mathbf{O}^q) = \max_\alpha (f_\alpha(\mathbf{O}^p) - f_\alpha(\mathbf{O}^q)),\ \forall \alpha \in \{F, T, S, C, H\}$. A small example in Appendix B illustrates how the function in (15) establishes Pareto-dominance on a given set of solution points. Thus, the TVWS sharing MOP in (13) is transformed into a max-min optimization problem in (15) for which an evolutionary algorithm is designed in the following section.

TABLE I
AN EVOLUTIONARY ALGORITHM FOR COEXISTENCE DECISION MAKING IN TVWS (EVCO)

Input: $\mathcal{W}, \mathcal{J}, \mathcal{T}, \mathcal{D}, M, P, \delta_g, \Omega = [0, T_j]$

Output: $\mathbf{O}^* \leftarrow \mathbf{O}$: minimum $(\mathbf{F}(\mathbf{O}) \in \mathbb{O})\, \forall \mathbf{O} \in C_k^*$

Step 1: ***Initialization:*** generate an initial population $\mathcal{P}$ as follows,
a) Define a rule to select a WSO subset, $\mathcal{W}_j \subseteq \mathcal{W}$, sharing a channel j, $\forall j \in \mathcal{J}$.
b) Define $O_{w,j} \forall w \in \mathcal{W}_j, \forall j \in \mathcal{J}$ randomly and uniformly distributed on Ω.
c) Define $O_{w,j} = 0, \forall w \in \mathcal{W} \setminus \mathcal{W}_j, \forall j \in \mathcal{J}$.

Step 2: ***Population engineering:*** for each solution points $\mathbf{O} \in \mathcal{P}$ do:
a) For each channel $j \in \mathcal{J}$, set $O_{w,j} = \beta_w, \forall w \in \mathcal{W}_j$ if (11c) or (11d) is violated.
b) If (11a) is violated, reduce allocated occupancy time $\forall O_{w,j} \in \mathbf{O}$, using Eq. (16).
c) If (11b) is violated, reduce allocated occupancy time $\forall O_{w,j} \in \mathbf{O}$, using Eq. (17).

Step 3: ***Clustering:*** Form a set of clusters C defined by $\mathcal{P}$ using cosine similarity as, $C_k = \{\max.S(\mathbf{O}^p, \mathbf{O}^q)\} \forall (\mathbf{O}^p, \mathbf{O}^q) \in \mathcal{P}$.

Step 4: ***Calculations:***
a) $\forall \mathbf{O} \in C_k, \forall C_k \in C$, compute $\mathbb{O} = \{\mathbf{F}(\mathbf{O}^p)\}$, using Eq. (2), (4), (7), (9), (10) and (12).
b) For each ordered pair cluster $(C_k, C_l) \in C$, compute indicator function, $\mathcal{I}_{\varepsilon^+}(C_k, C_l)$, using Eq. (15) and store in an indicator table $\mathcal{K}$.
c) Compute g^{th} generation indicator value as, $I_g = \sum_{k,l \in \{1,\ldots,K\}, k \neq l} \mathcal{I}_{\varepsilon^+}(C_k, C_l)$.

Step 5: ***Elitism and Replacement:***
While $g > M$ or $|I_g - I_{g-1}| > \delta_g$ do:
a) Identify an elite cluster set as, $\{C_k^*\} \leftarrow \min(\mathcal{I}_{\varepsilon^+}(C_k, C_l) \in \mathcal{K})$, and define suboptimal cluster set as, $C' \leftarrow C \setminus \{C_k^*\}$.
b) For each suboptimal cluster $C_k \in C'$, generate an offspring cluster as: $C_k^{\downarrow} = \{\mathbf{O}\}$, randomly on the domain $\Omega = [0, 1]$ such that $|C_k^{\downarrow}| = |C_k|$.
c) For all offspring clusters, $C_k^{\downarrow}$, apply Step 2 and Step 4.
d) For every $C_k^{\downarrow}$, if $\sum_{l \in \{1,\ldots,K\} \setminus k} \mathcal{I}_{\varepsilon^+}\left(C_k^{\downarrow}, C_l\right) < \sum_{l \in \{1,\ldots,K\} \setminus k} \mathcal{I}_{\varepsilon^+}(C_k, C_l)$ then:
i. Define next generation indicator value as: $I_{g+1} = I_g - \sum_{l \in \{1,\ldots,K\} \setminus k} \mathcal{I}_{\varepsilon^+}(C_k, C_l) + \sum_{l \in \{1,\ldots,K\} \setminus k} \mathcal{I}_{\varepsilon^+}\left(C_k^{\downarrow}, C_l\right)$,
ii. Next generation cluster set as, $C \leftarrow C_k^{\downarrow} \cup \{C\} \setminus C_k$,
iii. Update $\mathcal{K}$ as, $\mathcal{K} \leftarrow \mathcal{I}_{\varepsilon^+}\left(C_k^{\downarrow}, C_l\right) \cup \mathcal{K} \setminus \mathcal{I}_{\varepsilon^+}(C_k, C_l), \forall l \in \{1, \ldots, K\} \setminus k$.

