

# Design of Multiplierless Cosine Modulated Filterbank using Hybrid Technique in Sub-expression Space

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**Abstract:** In this paper, an optimal design technique for  $M$ -channel multiplierless cosine modulated filterbank (CMFB) is proposed using common sub-expression technique (CSE) and hybrid method with given roll-off factor (RF) and stopband attenuation ( $A_s$ ). The key feature of the proposed method is utilization of single optimization algorithm to generate optimal quantized and canonic signed digit (CSD) converted coefficients that satisfy the magnitude response of 0.7071 at frequency  $\omega = \pi/2M$ . CSE is employed to reduce the hardware requirement (adders) of a designed filter. Hybrid technique is based on the concept of particle swarm optimization (PSO) and artificial bee colony (ABC) algorithm. A comparative analysis of different CSE algorithms has been made, and performance of the proposed method is evaluated in term of adders.

**Keywords:** Multiplier-less  $M$ -channel filterbank, Roll-off-factor (RF), Common sub-expression elimination (CSE), hybrid optimization algorithm.

## I. INTRODUCTION

Multirate filter banks have emerged as an important field of research due to its miscellaneous applications in innumerable fields such as software defined radio (SDR) [1], filterbank multicarrier systems (FBMC) [2], satellite and image processing [3] etc.  $M$ -channel filterbank can be classified as tree structured, parallel and cosine modulation based [4]. Among these filter banks, cosine modulated based filterbank (CMFB) is mostly exploited due to its efficient design procedure. In CMFB, only one prototype filter is required to design efficiently, and then the rest of other filters are derived using cosine modulation [4]. The generalized block diagram of  $M$ -channel filterbank is given in Fig. 1 [4].

Efficiency of CMFBs in different applications relies on their efficient design. Therefore, several designs were proposed such as linear search optimization or iterative techniques [5-10], gradient based techniques [11-14], and closed form method [15, 16]. In these methods, either cut-off frequency or passband frequency has been optimized to improve the reconstruction error using either windowing or weighted least square technique for the filter design. Recently, many evolutionary algorithms such as particle swarm optimization (PSO), artificial bee colony (ABC) algorithm, cuckoo search algorithm (CSA), and memetic algorithm based techniques have also been proposed to design efficient filter and filterbank

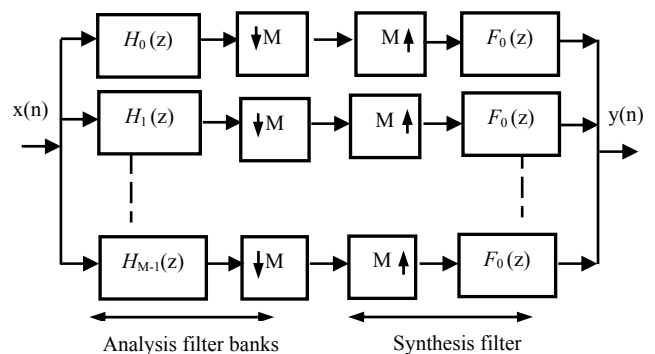


Figure 1. Block diagram of generalized  $M$ -channel filterbank

[17-21]. In general, design of digital filterbanks must be such that they utilize less hardware resources and provide high stopband attenuation with small channel overlapping for audio signal [23]; and for digital hearing aid, adjustable gain is required [24, 25]. For SDR, a reconfigurable filterbank with low complexity is preferred [26]. For realization of digital filters, adders and multipliers are needed; in which multipliers consume more area and high power as compared to adders. Therefore, multipliers are transformed into adders using add and shift method, and the circuit becomes multiplierless [27-30]. The complexity is further reduced using CSE techniques, where commonly appeared bit patterns (sub-expressions) are shared among them to eliminate the redundancy and so as the amount of adders [31]. For this purpose, firstly quantization process on continuous filter coefficients is performed, and then these coefficients are converted into CSD or binary. But, as a consequence, performance of designed filter is degraded, and given filter specifications are no longer satisfied [27, 31]. Initially, genetic algorithm (GA) was used to optimize the filter characteristics [32]. Since then, many approaches were proposed for optimal design of multiplierless digital filter and filter banks based on evolutionary algorithms (EAs) [33-37]. In these methods, two separate optimization techniques were employed, one for generating continuous filter coefficients, and another for optimizing the performance of CSD converted filterbank. Recently, authors [18, 38] have used single optimization technique for designing multiplierless filter and filterbank. In these techniques, optimization was performed to enhance the performance of designed filter and filterbank, respectively.

Therefore, in above context, this paper presents a simple design of multiplierless  $M$ -channel filterbank with CSE using hybrid optimization algorithm.

