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A new design method for FIR notch filter using Fractional Derivative and swarm intelligence

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Abstract. In this paper, a new design method for the finite impulse response (FIR) notch filters using fractional derivative (FD) and swarm intelligence technique is presented. The design problem is constructed as a minimization of the magnitude response error w.r.t. filter coefficients. To acquire high accuracy of notch filter, fractional derivative (FD) is evaluated, and the required solution is computed using the Lagrange multiplier method. The fidelity parameters like passband error, notch bandwidth, and maximum passband ripple vary nonlinearly with respect to FD values. Moreover, the tuning of appropriate FD value is computationally expensive. Therefore, modern heuristic methods, known as the constraint factor particle swarm optimization (CFI-PSO), which is inspired by swarm intelligence, is exploited to search the best values of FDs and number of FD required for the optimal solution. After an exhaustive analysis, it is affirmed that the use of two FDs results in 21% reduction in passband error, while notch bandwidth is slightly increased by 2% only. Also, it has been observed that, in the proposed methodology, at the most 66 iterations are required for convergence to optimum solution. To examine the performance of designed notch filter using the proposed method, it has been applied for removal of power line interference from an electrocardiography (ECG) signal, and the improvement in performance is affirmed.

Keywords. Notch filter; fractional derivative (FD); swarm intelligence.

1. Introduction

Filtering of any contaminated signal is the primary requirement in numerous signal processing applications. Thus, digital filters are the vital elements in digital signal processing, which have been classified as the finite impulse response (FIR) and infinite impulse response (IIR) filters. FIR filter having a transfer function with all zero's, results in always stable system functions, and are used extensively in noise filtering and filter banks [1–3]. Generally, the FIR notch filters are prominently used in elimination of interference, caused due to an individual frequency component. In early stage of research in the notch filter design [4], three methods were adopted such as: (i) windowed Fourier series approach; (ii) frequency sampling approach, and (iii) optimized FIR filter design approach [4, 5].

In the optimized FIR filter design approach [5], a reasonable amount of the passband ripples are introduced, and the frequency sampling method leads to large interpolation error as frequency response drastically changes across the notch point. Other familiar methods to lessen the minimum and maximum error in frequency response are McClellan-Parks-Rabiner (MPR) computer program and standard linear programming technique. MPR algorithm is generally used to design the Equiripple FIR filters, whereas standard linear programming is used for the design of Equiripple FIR notch filter, but this method fails due to huge memory requirement, and also takes more computational time for convergence. Another method for designing a FIR notch filter is the multiple exchange algorithm, also known as Equiripple FIR notch filter design method [6]. Recently, a new method has been proposed in which the 'Null width' is controlled by proper selection of zero odd order derivative constraints to obtain maximally flat linear phase FIR notch filter [7].

Fractional derivative (FD) has been employed for refining the performance in various signal processing applications like: image sharpening [8, 9], event detection in biomedical signals [10], filter design accuracy [11]. FD possesses the real time phenomena of memory effect of

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electrical circuits and chemical reaction, which helps in smooth tracking. Therefore, fractional derivative is extensively used by several researchers [12–20]. In [12–15], authors have proposed new methods for designing simple digital FIR filters, wideband fractional delay filters using fractional derivatives. However, in these techniques, the optimal value of order of fractional derivative is determined by trial and error method. In order to overcome this problem, authors have used different swarm based techniques such as particle swarm optimization (PSO), artificial bee colony (ABC) algorithm, cuckoo search (CS) optimization, etc. to determine the optimal value of order of FD for designing FIR filters and filter banks [16–20]. A new technique using fractional derivative and swarm based optimization has been proposed for designing IIR filters [21]. The expression of a fractional derivative consists of an integration, which is a non-local operator and that is why fractional derivative is also a non-local operator. Hence, the fractional derivative has a unique property of capturing the history of a variable. This is not easily conquerable by using only integer order derivative [22]. From the reviewed literature, it is evident that several methods have been proposed for designing FIR Notch filters. However, in these techniques, there is no provision for controlling the notch bandwidth and more accurate passband response. Therefore, there is strong motivation to develop a new design technique for FIR Notch filter that has improved passband response, and required for noise filtering for numerous signal processing applications [1–3].

Therefore, in the above context, this paper describes a new technique for designing a digital FIR Notch filter using fractional derivative and swarm intelligence with the improved passband response along with suitable notch bandwidth. For this purpose, the design problem of a digital FIR Notch filter is formulated as minimization of integral square error between the ideal response and actual response subjected to the fractional derivatives are evaluated at the prescribed frequency. For determining suitable value of order of FD, which controls the notch bandwidth and precise attenuation at the individual frequency, the constraint factor particle swarm optimization (CF-PSO) is used due to its simplicity and efficient implementation. The detailed experimental analysis has been carried out to produce an optimal choice iteration count. Statistical analysis is done, which confirms the robustness of the proposed method. To examine the efficacy of state-of-the- art with the proposed method, these algorithms have been tested for noise filtering of an ECG signal. Rest of the paper is organized as follows. Section 2 briefs the swarm intelligence based optimization methods, while section 3 contains an overview of fractional derivatives. Section 4 explains the design procedure of FIR notch filter using FDC. In esction 5, the proposed problem formulation is stated and a detailed explanation of the experimental set-up and the results are given in section 6. Finally, the conclusions are provided in section 7.