Step 6: Return $C_k^* \leftarrow C_k : \min(\mathcal{I}_{\varepsilon^+}(C_k, C_l) \in \mathcal{K})$

V. EVCO: AN EVOLUTIONARY ALGORITHM FOR COEXISTENCE DECISION MAKING IN TVWS

An EvCo algorithm, shown in Table I, is an update procedure that runs on the CDM system, proposed in Section IV. The EvCo uses its inputs – CDM system input parameters defined in (1), population size P, a number of generations M, generation indicator threshold δ_g and MOP domain $\Omega = [0, T_j]$ – to progressively improve the solutions in the set $\mathcal{P}$ using the optimizing function in (15). The EvCo output a solution $\mathbf{O}^* \in C_k^*$ that represents the best approximation of the Pareto-optimal point, as shown in the output section in Table I.

A. Explanation of Algorithm Steps

In the initialization step, a set of randomly generated solution points $\mathcal{P} = \{\mathbf{O}^1, \mathbf{O}^2, \ldots, \mathbf{O}^P\}$, called a population is defined. Each solution point $\mathbf{O} \in \mathcal{P}$ uniformly distributes the WSOs in the available TVWS, as follows. Let $\mathcal{W}_j \subseteq \mathcal{W}$ be the subset of WSOs sharing the channel j. Then, for each WSO in the set $\mathcal{W}_j$, the EvCo generates the channel occupancy time, $\forall O_{w,j} \in \mathbf{O}, \forall w \in \mathcal{W}_j, \forall j \in \mathcal{J}$, randomly and uniformly distributed in the domain Ω. The WSOs, not scheduled in channel j, get zero occupancy time, i.e., $O_{w,j} = 0, \forall w \in \mathcal{W} \setminus \mathcal{W}_j, \forall j \in \mathcal{J}$. The random generation of the occupancy time values, $\forall O_{w,j} \in \mathbf{O}$, may result in violating the constraints in (11). In such a case, the EvCo applies the population engineering to update the solution point $\forall \mathbf{O} \in \mathcal{P}$, as follows. If constraint (11c) or (11d) is violated, the EvCo updates the occupancy time $\forall O_{w,j} \in \mathbf{O}$ of each allotted WSO w on channel j, using its minimum allocable occupancy time, $\beta_w \in \mathcal{R}$. If constraint (11a) is violated, the EvCo computes an over-allocation as, $\sum_{w \in \mathcal{W}_j} O_{w,j} - T_j$, and calculates $\frac{O_w - O_{w,j}}{\sum_{w \in \mathcal{W}_j} (O_w - O_{w,j})}$ to compute normalized unsatisfied allocation. Next, the occupancy time of each WSO sharing a channel j is updated as,

$$O_{w,j} \longleftarrow O_{w,j} - \left(\sum_w O_{w,j} - T_j\right) \frac{O_w - O_{w,j}}{\sum_w (O_w - O_{w,j})}, \quad \forall w \in \mathcal{W}_j, \forall j \in \mathcal{J}. \tag{16}$$

If constraint (11b) is violated, the over-allocation, $\sum_{\forall j \in \mathcal{J}_w} O_{w,j} - n_w O_w$, is reduced in proportion to O_w as,

$$O_{w,j} \longleftarrow O_{w,j} - \left(\sum_{j \in \mathcal{J}} O_{w,j} - n_w O_w\right) \frac{O_{w,j}}{O_w}, \forall w \in \mathcal{W}_j, \quad \forall j \in \mathcal{J}. \tag{17}$$

The EvCo then forms a cluster of solution points in $\mathcal{P}$ with enough similarity. Clustering the solution points helps the EvCo to rank a set of non-comparable solution points rather than a single solution point. This property improves the convergence speed of the algorithm as discussed in Section VI. The similarity among solution points in $\mathcal{P}$ is measured using a cosine similarity function. Briefly, the cosine similarity measures the angular similarity between two vectors as [24],

$$S(\mathbf{O}^p, \mathbf{O}^q) = \frac{\langle \bar{\mathbf{O}}^p, \bar{\mathbf{O}}^q \rangle}{\|\mathbf{O}^p\| \|\mathbf{O}^q\|}, \forall \mathbf{O}^p, \mathbf{O}^q \in \mathcal{P} \tag{18}$$

where $\bar{\mathbf{O}}^p$ and $\bar{\mathbf{O}}^q$ represent the one-dimensional transformation of 2-D vectors $\mathbf{O}^p$ and $\mathbf{O}^q$, respectively. The solution points with the maximum cosine similarity are grouped in the same cluster, e.g., C_k. A cluster set is then defined as, $\mathcal{C} = \{C_1, C_2, \ldots, C_k\}$. The EvCo then computes the fitness function of each cluster in $\mathcal{C}$ as define in Step 4 in Table I. In the fitness function calculation, an indicator value is calculated for every ordered cluster pair $(C_k, C_l) \in \mathcal{C}$ and stored in an indicator table $\mathcal{K}$.