## II. HYBRID SWARM OPTIMIZATION ALGORITHM

Swarm algorithms are nature's inspired evolutionary optimization approach, comprised of collective behaviour of biological species such as fish schooling, birds flocking and food searching behaviour of honey bee swarm, etc. [17-22, 32-39]. These algorithms are used for optimization in different field due to their unique strategy of solving linear, nonlinear and complex problems in multidimensional space. Recently, authors [17] have modified the concept of different evolutionary techniques to enhance the exploration capabilities, for efficient design of multirate filterbank and IIR filters [17, 39].

Therefore, in this paper, hybrid swarm algorithm is exploited for optimization, utilizing the key concept of particle swarm optimization (PSO) and artificial bee colony algorithm (ABC) [17-22]. In PSO, optimization is done through iterative updating of the fitness function (cost function) according to an optimal position. For this purpose,  $M$  particles (parameter of optimization) are initialized with random positions and velocity in a  $D$ -dimensional space defined in each coordinate of the positions with upper and lower boundaries [17, 18], whereas, in ABC, optimization is done through iterative updating of a fitness function, (nectar amount) on the basis of performances of employed bee, scout bee, and onlooker bee respectively [17, 18, 38]. In the hybrid method, to improve the performance of earlier methodologies, the concept of limit and trials (from ABC) are commendably employed on updated particles (PSO) such as to keep the optimum value of fitness function in limit numbers of a trial. If it fails, then the position and velocity are replaced with optimal particles, which correspond to the best solution [17-19, 38]. For this purpose, control parameters are specified and  $M$  numbers of parameters are initialized with random position and velocity. The positions of particles are updated according to their current and previous position, given as [17-19]:

$$X_i(k+1) = X_i(k) + V_i(k+1), \quad (1)$$

while, velocity is updated as [17-19]:

$$V_{ik}(j+1) = wV_{ik}(j) + c_1r_1[p^{best}_{ik} - x_{ik}(j)] + c_2r_2[g^{best}_{ik} - x_{ik}(j)] \quad (2)$$

Here,  $V_{jk}$  represents the velocity of  $j^{th}$  particle in  $k^{th}$  dimension at  $i^{th}$  time step;  $r_1$  and  $r_2$  are the randomly generated numbers within a range of 0 and 1.  $C_1$  is a cognitive coefficient, defines the attraction towards its current best value,  $C_2$  is social parameters, determines the influence towards the swarm [17-19]. Then, the best position is selected by evaluation of objective function through comparison. If the new solution provides a minimum value of objective function than previous one, then the old position is replaced corresponding to that solution. The detailed description can be found in reference therein [17-21].

## III. OVERVIEW OF COMMON SUB-EXPRESSION ELIMINATION

In common sub-expression elimination (CSE) algorithms, the sub-expressions that have commonly occurred are eliminated to improve speed and performance of the system [27-29]. Initially, CSE was employed in compiler design to enhance the speed and to reduce the memory requirement of a system [27-29]. Later on, many researchers have utilized this concept for efficient realization of filters by minimizing the requirement of adders. Hence, efficient structure formulation is equally important in the realization of a filter. Moreover, the inputs that are to be fed are multiplied by constant (coefficients) of a filter, which required a large amount of memory (registers) to store the data. This problem is termed as multiple constant multiplications (MCM), which is also resolved through CSE, where the multipliers are realized in terms of adders, and these adders are further reduced using efficient sharing of sub-expressions [27-29]. Here the sub-expressions are formed by clubbing of redundantly presented bit patterns in a set of filter coefficients. The basic principle involved in CSE is iterative matching, where bitwise matches are linearly searched and eliminated [27-29]. On the basis of search mechanism, CSE is classified as horizontal CSE (HCSE), and vertical CSE (VCSE). In case of HCSE, bit patterns are searched and eliminated row wise or between the coefficients, while in VCSE, bit patterns are searched and eliminated column wise or across the coefficients [27-29].

## IV. DESIGN OF MULTIPLIERLESS $M$ -CHANNEL FILTERBANK

In this paper, an optimal multiplierless prototype filter has been designed for  $M$ -channel filterbank with CSE algorithms to minimize the requirement of adders, and to design resource efficient digital filter using hybrid optimization algorithm. CMFBs are designed using Kaiser window for a given stopband attenuation ( $A_s$ ) and channel overlapping. For this purpose, concept of roll off factor (RF) is utilized, a detailed description is given in references therein [10, 13-18]. In CMFB, only one prototype filter is required to design, while the rest of other filters are derived through cosine modulation.