2. Swarm intelligence based optimization

The modern heuristic search methods are proven as the robust in problem solving of non-differentiable, multi-modelled, and non-convex problems. Particle swarm optimization (PSO) [23, 24], artificial bee colony (ABC) [25], Hybrid algorithm [26], cuckoo search optimization (CSO) [27], and similar other methods are most prominent swarm intelligence based techniques. In all these techniques, solution of non-differentiable problem is searched from a search space matrix (***U***), which is continuously updated. PSO is inspired by the communication of biological organism, and extensively used in numerous optimization problems due to its simple structure, efficient exploration and exploitation ability [24]. The principle equations in PSO are [23]:

$$\mathbf{V}^{k+1} = \chi\left[W \cdot \mathbf{V}^k + \psi_1 \cdot \left(\mathbf{PB}^k - \mathbf{U}^k\right) + \psi_2 \cdot \left(GB^k - \mathbf{U}^k\right)\right]. \tag{1}$$

In the above Eq., k is the iteration cycle count, $\mathbf{V}^k$ represents the current velocity matrix, associated with search space matrix ($\boldsymbol{U}$), W is the inertia weight, C_1 and C_2 are the learning coefficients rates, which evaluate following as; $\psi_1 = C_1 \cdot rand(\cdot)$ and $\psi_2 = C_2 \cdot rand(\cdot)$, while χ is the constrained factor. ***PB*** represents archive of personal best solutions discovered till k^{th} iteration, whereas GB is the global best solution at k^{th} iteration. New velocity is computed by using Eq. (1), which is used for updating of ***U*** as [23]:

$$\mathbf{U}^{k+1} = \mathbf{V}^{k+1} + \mathbf{U}^k. \tag{2}$$

During the course of modification, if either value of ***U*** or ***V*** gets beyond the limit, then the respective values are restored. For restoration, either new suitable value, which is either in the predefined range or ultimate value of range is assigned to out of range elements of ***U*** or ***V***.

3. Fractional derivative (FD)

The exhaustive research in numerous signal processing applications using fractional derivatives (FD) has been fascinated [11–22]. Riemann–Liouville, Grünwald–Letnikov and Caputo are the three most prominent definitions of FD, and Grünwald–Letnikov fractional derivative is mostly used [11, 16–19].

$$D^u y(m) = \frac{d^u y(m)}{dm^u} = \lim_{\Delta \to 0} \sum_{l=0}^{\infty} \frac{(-1)^l T_l^u}{\Delta^u} y(m - l\Delta), \tag{3}$$

and the coefficient T_l^u is computed as:

$$T_l^u = \frac{\Gamma(u+1)}{\Gamma(l+1)\Gamma(u-l+1)} = \begin{cases} 1, & l = 0 \\ \dfrac{[u(u-1)(u-2)\ldots(u-l+1)]}{1,2,3\cdots l}, & l \geq 1 \end{cases}. \tag{4}$$

In the above Eq., $\Gamma(\cdot)$ represents a gamma function. Based on this, FDs of trigonometric function may be computed as:

$$D^u\{A\cdot\sin(\omega x+\varphi)\}=A\cdot\omega^u\cdot\sin\left(\omega x+\varphi+\frac{\pi}{2}u\right), \quad (5)$$

and

$$D^u\{A\cdot\cos(\omega x+\varphi)\}=A\cdot\omega^u\cdot\cos\left(\omega x+\varphi+\frac{\pi}{2}u\right). \quad (6)$$

4. Design of FIR Notch Filter using FD

The design problem of a digital filter is to evaluate the coefficients of a transfer function, which reasonably satisfy the approximation to the desired response. The notch filter function is to attenuate an individual frequency component decidedly, while other frequency components are kept intact. Therefore, the ideal response of a notch filter is given by:

$$H_d\left(e^{j\omega}\right)=\begin{cases}0, & \omega=\omega_{notch}\\ 1, & \omega\neq\omega_{notch}\end{cases}. \quad (7)$$

4.1 *Design procedure of FIR notch filter*

The transfer function of a causal FIR filter with order of N is defined as [11]:

$$H\left(e^{j\omega}\right)=\sum_{n=0}^{N}h(n)\cdot e^{-j\omega n} \quad (8)$$

The filter transfer function, defined by the above equation has a linear-phase response, if the impulse response $\{h(n)\}$ is symmetric. On this basis, whether $h(n)$ is the symmetric or anti- symmetric, FIR filters are categorized into four types as Type-1 to Type-4 [11]. In this paper, Type-1 filter, whose impulse response is symmetric with even order (N) is considered. Due to symmetric response, Eq. (8) may be reframed as:

$$\begin{aligned}H\left(e^{j\omega}\right)&=e^{-j\omega L}\left\{h(L)+2\cdot\sum_{n=0}^{L-1}h(n)\cdot\cos(\omega(L-n))\right\},\\ &=H_o(\omega)\cdot e^{-j\omega L}.\end{aligned} \quad (9)$$

Here, $L={}^{N}\!/_{2}$, $H_o(\omega)$ is the magnitude response of a FIR filter, which can be rewritten as:

$$H_o(\omega)=\sum_{l=0}^{L}b(l)\cos(\omega\, l), \quad (10)$$

where

$$b(l)=\begin{cases}h(L) & l=0\\ 2\cdot h(L-l) & 1\leq l\leq L\end{cases} \quad (11)$$

Eq. (10) may also be represented in matrix form as:

$$H_o(\omega)=\mathbf{b}^T\cdot\mathbf{C}(\omega), \quad (12)$$

where

$$\mathbf{b}=[b(0)\,b(1)\ldots b(L)], \quad (13)$$

and

$$\mathbf{C}(\omega)=[1\ \cos(\omega)\ldots\cos(L\omega)]. \quad (14)$$

In Eq. (12), T denotes the transpose of a vector. Now, in case of notch filter, the design problem is reduced to evaluate the coefficients of filter (***b***) such that it should eliminate the desired individual frequency component and has unity magnitude for the rest of other frequencies. Now, the filter coefficients are determined by minimizing an error function, defined as:

$$\begin{aligned}J(\mathbf{b})&=\int_{\omega\in ROI}(H_d(\omega)-H_0(\omega))^2d\omega,\\ &=\mathbf{b}^T\mathbf{Q}\mathbf{b}-2\mathbf{p}^T\mathbf{b}+\alpha,\end{aligned} \quad (15)$$

where, *ROI* is the region of interest, matrix ***Q***, vector ***p***, and scalar α are given by [11]:

$$\mathbf{Q}=\int_{\omega\in ROI}\mathbf{C}(\omega)\cdot\mathbf{C}(\omega)^Td\omega, \quad (16)$$

$$\mathbf{p}=\int_{\omega\in ROI}(H_d(\omega)\cdot\mathbf{C}(\omega))d\omega, \quad (17)$$

and

$$\alpha=\int_{\omega\in ROI}\{H_d(\omega)\}^2d\omega. \quad (18)$$

Now on the differentiation of Eq. (15) w.r.t. ***b,*** and equating to zero, results in the conventional least squares design solution as $\mathbf{b}_{LS}=\mathbf{Q}^{-1}\cdot\mathbf{p}$. To yield more accuracy at notch frequency, the following constraints are employed on the response $H_o(\omega)$ at the given frequency as [11]:

$$H_o(\omega_0)=H_d(\omega_0)=0, \quad (19)$$

and

$$DH_o(\omega)|_{\omega=\omega_0}=0. \quad (20)$$

In case of a notch filter, the fractional derivative evaluated atω_0 must satisfy the constraint defined as [11]:

$$D^uH_o(\omega)|_{\omega=\omega_0}=\beta(u-1) \quad (21)$$

In Eq. (21), u is the order of FD and β is the recommended constant, and for this work, it is taken as 30 [11].

By using Eqs. (6) and (10), the fractional derivative of $D^u H_o(\omega)$ can be computed as:

$$D^u H_o(\omega) = \frac{d^u\left(\sum_{l=0}^{L} b(l)\cos(\omega l)\right)}{d\omega^u} = \sum_{l=0}^{L} b(l)\frac{d^u \cos(\omega l)}{d\omega^u},$$
$$= \sum_{l=0}^{L} b(l)\cdot l^u \cdot \cos\left(\omega l + \frac{\pi u}{2}\right) = \mathbf{b}^T \cdot \mathbf{c}(\omega, u), \tag{22}$$

where, the vector $\boldsymbol{c}(\omega, u)$ is defined as:

$$\mathbf{c}(\omega, u) = \begin{bmatrix} 0 \\ 1^u \cdot \cos\left(\omega + \frac{\pi u}{2}\right) \\ 2^u \cdot \cos\left(2\omega + \frac{\pi u}{2}\right) \\ \vdots \\ L^u \cdot \cos\left(L\omega + \frac{\pi u}{2}\right) \end{bmatrix} \tag{23}$$

By using the equations (12), (22) and (23), the constraint equations (19), (20), and (21) are rewritten in matrix form as:

$$\mathbf{CB_x} \cdot \mathbf{b} = \mathbf{F}. \tag{24}$$

where

$$\mathbf{CB_x} = \begin{bmatrix} \mathbf{C}^T(\omega_0) & \mathbf{c}^T(\omega_0, 1) & \mathbf{c}^T(\omega_0, u) \end{bmatrix}^T, \tag{25}$$

and

$$\mathbf{F} = [0 \quad 0 \quad \beta(u-1)]^T. \tag{26}$$

The constraints defined in Eq. (19) is used for achieving exact zero magnitude response at the reference notch frequency (ω_0), while Eq. (20) is used to make first order derivative equal to be zero [11]. And the constraint defined by Eq. (21) aids in controlling 3-dB notch bandwidth [11]. Therefore, it is possible to adjust the notch bandwidth by tuning the value of u.

On merging the objective function given by Eq. (15), with constraints in Eq. (24), the definition of design problem of notch filter is expressed as:

$$\begin{aligned} &\text{Minimize } J(\mathbf{b}) = \mathbf{b}^T\mathbf{Q}\mathbf{b} - 2\mathbf{p}^T\mathbf{b} + \alpha, \\ &\text{subjected to } \mathbf{CB_x} \cdot \mathbf{b} = \mathbf{F}. \end{aligned} \tag{27}$$

The Lagrange multiplier method [11, 16] gives the optimal solution of such constrained optimization problem, and is computed as:

$$\mathbf{b}_{opt} = \mathbf{Q}^{-1} \cdot \mathbf{p} - \mathbf{Q}^{-1} \cdot \mathbf{CB_x}^T \cdot \left(\mathbf{CB_x} \cdot \mathbf{Q}^{-1} \cdot \mathbf{CB_x}^T\right)^{-1} \left[\mathbf{CB_x} \cdot \mathbf{Q}^{-1} \cdot \mathbf{p} - \mathbf{F}\right]. \tag{28}$$

This is a closed-form solution and effortlessly computable. The computational complexity of this method includes two terms, one is the computation of conventional least squares solution, which is $\mathbf{Q}^{-1} \cdot \mathbf{p}$. Second term is the product of $\mathbf{Q}^{-1} \cdot \mathbf{CB_x}^T\left(\mathbf{CB_x} \cdot \mathbf{Q}^{-1} \cdot \mathbf{CB_x}^T\right)^{-1}$ and $\left[\mathbf{CB_x} \cdot \mathbf{Q}^{-1} \cdot \mathbf{p} - \mathbf{F}\right]$, in which the computation of inverse of a matrix $\left(\mathbf{CB_x} \cdot \mathbf{Q}^{-1} \cdot \mathbf{CB_x}^T\right)^{-1}$ is expensive task. However, size of $\mathbf{CB_x} \cdot \mathbf{Q}^{-1} \cdot \mathbf{CB_x}^T$ is small of $i \times i$, where i = (integral order) + (order of FD terms), which are user defined and smaller. Therefore, the computational complexity of second term is also small. Authors in [11], have designed a notch filter using single FD term and have shown its effect on notch bandwidth (W_{notch}) as depicted in figure 1(a), and the corresponding frequency response in figure 1(b). The fidelity parameter, defined as passband error (er_p) is computed as:

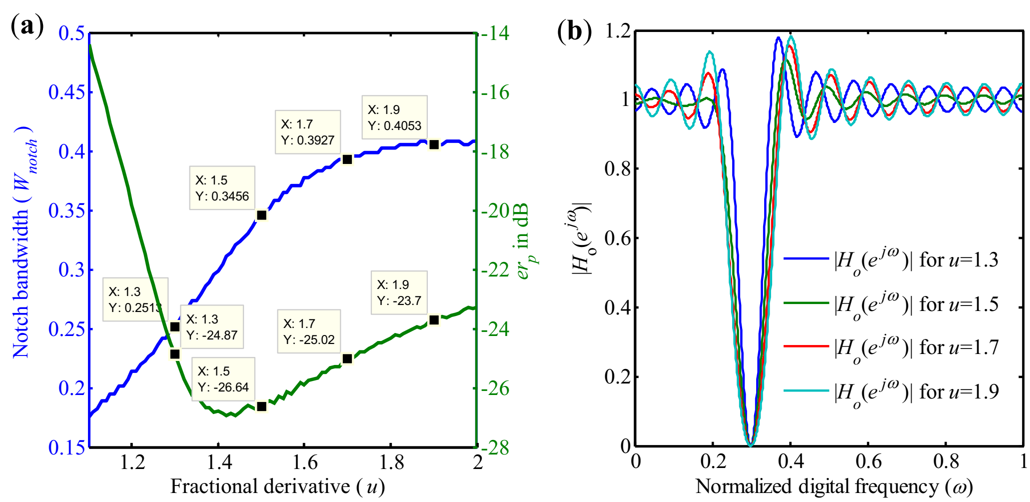

Figure 1. (**a**) Variation of notch bandwidth and er_p using single FD (u), (**b**) FIR notch filter frequency response for different FD value (u) =1.3, 1.5, 1.7, and 1.9.

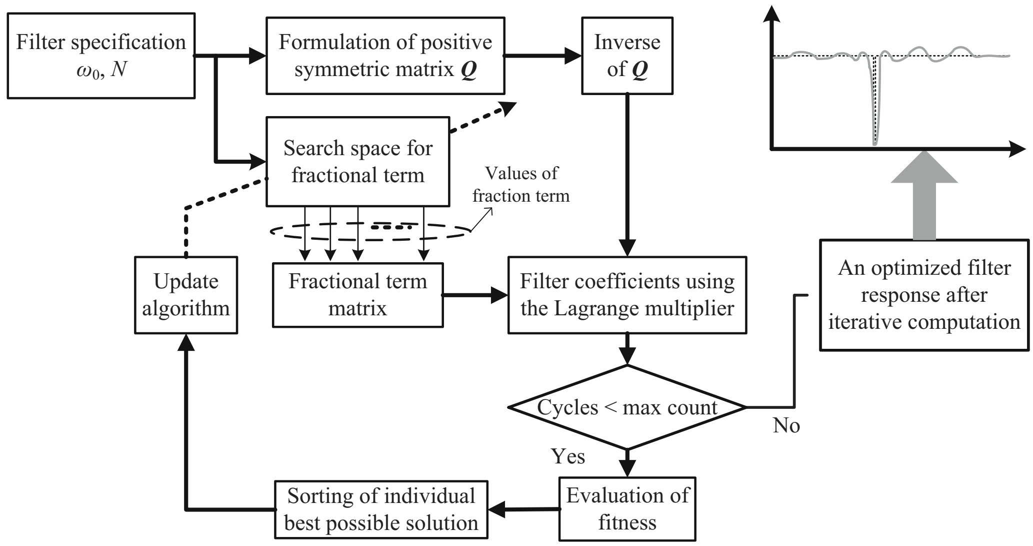

Figure 2. Block diagram of the proposed method.

$$er_p = \frac{1}{\pi}\left(\int_0^{\omega_c^1}\left(H_d\left(e^{j\omega}\right) - H_o\left(e^{j\omega}\right)\right)^2 d\omega + \int_{\omega_c^2}^{\pi}\left(H_d\left(e^{j\omega}\right) - H_o\left(e^{j\omega}\right)\right)^2 d\omega\right), \tag{29}$$

where, ω_c^1 and ω_c^2 are the lower and upper cut-off frequencies, given by:

$$H_o\left(\omega_c^l\right) = \sqrt{2}, \text{ where } l = 1 \text{ or } 2. \tag{30}$$

It is found that er_p varies with W_{notch} such that at u =1.01, W_{notch} is the minimum, however er_p is having maximum value. When u is increased, W_{notch} starts increasing with the reduction in er_p. Authors in [11], have used the step size of 0.1 for FD values, however it is observed that the step size with 0.01 attains more accurate results as shown in figure 1. When FD value is incremented with 0.01 and u is equal to 1.42, er_p attains it's the best value of -26.96 dB, which is the minimum and starts increasing, when u is greater than 1.45. Therefore, more accuracy with less W_{notch} may be achieved with high precision value of u, however it would be computational expensive in such approach. Therefore, swarm intelligence based modern heuristic approach is a suitable choice for obtaining the optimal solution, which simultaneously reduce the W_{notch} and er_p.

5. Problem formulation

In this work, the design problem of a notch filter response with less er_p and narrow W_{notch} is constructed as a minimization of Eq. (29). Here, W_{notch} is controlled by the value and number of u. Therefore, particle swarm optimization (PSO) is used for finding the suitable FD value and number of FD used. The proposed method using FD and swam intelligence technique is completely described in figure 2.

5.1 *Particle swarm optimization*

In PSO, the optimal solution is found by controlling the local and global search using search space, which is formed in the initial stage. In the proposed methodology, for acquiring more accurate solution, FD values are searched using CI-PSO. Therefore, search space ($\boldsymbol{U}$) is formulated by a matrix containing elements uniformly distributed in the range of lower (U_l) and upper (U_u) bound, defined as:

$$\mathbf{U} = U_l - (U_u) \oplus rand(0, 1). \tag{31}$$

Each set of a row vector of $\boldsymbol{U}$ is the possible combination of FD values for the evaluations (25) and (26). This approach ensures the independency on step size, and the self-learning mechanism of PSO helps in finding the appropriate best value. Also, with this approach, more fractional order based design can be tested with less computational cost.