The EvCo then iterates for a number of generations to improve the quality of the solution points. This process is achieved through elitism and replacement operators of the evolutionary theory. In elitism, a set of clusters with the best indicator value in generation g, denoted as $\{C_k^*\}$ is identified and passed to the next generation cluster set. The elite set size is defined as, $|\{C_k^*\}| = |\mathcal{C}| - |\mathcal{C}|^\alpha$ where $\alpha \in [0, 1]$ is a scaling factor to control the rate of elitism. The elitism can increase the performance of the EvCo because it prevents losing the best-found solutions in the current generation. The EvCo then generates an offspring cluster, $C_k^\downarrow$ against all worst valued clusters in the set $\mathcal{C}' = \mathcal{C} \setminus \{C_k^*\}$. Next, the EvCo applies hill-climbing based replacement operator for each offspring cluster as follows. It computes an indicator function value $I_{\varepsilon^+}(C_k^\downarrow, C_l)$ for an ordered cluster pair, $(C_k^\downarrow, C_l), \forall C_l \in \mathcal{C}$. If the indicator value of the offspring cluster $C_k^\downarrow$ is lower than that of the corresponding cluster $C_k \in \mathcal{C}'$, the $C_k^\downarrow$ replaces C_k in the next generation cluster set and indicator table is updated with the indicator value of $C_k^\downarrow$, otherwise C_k is passed to the next generation cluster set and indicator table remains unchanged. The elitism selection and hill climbing replacement process continues until a stopping criterion, as defined in Step 5.

B. Complexity Analysis of EvCo

In this section, we make some comments about the computational cost of the EvCo. Let W, J, P=$|\mathcal{P}|$ be the number of WSOs, the number of channels and the population size, respectively. Generating an initial population, performing population engineering and computing objective functions in (2), (4), (7), (9), (10), (12), (16), and (17) all are linear operations in W, J, and P having complexity, $O(PWJ)$. The cosine similarity in (18) is defined by computing the Euclidean dot product and Euclidean distance operator, both of which have a computational complexity of the order of population size P, defined as $O\left(\frac{P(P-1)}{2}\right)$. Computing an indicator function involves finding a minimum epsilon so that a cluster C_k becomes weakly Pareto-optimal to cluster C_l for each ordered pair of clusters $(C_k, C_l) \in \mathcal{C}$. It requires to compute functions in (2), (4), (7), (9),(10), and (12) each of which requires $O(PWJ)$ complexity.

In the elitism step, EvCo identifies an elite cluster C_K^* in an arbitrary array (indicator table) of length $K \times K$ which is a linear time operation requiring $O\left(M\left|\{C_k^*\}\right|\right)$ complexity where M is the total number of generations. Let $N \leq P$ be the number of solution points of all offspring clusters $C_k^\downarrow$ then, population generation, population engineering, objective function calculations, and the indicator function in (15) all require $O(MNWJ)$ computational complexity. The cosine similarity function and clustering the N solution points require $O\left(\frac{MN(N-1)}{2}\right)$ and $O\left(M(N-1)^2\right)$ complexity, respectively. Finally, the overall computational complexity of the proposed algorithm is a function of the number of generations, M, the elitism rate, N , the number of WSOs, W, and the number of channels J, defined as, $O(MNWJ)$.

VI. Simulations Results

In this section, we describe our simulation setup, the summarized allocation policies of comparative algorithms, and the comparative results of the simulation.

A. Simulation Setup

Consider an 802.19.1 coexistence system deployed in a geographic region. The number of coexisting WSOs in the area is W=32 and the number of available TV channels in the area varies as, J=$\{5, 6, \ldots, 16\}$. The system has eight CMs, each serving four WSOs. The MATLAB® is used as a simulation platform to model the WSOs and their channel demand parameters as follows. Each WSO is defined as a group of unlicensed TV band devices. These devices are modeled using FCC regulations. The FCC defines four types of TV band devices, such as fixed, portable Mode 1, portable Mode 2 and sensing only [25]. In this simulation, we model first three types of devices. The channel demands and channel characteristics of WSOs are randomly generated. For example, the additive white Gaussian noise channels are considered. The transmission power of each WSO is generated randomly on $(0, P_{\max}]$ where $P_{\max}$ is the maximum allowed transmission power which is set based upon WSO type. For example, a WSO with fixed transmitter like AP in IEEE 802.22 network can radiate at a maximum of 4 W. A WSO comprising portable mode devices like in IEEE 802.15.4 can transmit at a maximum of 100 mW. An initial population of 50 solution points,$\mathcal{P}\{\mathbf{O}^1, \mathbf{O}^2, \ldots, \mathbf{O}^{50}\}$ is randomly generated in the domain of (11), $\Omega = [0, T_j]$, using rand function in MATLAB, where $T_j = 1, \forall j \in \mathcal{J}$. The solution points in $\mathcal{P}$ are then grouped into 25 clusters based on maximal cosine similarity values. The worst fitness valued chromosome in each generation is replaced with randomly generated new offspring cluster. The number of generations is set as, M=300.

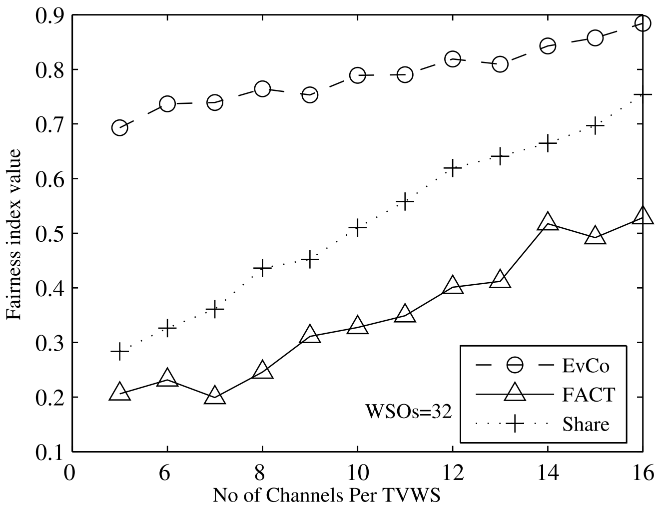

Fig. 2. Fairness index value of coexisting WSOs for a variable number of TV channels in the system.