The generated coefficients known as continuous coefficients are converted into quantized coefficient, and then CSD coefficients. CSE algorithms are employed to reduce the requirement of adders through HCSE, VCSE, and mixed CSE approaches for both binary represented, and CSD converted filter coefficients. To begin with the algorithms, initialization of parameters are done. The parameter of optimization is cut-off frequency ( $\omega_c$ ), while, upper limit has been set as passband edge frequency ( $\omega_p$ ), and lower limit has been set as stopband edge frequency ( $\omega_s$ ), computed as [10, 13-18]:

$$\omega_s = \frac{(1+RF)\pi}{2M}, \text{ and } \omega_p = \frac{\pi}{2M}, \quad (3)$$

Here,  $M$  is the number of band. In  $M$ -channel CMFB bank, condition for perfect reconstruction (PR) is given as [10, 13-18]:

$$\left| H_0(e^{j\omega}) \right|^2 + \left| H_0(e^{j(\omega-\pi/2M)}) \right|^2 = 1, \text{ for } 0 < \omega < \pi/2M \quad (4)$$

Here,  $H_0(z)$  represents the magnitude response of prototype filter. This PR condition is reduced to 0.7070 at quadrature frequency of  $\omega = \pi/2M$  representing ideal magnitude response (IMR) of prototype filter that is determined by varying the  $\omega_c$  using hybrid method [17, 38]. For this purpose, formulated objective function is given as:

$$\varphi = abs[MRI - MRD]; \quad (5)$$

Here, MRD represents the magnitude response of designed filter. This function is evaluated for minimum value, and corresponding solution is obtained as best solution. The

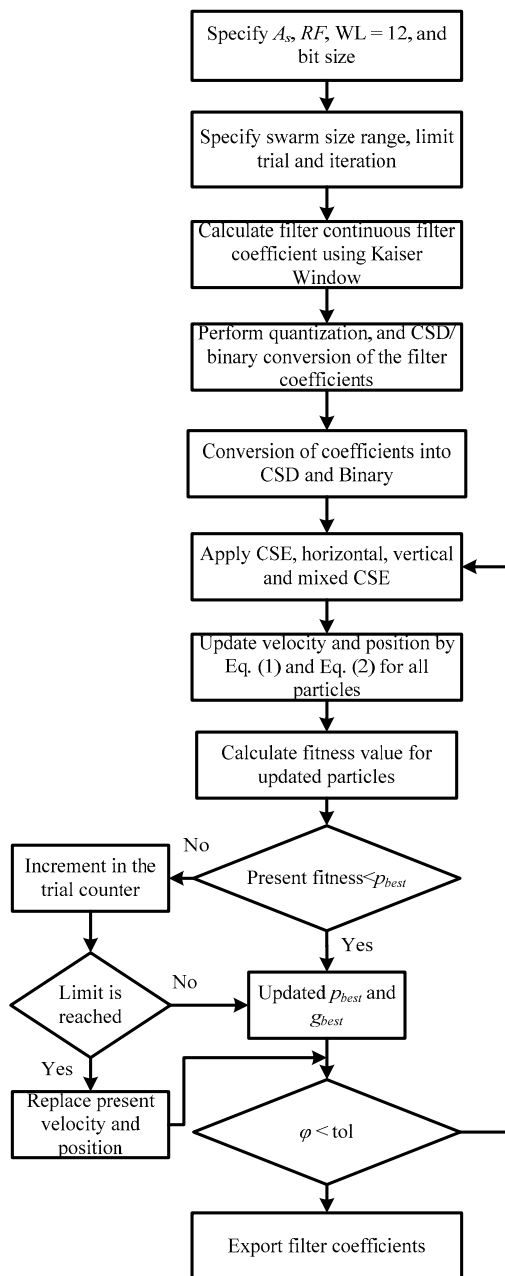


Figure 2. Flowchart of proposed methodology [17, 38].

procedure to describe this step is given in a flowchart, depicted as Fig. 2.

## V. RESULT AND DISCUSSION

In this section, design example with simulation results is discussed to analyze the performance of proposed algorithm using CSE. Various performance evaluation parameters such as amplitude distortion, aliasing distortion, CPU time, and saving of adders are computed [4, 18]. The mathematical formulas for computing errors are given as [4, 18]:

$$e_{am} = \max \left( 1 - \left| T_0(e^{j\omega}) \right| \right) \quad (6)$$

$$e_{t\_alias} = \left[ \sum_{s=1}^{M-1} |T_s(e^{j\omega})|^2 \right]^{1/2} \quad (7)$$

*Design Example:*

In this example, 8-channel CMFB has been designed with  $N=160$  i.e.  $M$  is taken as 8, value of  $RF$  is 1.29 with  $A_s = 80$ dB similar to [18]. Performance of the proposed filter is also compared with PSO, ABC and cuckoo search algorithm (CSA) as summarized in Table I, while comparison with magnitude response is depicted in Fig. 3.