5.2 *The algorithmic steps to be followed for the proposed method based on FD using PSO*

The complete design scheme can be framed using following steps:

Step 1: Declare the filter specifications like: filter order (K), notch frequency (ω_{notch}), and FD order.
Step 2: Define the ideal repose ($H_d(e^{j\omega})$) on the basis of Eq. (7).

Step 3: Compute a vector $\boldsymbol{C}(\omega)$, matrix $\boldsymbol{Q}$, and vector $\boldsymbol{p}$ using Eqs. (14), (16), and (17). Also, evaluate the inverse of $\boldsymbol{Q}$ and store for further computations.
Step 4: Initialize the search space matrix ($\boldsymbol{U}$) and velocity matrix ($\boldsymbol{V}$) with uniformly distributed random matrix within the limit of U_l to U_u as defined by Eq. (31).
Step 5: Evaluate the matrix $\boldsymbol{CB_x}$ and vector $\boldsymbol{F}$ for each solution containing FD value in a row vector of $\boldsymbol{U}$. After then, compute an optimal filter coefficients $\boldsymbol{b}_{opt}$ using Eq. (28).
Step 6: Compute the frequency response, followed by the computation of error function, defined by Eq. (29). Store these error values as ***Local Best Error***, and find out the least value of error from these. Assign this least value to *Global Best Error*. Assign the content of matrix $\boldsymbol{U}$ into matrix $\boldsymbol{PB}$ (personal best solutions). In the end, the solution corresponding to *Global Best Error* is kept in $\boldsymbol{GB}$ (global best solutions).
Step 7: Update $\boldsymbol{V}$ using Eq. (1), followed by the update of $\boldsymbol{U}$ using Eq. (2).
Step 8: Restore these values, if they move beyond U_l and U_u.
Step 9: Using the updated $\boldsymbol{U}^{n+1}$, evaluate $\boldsymbol{CB_x}$, $\boldsymbol{F}$, then compute the filter coefficients $\boldsymbol{b}_{opt}$ using Eq. (28) similarly as performed in step 5.
Step 10: Consider those solutions of updated $\boldsymbol{U}$, whose er_{p} is less than that of ***Local Best Error***. After this, sort out the minimum value of ***Local Best Error***, and if it is smaller than the current ***Global Best Error***, then update the ***Global Best Error*** and $\boldsymbol{GB}$ with respective value and solution.
Step 11: Repeat steps 4 to 9 till maximum number iteration are completed or er_p is dropped beyond tolerable limit.

6. Experimental set-up, results and discussion

This section elaborates the experimental set made for the design of a FIR notch filter using FD with PSO. For this purpose, MATLAB® 2014 is used on Genuine Intel (R) CPU i7 3770 @ 3.40 GHz, 4GB RAM. The grid size of 500 equally spaced sample for normalized digital frequency is taken during the experiments.

6.1 *Statistical analysis of the proposed method*

In PSO, size of search space is the key factor and depends on computation time. If $\boldsymbol{U}$ is smaller, it results in less computation time and grows almost abruptly as the size increases. The size of $\boldsymbol{U}$ is defined by the dimensionality (D) and number of solution (NS). Therefore, for obtaining the solution in reasonable computational time (t), it is required to set NS reasonably best by experimental evaluation. Therefore, in this section, various experiments have been performed to demonstrate the effect of D and NS on er_p, W_{notch}, t and convergence. Number of FDs is considered as D, and 30 trials of experiments are performed for possible combinations of D and NS. The mean of different fidelity parameters such as er_p, W_{notch}, is computed for analysis, and it is observed that the er_p and W_{notch} are increased for D greater than 2 as illustrated in figures 3(a) and 3(b). The mean value of t is increased with the increase in number of search space solution (NS) and D as shown in figure 3(c). However, the performance measured in terms of er_p and W_{notch} are being intact irrespective of the value of NS. Therefore, NS = 10 is an optimal choice for acquiring the best results in reasonable computation time. The mean value of rate of decay for er_p w.r.t. iteration for different number of FD, denoted by D is shown in figure 3(d).

The computation time taken during optimization process depends on order of filter, number of fractional derivatives and search space size. From the above discussion, it is clear that two fractional derivatives are the best suited for minimum er_p. Also, swarm size equal to 10 achieves the same performance as achieved with other higher value of it, which is also observed in figure 4 that shows the variation in the best and worst performance for different swarm size values. It can be perceived that er_p quickly converges into steady state as shown in figure 3(d). To find the practical value of iteration cycles, the convergence profile is differentiated w.r.t. number of iteration (n). The value of n after, which is:

$$\frac{\partial\left(10 \cdot \log_{10}\left[\mathrm{mean}\left(er_p\right)\right]\right)}{\partial k} = \mathrm{constant}, \tag{32}$$

and this is the best appropriate choice. On performing the above operation, the suitable value of n is found to be 13 and 66 for $D = 1$ and 2, respectively as shown in figure 5(a). It can also be observed that $D = 2$ archives 21% reduction in $er_{p,}$ when compared with $D = 1$; however, slight increment in W_{notch} of 2% has occurred as depicted in figure 5(b). The frequency response of FIR notch filter designed by the proposed method is shown in figure 5(c) with notch at $\omega = 0.3$.

6.2 *Comparative analysis*

Based on the results obtained in the above analysis, robustness of the proposed methodology has been tested by designing different order notch filter with different notch frequencies. The maximum iteration is kept to be 70, and order is varied from 10 to 80 with increment of 10. The proposed method has been tested using single FD and two FDs, the results obtained in single FD is summarized in table 1, while table 2 summarizes the results obtained in case of two FDs.