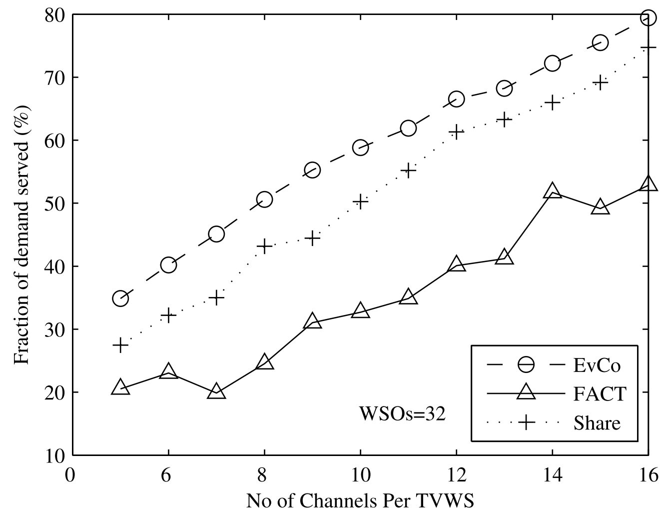

Fig. 3. WSO satisfaction from TVWS allocation in terms of WSO fraction of channel demands for a variable number of TV channels in the region.

B. Comparative Algorithms

The proposed algorithm is compared to FACT [9] and Share [10]. The TVWS sharing mechanism of FACT is summarized in Section II. Note that for comparative purposes; the channel allocation strategy is the same as that defined in [9]; however, the objective functions used to evaluate the FACT performance are as defined in (2), (4), (7), (9), and (10).

The TVWS sharing problem in [10] is modeled as a lexicographic ordering of throughputs of coexisting networks, as summarized in Section II. The *Share* algorithm in [10] operates in three phases. In the first phase of allocation, *Share* orthogonalizes the WSOs in the available TV channels. In the second phase, channel sharing is performed mutually among allotted WSOs of the first phase under the condition that their throughputs achieved in the first phase do not decrease. The fairness is improved in the third phase by sharing the channel with WSOs that do not obtain channels in the previous phases such that lexicographically ordered throughputs does not decrease. We show graphs of the simulation results of the three algorithms in Fig. 2 to Fig. 5. The dashed, dotted, and solid lines in the figures represent the behaviors of EvCo, FACT, and *Share*, respectively, as explained in the following subsection.

C. Results and Discussions

The three allocation algorithms are compared using the following performance metrics: fairness in allocation, system throughput, WSO satisfaction in terms of percentage of their demand served, and resource utilization in terms of spectral efficiency (SE).

1) Fairness in Allocation: Fig. 2 shows the fairness in sharing the TVWS, measured by Jain's fairness index, as defined in (2). A higher index value indicates fairer allocation. As can be seen in Fig. 2, the EvCo achieves a higher fairness index value. This is due to the flexible allocation policy of EvCo. In this policy, the channel occupancy allocation of WSOs sharing a channel is adjusted such that their normalized throughputs are equalized maximally, i.e., $T_w \approx T_m, \forall w, m \in \mathcal{W}$, as defined in Section IV-A. FACT also considers the normalized throughput as a fairness metric; however, it's strict allocation policy is discriminating, as explained in Section II. As a result, a decreased fairness index value is observed, especially when the number of TV channels in the system is low, as shown in Fig. 2.

The *Share*, on the other hand, achieves a better fairness index value than the FACT. This improvement is because it does not strictly satisfy the channel occupancy demand of each allotted WSO. Rather it enables WSOs to share a channel in second and third phase of allocation. However, a channel is shared among coexisting WSOs only if the system throughput is improved. This constrained sharing reduces the fairness in allocation among coexisting WSOs. As a result, the fairness in allocation of *Share* is comparatively lower than the EvCo algorithm as shown in Fig. 2.

2) WSO Satisfaction From Allocation: WSO satisfaction is defined in terms of their fraction of channel demand served, defined as, $\frac{1}{W}\sum_{w\in\mathcal{W}}\frac{r_w}{d_w}$. Fig. 3 shows that the EvCo achieves the highest average WSO satisfaction as compared to the comparative algorithms. This improvement is due to maximally satisfying the channel demands of the WSOs by optimizing their achieved data rates, as defined in (10). Moreover, the EvCo readjusts the channel occupancy time of WSOs to schedule a greater number of WSOs in a channel. These allocation steps improve the fraction of the demand served to the WSOs in the system. On the other hand, the FACT achieves the least WSO satisfaction from the allocation as shown in Fig. 3. This decrease in satisfaction is due to it's an unequal channel slot allocation among WSOs sharing a channel, as discussed in Section II. Consequently, a higher variation in WSO fraction of demand serve is observed which leads to a lower overall WSO satisfaction in the system. The *Share*, however, achieves a higher WSO satisfaction from the allocation than that of the FACT as shown in the figure. This is because the *Share* enables the coexisting WSOs to share the