It is observed that the proposed filter gives better result with hybrid technique. Hence, the complete performance of the designed filter using hybrid technique is summarized in Table II, and simulation result is depicted in Fig. 4. Here, in Table II, AMD represents amplitude distortion, while ALD represents aliasing distortion. In Table III, performance of the proposed prototype using CSE is analyzed; here, VCSE and HCSE represents vertical and horizontal CSE approaches while, HVCSE represents mixed vertical and horizontal CSE approaches. These CSE are applied on binary and CSD filter coefficients respectively. It is observed that mixed CSE technique minimizes the number of adders significantly, while in Table 5, comparative study of the designed filter using proposed methodology with other published algorithms has been made. It has observed that performance of proposed filter has almost comparable performance after quantization and CSD conversion also, while with the aid of CSE requirement of adders are further reduced at the same time, hence only single optimization technique is utilized to generate optimized continuous filter coefficients and quantized coefficients with minimum number of adders for realization.

TABLE I PERFORMANCE EVALUATION OF DESIGNED PROTOTYPE FILTER USING DIFFERENT OPTIMIZATION ALGORITHMS

Parameters	ABC	PSO	CSA	Hybrid
$A_s$ (dB)	76.36	77.12	78.6	78.54
AMD ( $e_{am}$ )	$5.21 \times 10^{-2}$	$9.4 \times 10^{-3}$	$6.23 \times 10^{-3}$	$2.67 \times 10^{-3}$
RE	$2.78 \times 10^{-2}$	$8.19 \times 10^{-3}$	$4.54 \times 10^{-3}$	$2.53 \times 10^{-3}$
ALD ( $e_a$ )	$7.85 \times 10^{-4}$	$7.21 \times 10^{-5}$	$5.77 \times 10^{-6}$	$4.31 \times 10^{-6}$
CPU (s)	29.22	26.022	24.89	23.06
ADDERS	126	124	112	108

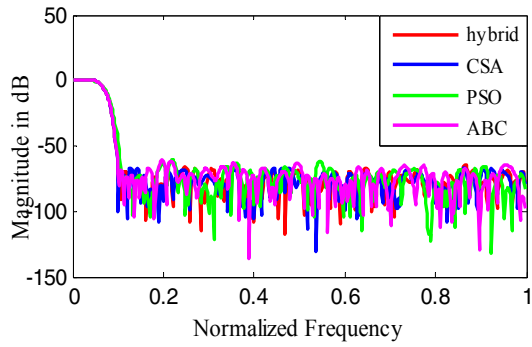


Figure 3. Comparing the magnitude response of prototype filter

TABLE II PERFORMANCE OF DESIGNED PROTOTYPE USING HYBRID TECHNIQUE

length	Bands	$A_s$ (dB)	RF	AMD ( $e_{am}$ )	ALD ( $e_a$ )	CPU (s)
144	8	80	1.12	$1.62 \times 10^{-3}$	$4.08 \times 10^{-5}$	24.44
144	8	100	1.42	$3.45 \times 10^{-3}$	$7.14 \times 10^{-6}$	24.41
160	8	80	1.29	$2.49 \times 10^{-3}$	$8.67 \times 10^{-5}$	24.03
224	16	100	1.84	$2.44 \times 10^{-3}$	$7.67 \times 10^{-7}$	24.37
288	16	95	1.35	$2.46 \times 10^{-3}$	$8.51 \times 10^{-7}$	24.17
448	32	100	1.83	$3.24 \times 10^{-3}$	$3.23 \times 10^{-7}$	25.32
512	32	100	1.61	$2.11 \times 10^{-3}$	$4.54 \times 10^{-7}$	25.92
576	32	110	1.58	$3.38 \times 10^{-3}$	$6.05 \times 10^{-7}$	26.18