6.3 *Application in an electrocardiogram signal filtering*

ECG signal processing is the most eminent and consistently evolving stream in bio-medical signal processing [28, 29].

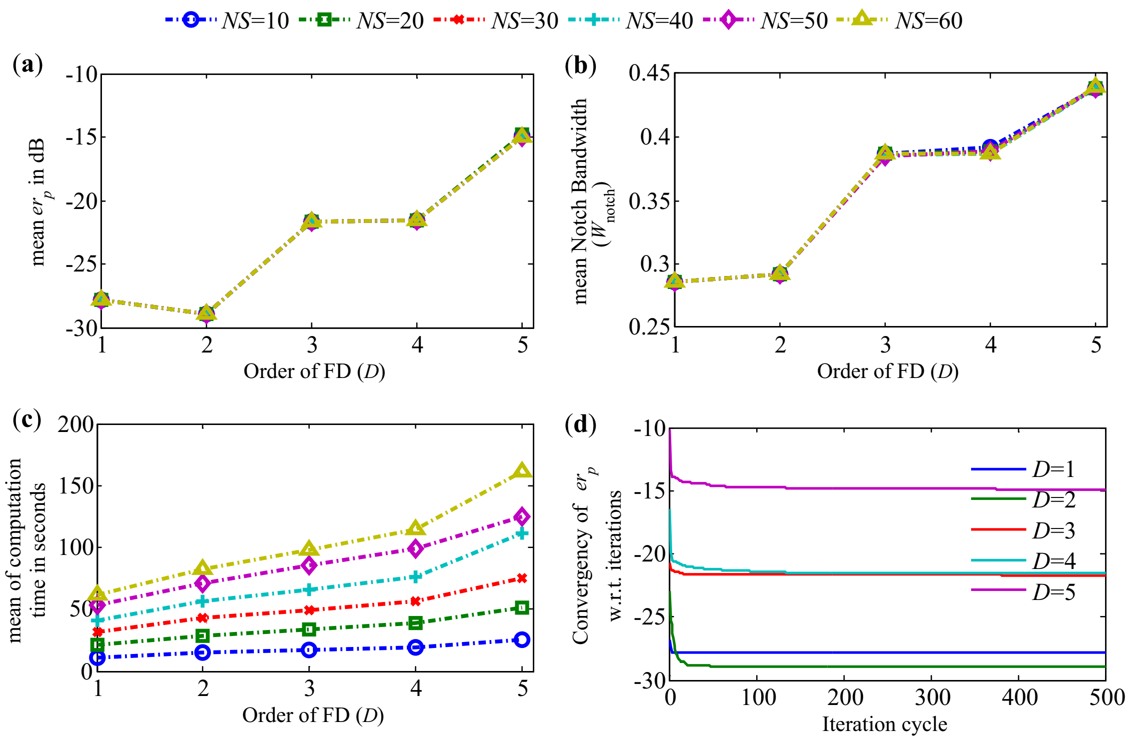

Figure 3. (**a**) Variation on mean value of er_p w.r.t. number of FDs (D) with different search space size (NS), (**b**) Variation on mean value of W_{notch} w.r.t. D with N, (**c**) Variation on mean value of computation time w.r.t. D with NS, (**d**) Convergence of er_p w.r.t. to iteration cycle for NS = 10 for different values of D.

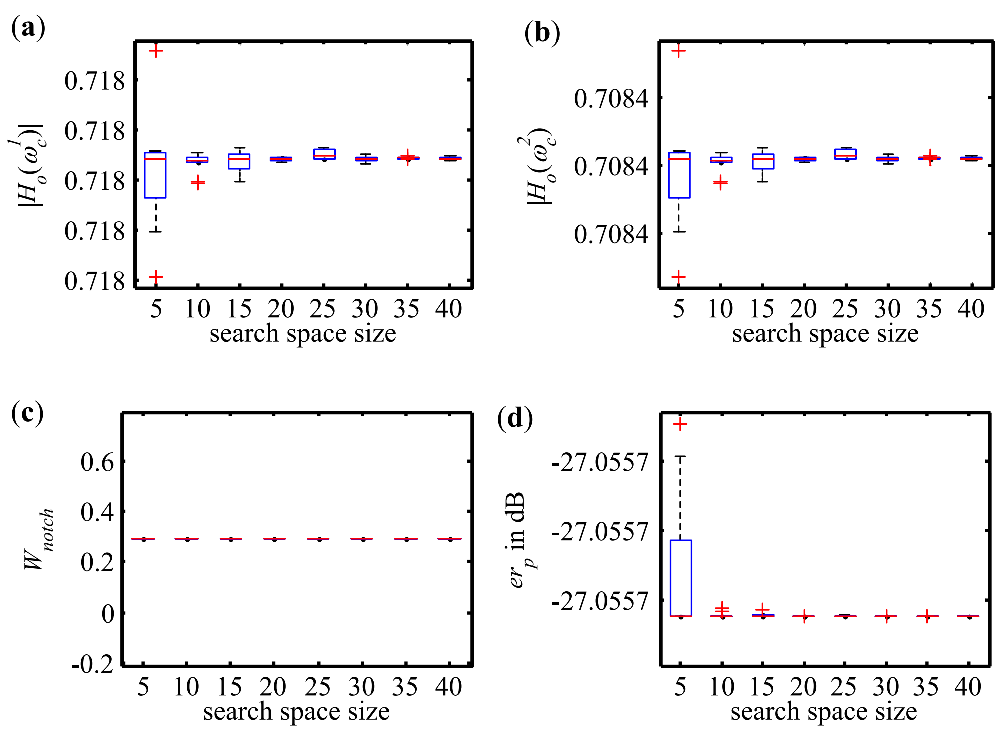

Figure 4. Variations in values of worst, median and best values obtained in 30 independent experiments performed. (**a**) Magnitude at lower cutoff frequency $|H_o(\omega_c^1)|$. (**b**) Magnitude at upper cutoff frequency $|H_o(\omega_c^2)|$. (**c**) W_{notch}. (**d**) er_p in dB.