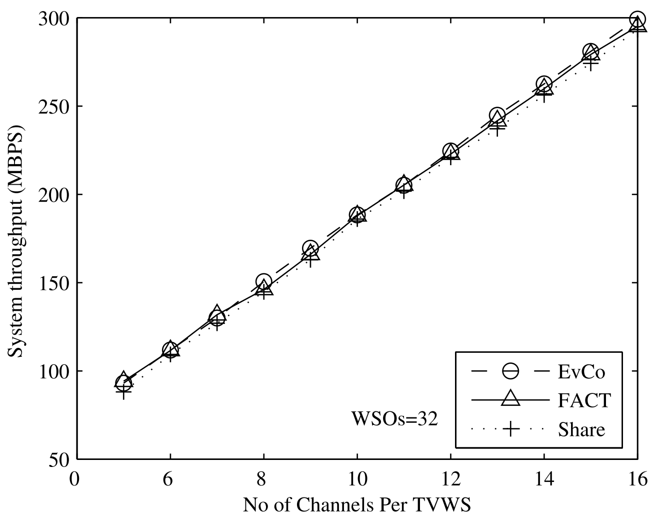

Fig. 4. System throughput for 32 WSOs on a varying number of TV channels.

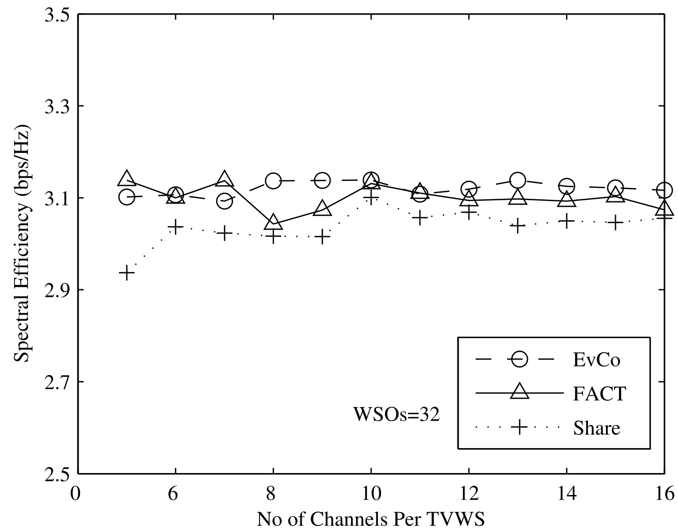

Fig. 5. The spectral efficiency of coexisting WSOs averaged over a number of available TV channels in the system.

channels during the second and third phase of allocation. This sharing process improves their fraction of channel demand serve thus, leading to a comparatively higher WSO satisfaction in the system as shown in Fig. 3.

3) System Throughput: Fig. 4 shows the system throughput achieved by the three algorithms. The system throughput (ST) is measured using the Shannon-Hartley capacity theorem [26] as follows.

$$ST = \sum_{j \in \mathcal{J}} \sum_{w \in \mathcal{W}} O_{w,j} b_{w,j} \log_2\left(1 + SINR_{w,j}\right) \tag{19}$$

Fig. 4 shows that the EvCo and FACT exhibit competitive behavior. At some instance of the number of TV channels in the system, the EvCo results in higher system throughput while in some other cases, the FACT gives higher throughput. The reason is that both algorithms make use of optimization parameters such as homogeneity and contiguous channel allocation in their MOP formulation. These optimization steps result in lower scheduling delays in sharing the channel among the WSOs and help the WSOs to use adaptive channel widths. These achievements improve the channel utilization thus, leading to a higher system throughput. On the other hand, *Share* gives a lower system throughput than the EvCo and the FACT, as shown in Fig. 4, which is due to an orthogonal channel allocation in the first phase of allocation. In such allocation, it is quite possible that if a WSO with poor signal to interference and noise ration (SINR) happen to get a channel and the WSOs with good SINR value may not able to share the channel in the second or third phase due to the constraint of maintaining the lexicographic ordering of throughput. Consequently, the system throughput achieved by the *Share* is decreased, as shown in Fig. 4.

4) SE: According to ITU-R [27], the SE of a radio communication system can be defined as follows:

$$\eta = \frac{M}{B \times S \times T} \tag{20}$$

where M is the amount of information transferred over distance S using spectrum width B in time T. Keeping all other parameters constant, we define M as the data rate achieved by WSOs sharing available TVWS, as described in (19), and the distance is taken as one without loss of generality. The parameter T is set equal to the window time of the channel which is taken as one without loss of generality, and the channel width is 6 MHz. Fig. 5 shows the effect of heterogeneity on resource utilization regarding the SE of the three algorithms. In the figure, the bps/Hz value is averaged over the number of channels in the system.

Fig. 5 shows that both EvCo and FACT show competitive behavior. However, as compared to *Share*, EvCo yields a better SE result. This improvement is due to optimizing SINR and contiguous channel allocation parameters in the MOP formulation in EvCo, as defined in (4) and (7), respectively. Fig. 5 also shows that the SE values, especially those of FACT, are higher when the number of channels in the system is small and gradually decrease as the number of channels increases. The reason is that when the number of available channels is low, the WSOs with optimal channel utilization are prioritized in channel allocation over the WSOs with suboptimal channel utilization. However, as the number of available channels increases, suboptimal WSOs can get a larger share of the channels. These suboptimal WSOs have a detrimental effect on achievable SE due to the poor channel conditions. This effect intensifies as the number of available channels exceeds 19. At this point, almost all coexisting WSOs in the system obtain a plentiful share of the available spectrum, leading to a sharp decline in the SE values of the three algorithms.