TABLE III PERFORMANCE EVALUATION OF DESIGNED PROTOTYPE FILTER IN TERMS OF ADDERS USING CSE

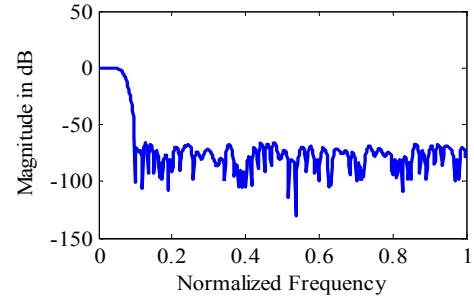
$N$	$A_s$ (dB)	CSD	CSD-VCSE	CSD-HCSE	CSD-HVCSE	Bin-VCSE	Bin-HCSE	Bin-HVCSE
144	80	103	98	86	84	87	81	73
144	100	108	96	80	88	85	83	72
160	80	149	126	119	108	94	90	76
224	100	168	120	116	106	102	98	86
288	95	198	148	124	114	108	102	90
448	100	252	229	212	198	176	160	108
512	100	248	224	218	202	189	173	112
576	110	296	260	247	234	196	189	135

TABLE IV PERFORMANCE COMPARISON OF DESIGNED PROTOTYPE FILTER WITH OTHER PUBLISHED ALGORITHM

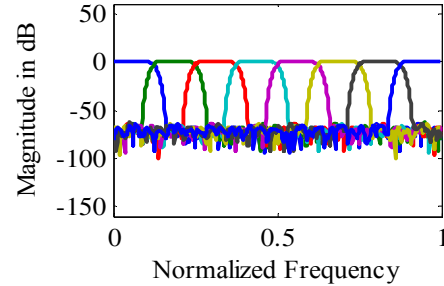
Type of Algorithm	Taps ( $N+1$ )	AMD ( $e_{am}$ )	ALD ( $e_a$ )
Algorithm in [15]	440	$3.41 \times 10^{-3}$	$2.60 \times 10^{-7}$
Algorithm in [5]	440	$5.30 \times 10^{-3}$	$2.57 \times 10^{-6}$
Algorithm in [7]	440	$3.44 \times 10^{-3}$	$2.61 \times 10^{-7}$
Algorithm in [10]	440	$2.81 \times 10^{-3}$	$2.64 \times 10^{-7}$
Algorithm in [13]	512	$2.1 \times 10^{-3}$	$4.00 \times 10^{-8}$
Algorithm in [21]	512	$1.14 \times 10^{-3}$	$2.29 \times 10^{-6}$
Algorithm in [18]	512	$1.14 \times 10^{-3}$	$2.29 \times 10^{-6}$
Hybrid method	512	$3.6 \times 10^{-3}$	$4.07 \times 10^{-7}$

## VI. CONCLUSION

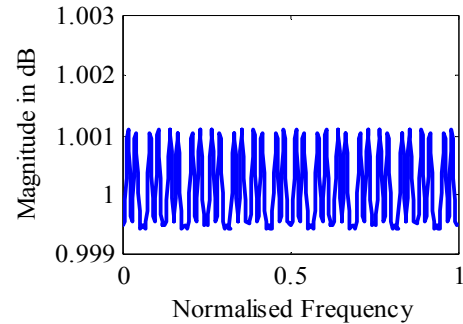
In this paper, a simple design of multiplierless M-channel CMFB is presented using hybrid technique and CSE. Hybrid



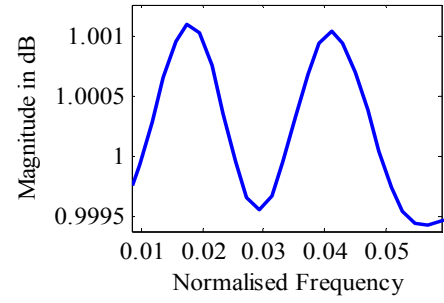
(a)



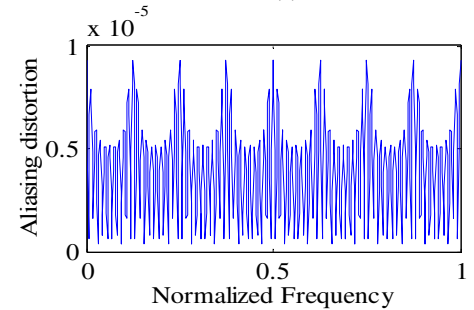
(b)



(c)



(d)



(e)

Figure 4. Simulation result of  $M$ -channel CMFB for  $N=160$  and  $A_s=80$  dB using hybrid technique, (a) Magnitude response of a prototype filter in dB, (b) Magnitude responses of analysis filters in dB, (c) Amplitude distortion, (d) Zoom plot of amplitude distortion, (e) Aliasing distortion

optimization technique, based on PSO and ABC, has provided better performance for the designed prototype filter when compared to ABC, PSO, and CSA. Several CSE algorithms i.e. horizontal, vertical, and mixed CSE have also been examined, among which prototype filter using hybrid CSE has achieved significant reduction of adders.

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