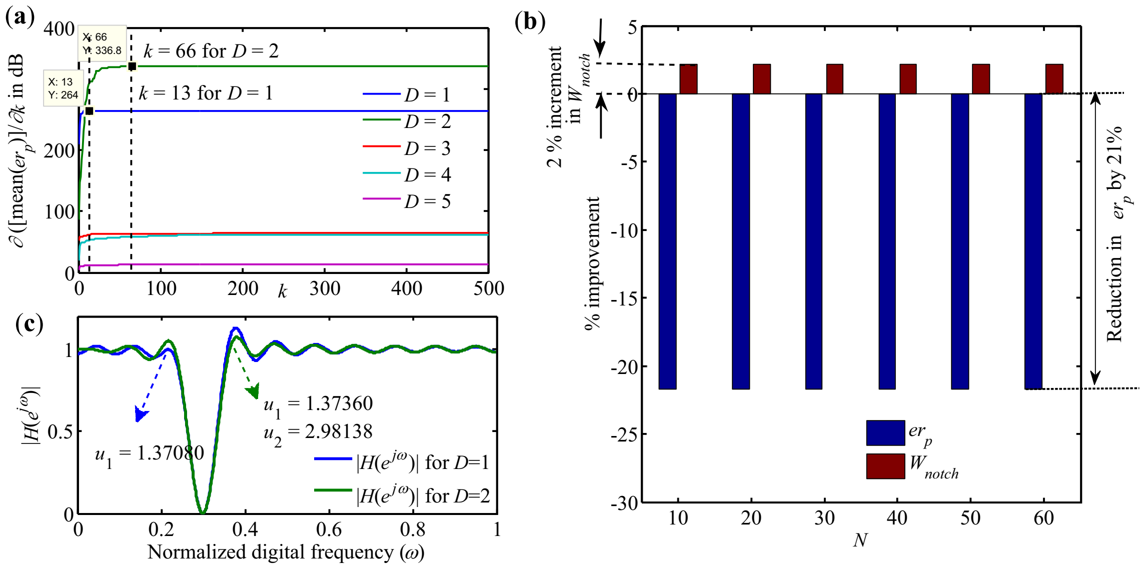

Figure 5. (**a**) Computation of optimal iteration count, (**b**) comparison of improvement in er_p and W_{notch} between design methodology of proposed method for D = 1 and D = 2, respectively, and (**c**) frequency response of notch filter designed using proposed methodology for D = 1 and D = 2.

Table 1. Performance of the proposed method with different order and notch frequencies for single FD.

	ω_{notch} = 0.25		ω_{notch} = 0.60		ω_{notch} = 0.80	
Filter Order	er_p	W_{notch}	er_p	W_{notch}	er_p	W_{notch}
10	0.4672	0.4618	4.4578	0.4492	-1.8395	0.4650
20	-16.7001	0.5027	-17.8581	0.4681	-17.9987	0.4650
30	-22.7098	0.3644	-22.9152	0.3613	-24.0427	0.3424
40	-27.7795	0.2890	-27.8404	0.2859	-27.9608	0.2859
50	-29.2317	0.2419	-28.7865	0.2482	-29.2498	0.2450
60	-29.6564	0.2168	-30.0488	0.2105	-29.9702	0.2073
70	-30.7874	0.1948	-30.6193	0.1916	-30.7997	0.1854
80	-31.5196	0.1696	-31.4998	0.1696	-31.4098	0.1696

Table 2. Performance of proposed method with different order and notch frequencies for two FDs.

	ω_{notch} = 0.25		ω_{notch} = 0.60		ω_{notch} = 0.80	
Filter Order	er_p	W_{notch}	er_p	W_{notch}	er_p	W_{notch}
10	0.4703	0.4609	4.3841	0.4496	-1.9289	0.5759
20	-16.6999	0.5023	-17.8597	0.4681	-18.0006	0.4618
30	-22.8581	0.3644	-23.0752	0.3641	-24.2116	0.3456
40	-28.8163	0.2922	-28.9032	0.2922	-29.0729	0.2890
50	-31.3496	0.2532	-30.9587	0.2576	-31.2656	0.2513
60	-31.5176	0.2262	-31.5415	0.2205	-31.6355	0.2212
70	-31.4722	0.1970	-31.5991	0.1963	-31.6769	0.1910
80	-31.6336	0.1715	-31.6621	0.1696	-31.7297	0.1715

One of the important parts is extraction of QRS complex and analyzing its characteristics to diagnose the irregularities in the heart rhythm. The notch filters are widely used in application, where an individual harmonic elimination is required such as interference of power line in an electrocardiogram (ECG) recording, open-loop voltage across the

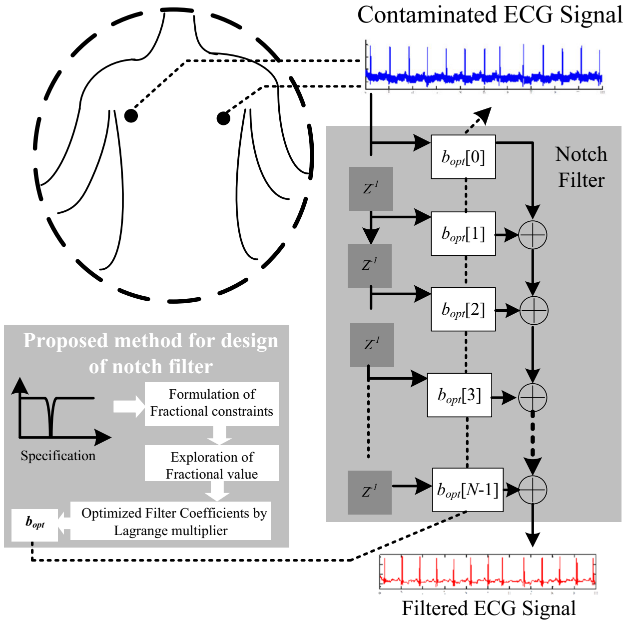

Figure 6. Proposed methodology for design of notch filter for power line interference removal from ECG signal.