D. Complexity Graph of EvCo, FACT, and Share

In this section, we empirically compare the performance of the three algorithms using performance metrics like accuracy and speed. The performance study is done by measuring how well each algorithm approximates an utopia point. An utopia point in multiobjective optimization is a point where every objective function achieves an optimal value. As objective functions $f_\alpha \in \mathbf{F}, \forall \alpha \in \{F, T, S, C, H\}$ in (13)

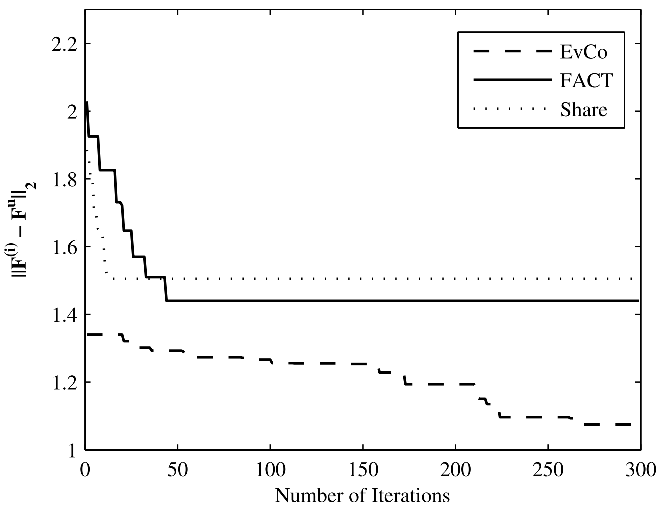

Fig. 6. Accuracy in the solution obtained at each iteration.

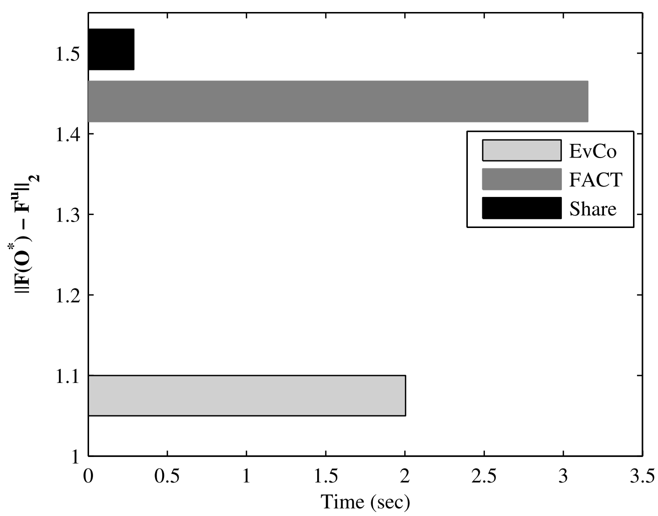

Fig. 7. Run-time of the three algorithms to identify the best solution.

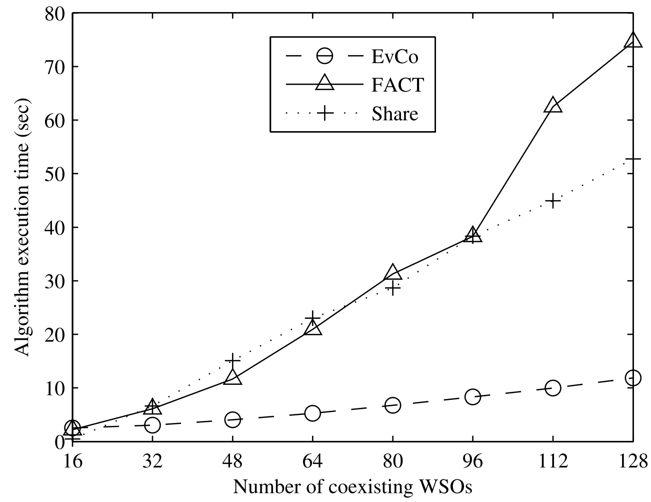

Fig. 8. Run-time of the three algorithms to identify an optimal solution for larger denser networks.

TABLE II
DATA SET FOR NON-CONVEXITY PROOF

WSO No.	$\mathbf{O}^p$	$\mathbf{O}^q$	d_w	SINR
1	0.2240	0.1634	10.6420	2.7287
2	0.1763	0.4756	48.7117	2.5371
3	0.5997	0.3610	9.4433	1.9674

TABLE III
INPUT PARAMETERS TO EVCO: DATA SET FOR EXAMPLE SETUP

WSO No(w)	Input parameters to the algorithm				Achieved rate
	O_w	n_w	$SINR_{w,j}$	d_w (MBPS)	r_w (MBPS)
1	0.95	1	6.7799	16.8706	10.1353
2	0.50	1	6.5284	8.7370	3.8802
3	0.40	2	7.8409	15.0921	10.4611
4	0.70	1	4.8911	10.7459	3.7024
5	0.90	1	5.4754	14.5528	6.4049

are contradicting, therefore, a single solution point $\mathbf{O}^*$ cannot optimize all of them. An optimal solution $\mathbf{O}^{\alpha*}$ is obtained. It is point where an objective function f_α is individually optimized. Let $\mathbf{F}^{\mathrm{u}}$ be an utopia point for $\mathbf{F}$ in (13) defined as, $\mathbf{F}^{\mathrm{u}} = \left[f_F\left(\mathbf{O}^{F*}\right), f_T\left(\mathbf{O}^{T*}\right), f_S\left(\mathbf{O}^{S*}\right), f_C\left(\mathbf{O}^{C*}\right), f_H\left(\mathbf{O}^{H*}\right]^T\right.$. Since, each objective functions in $\mathbf{F}$ in (13) is a non-negative minimization function, as defined in Section IV-*C*, therefore, we define the utopia point for MOP in (13) as, $\mathbf{F}^{\mathrm{u}} = [0, 0, 0, 0, 0]$. Then, using $\mathbf{F}^{\mathrm{u}}$, the accuracy and the convergence test of the three allocation algorithms are performed as follows.

In the accuracy test, we compute measurement error to determine how good each of the three comparative algorithms approximates the Utopia point. The measurement error function is defined as, $\left\|\mathbf{F}^{(\mathrm{i})} - \mathbf{F}^{\mathrm{u}}\right\|_2$ where $\mathbf{F}^{(\mathrm{i})}$ is the the i^{th} iteration objective function value of each algorithm. The lower is the measurement error, the better is the solution point. The result of the accuracy study is shown in a graph in Fig. 6. The figure shows that the proposed algorithm gives the least measurement error. The difference between lines of the EvCo and the comparative algorithms in Fig. 6 attributes to the true multiobjective optimization property of the proposed CDM system, as discussed in the Introduction section. Moreover, although the EvCo converges to an optimal point at a much higher number of iterations, more than 250 in Fig. 6, yet, it is faster than the FACT as shown in Fig. 7, as discussed follows.

In the convergence test, we measure how quickly the three algorithms converge to an optimal solution point. The test results shown in Fig. 7 and Fig. 8 are defined as follows. Let $\left\|\mathbf{F}\left(\mathbf{O}^*\right) - \mathbf{F}^{\mathrm{u}}\right\|_2$ gives measurement error, defined on function $\mathbf{F}$ using an optimal solution point, $\mathbf{O}^*$. The time taken to identify $\mathbf{O}^*$ by each algorithm is shown in Fig. 7. The result in the figure shows that the *Share* finds $\mathbf{O}^*$ quickly than the comparative schemes. The reason is that the complexity of *Share* is a function of the number of WSOs getting a channel in the first phase of allocation. Since, the allocation process in *Share* is orthogonal in the number of channels where the maximum number of channels in the simulation setup are 16, therefore, the *Share* run-time is considerably short. However, as the number of WSOs in the system increases, the *Share* takes the comparatively higher time to identify $\mathbf{O}^*$ as shown in Fig. 8. On the other hand, the FACT takes the highest

TABLE IV
INITIAL POPULATION FOR EXAMPLE SETUP

WSO Number(w)	C_1				C_2			
	Solution $\mathbf{O}^3$		Solution $\mathbf{O}^4$		Solution $\mathbf{O}^1$		Solutino $\mathbf{O}^2$	
	$O_{w,1}$	$O_{w,2}$	$O_{w,1}$	$O_{w,2}$	$O_{w,1}$	$O_{w,2}$	$O_{w,1}$	$O_{w,2}$
1	0.6979	0	0.5757	0	0.4972	0	0.3418	0
2	0	0.1672	0	0.2223	0	0.3161	0	0.3766
3	0.3021	0.2235	0.4243	0.1352	0.1591	0.2442	0.3066	0.3655
4	0	0.2424	0	0.2462	0	0.4397	0	0.2579
5	0	0.3669	0	0.3963	0.3437	0	0.3516	0

time to identify a solution point $\mathbf{O}^*$ as shown in Fig. 7. The reason is that computing the weight of neurons in this scheme requires high run-time complexity of the order of $O\left(J^2W^2T\right)$ where T is the number of time slots per channel. Moreover, as the number of WSOs or the number of channels in the system increases, the complexity of FACT increases quadratically as shown in Fig. 8. On the other hand, the EvCo outputs optimal solution $\mathbf{O}^*$ more quickly because computing an indicator function is fast. Moreover, the EvCo finds the optimal solution point by simply calculating the functions defined in (2), (4), (7), (9), and (10). Such computations require linear time complexity. Thus, these results conclude that the EvCo is highly versatile in highly congested areas and completes the TVWS scheduling process in a quick run-time for larger, denser networks.

VII. CONCLUSION

In this paper, we design an 802.19.1-compliant coexistence decision making (CDM) system that implements a multiobjective optimization problem (MOP) for channel sharing in TVWS. We also design an evolutionary algorithm, called EvCo, to schedule a set of hetero-WSO on a set of available TV channels in the system. We evaluate the performance of the EvCo on 802.19.1-compliant CDM system and compare its performance with existing TVWS sharing algorithms. Our evaluation results show that the EvCo is superior to the comparative algorithms regarding fairness and WSO satisfaction from the allocation. Moreover, the EvCo can be readily implemented in an 802.19.1-based CDM system without requiring any significant changes to the architecture of the baseline system.

APPENDIX A
NON-CONVEXITY OF F

In this section, we show that the objective function in (11) is non-convex on $\mathcal{P}$.

Definition 2: The function $\mathbf{F}$ in (11) is considered convex if and only if $f : \mathbb{R}^{W\times J} \rightarrow \mathbb{R}_0^+, \forall f \in \mathbf{F}$, is convex, $\mathcal{P}$ is convex set, and $\forall \mathbf{O}^p, \mathbf{O}^q \in \mathcal{P}$ using $\theta \in [0, 1]$ if the following inequality holds:

$$f\left(\theta\mathbf{O}^p + (1-\theta)\mathbf{O}^q\right) \leq \theta f\left(\mathbf{O}^p\right) + (1-\theta) f\left(\mathbf{O}^q\right).$$

Using a counterexample, we show that function $\mathbf{F}$ is not a convex function. The set $\mathcal{P}$ is convex if $\forall \mathbf{O}^p, \mathbf{O}^q \in \mathcal{P}$ implies that $\left(\theta\mathbf{O}^p + (1-\theta)\mathbf{O}^q\right) \in \mathcal{P}$ with $0 \leq \theta \leq 1$. Let $\exists \mathbf{O}^p, \mathbf{O}^q \in \mathcal{P}, \theta = 0.95$ and the parameters like d_w and SINR, as shown in Table II, we compute $f_F\left(0.95\mathbf{O}^p + 0.05\mathbf{O}^q\right) = 0.349$ and $(0.95 f_F\left(\mathbf{O}^p\right) + 0.05 f_F\left(\mathbf{O}^q\right)) = 0.346$. These results show that $f\left(\theta\mathbf{O}^p + (1-\theta)\mathbf{O}^q\right) > \theta f\left(\mathbf{O}^p\right) + (1-\theta) f\left(\mathbf{O}^q\right)$, which violates the inequality defined in Definition 2. An MOP is convex if all objective functions and feasible regions are convex [28], [29]. However, f_F has been shown to be a non-convex function; therefore, $\mathbf{F}$ is non-convex.

APPENDIX B

Following example shows how the EvCo, schedules a set of WSOs $\mathcal{W} = \{1, 2, \ldots, 5\}$, on a set of channels, $\mathcal{J} = \{1, 2\}$. Let the input parameters to the CDM system be as shown in Table III. Let $\mathcal{P} = \left\{\mathbf{O}^1, \ldots, \mathbf{O}^4\right\}$ be a randomly generated set of four solution points, as shown in Table IV. These solution points are arranged into two clusters, $C_1 = \left\{\mathbf{O}^3, \mathbf{O}^4\right\}$ and $C_2 = \left\{\mathbf{O}^1, \mathbf{O}^2\right\}$, based upon maximal cosine similarity values calculated using (18). Then, for all solution points in Table IV, the function $\mathbf{F}$ in (13) is computed as, $\mathbf{F}\left(\mathbf{O}^1\right) = (0, 1, 0, 0, 1)^{\mathrm{T}}$, $\mathbf{F}\left(\mathbf{O}^2\right) = (1, 0, 0.4908, 0, 1)^{\mathrm{T}}$, $\mathbf{F}\left(\mathbf{O}^3\right) = (0.72, 0.1271, 1, 0, 0)^{\mathrm{T}}$, $\mathbf{F}\left(\mathbf{O}^4\right) = (0.2851, 0.1750, 0.5664, 0, 0)^{\mathrm{T}}$. Then, the distance function for each solution point pair $\left(\mathbf{O}^p, \mathbf{O}^q\right) \forall \mathbf{O}^p \in C_1, \forall \mathbf{O}^q \in C_2$ is computed using (15) as, $d_\varepsilon\left(\mathbf{O}^p, \mathbf{O}^q\right) = \max\left(\mathbf{F}\left(\mathbf{O}^p\right) - \mathbf{F}\left(\mathbf{O}^q\right)\right)$. For example, $d_\varepsilon\left(\mathbf{O}^3, \mathbf{O}^1\right) = 0.2806$, $d_\varepsilon\left(\mathbf{O}^3, \mathbf{O}^2\right) = 0.1429$, $d_\varepsilon\left(\mathbf{O}^4, \mathbf{O}^1\right) = 0.1590$, $d_\varepsilon\left(\mathbf{O}^4, \mathbf{O}^2\right) = 0.1234$, $d_\varepsilon\left(\mathbf{O}^1, \mathbf{O}^3\right) = 0.6153$, $d_\varepsilon\left(\mathbf{O}^1, \mathbf{O}^4\right) = 0.5816$, $d_\varepsilon\left(\mathbf{O}^2, \mathbf{O}^3\right) = 0.4530$, $d_\varepsilon\left(\mathbf{O}^2, \mathbf{O}^4\right) = 0.4530$.

The indicator function value for an ordered pair cluster (C_1, C_2) is then defined using Eq. in (15) as, $I_{\varepsilon^+}(C_1, C_2) = \min\{0.2806, 0.1429, 0.1590, 0.1234\} = 0.1234$. Similarly, for ordered pair (C_2, C_1), indicator function value is calculated as, $I_{\varepsilon^+}(C_2, C_1) = \min\{0.6153, 0.5816, 0.4530, 0.4530\} = 0.4530$. Since, $I_{\varepsilon^+}(C_1, C_2) < I_{\varepsilon^+}(C_2, C_1)$ thus C_1 Pareto-dominates C_2. The EvCo next iterates for the number of generations M=300 and produces an optimal solution $\mathbf{O}^*$. The rate achieved by each WSO, $w \in \mathcal{W}$ is then calculated using $\mathbf{O}^*$ as shown in Table III.

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