input of an analog instrument and many such [7]. In this section, power line interference in an ECG is filtered by using the designed notch filter as shown in figure 6. First, an artificial ECG is generated, and then contaminated with power line interference, and finally filtered using the designed notch filter. This experiment is also performed on ECG recorded signals from MIT-BIH [30]. The quality of filtering is judged by finding the value of following [31, 32]: mean squared error (*MSE*):

$$MSE = \frac{1}{N_s}\sum \left|x(n) - \widehat{x}(n)\right|^2, \text{ where } N_s \text{ number of samples}, \tag{33}$$

where N_s number of samples. Percent root mean square difference (*PRD*):

$$PRD = \left(\frac{\sum \left|x(n) - \widehat{x}(n)\right|^2}{\sum [x(n)]^2}\right)^{1/2} \cdot 100, \tag{34}$$

and signal to noise ratio (*SNR*):

$$SNR \text{ dB} = 10 \cdot \log_{10}\left(\frac{\sum [x(n)]^2}{\sum \left|x(n) - \widehat{x}(n)\right|^2}\right) \tag{35}$$

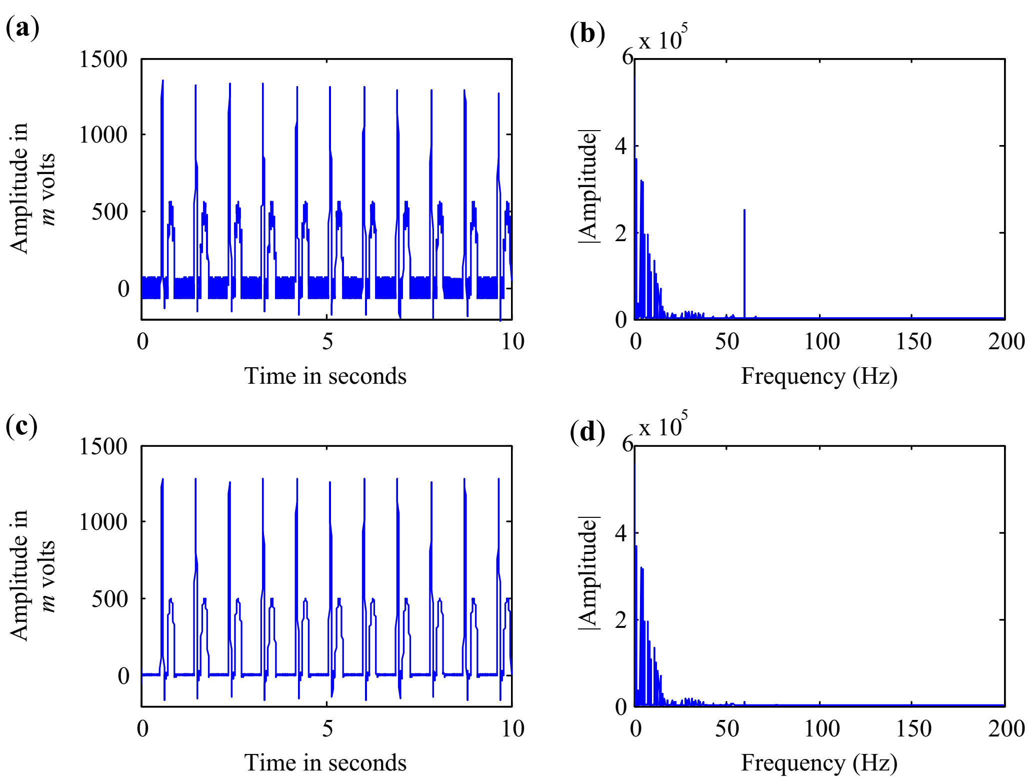

Figure 7. **(a)** Contaminated synthesized ECG signal, using 400 Hz sampling frequency, with 60 Hz interference, **(b)** Contaminated synthesized ECG signal spectrum, **(c)** filtered synthesized ECG signal by filter designed using proposed methodology, and **(d)** spectrum of filtered ECG signal.

An ECG signal with the sampling rate of 400 Hz is synthesized, and 60 Hz interference has been introduced as shown in figures 7(a) and 7(b). This contaminated signal has been filtered using the notch filters designed using FD approach as suggested in [11], and by the proposed design approach. It can be perceived from table 3 that filtering using notch filter designed by the proposed methodology using second order FD obtains better value of fidelity parameters. This is possible due to the filter designed with second order FD, which has better passband accuracy with optimal notch bandwidth. The obtained performance has been compared and summarized in table 3. It is evident that with the second order fractional derivative approach, filter achieves more accurate filtering results when compared to single order fractional derivative approach as given in [11].

The designed filters are also tested for real time ECG signal taken from [30]. These signals are mixed with 50 and 60 Hz power line signal. The sampling frequency of ECG signal is 360 Hz, and if these signals are interfered by 50 Hz power line signal, then notch filter with $\left({}^{50}\!/_{360}\right) \times 2 = 0.2778$, normalized digital frequency is required. Where as in case of 60 Hz power line signal interference, it is required that notch frequency should be 0.3333 normalized. In trial based approach [11], it took 3.7813 seconds for completion, and then additional time in sorting of best solution from entire listed output solutions. If same approach is adopted for two derivatives, then it would take more computation time. Whereas the proposed technique takes maximum of 7.39 seconds for obtaining the optimal values of u for $D = 2$. ECG signals are

Table 3. Performance evaluation of notch filter designed by proposed technique in filtering of synthesize ECG signal.

Design Technique	FD Order	Fractional value (u)	MSE	PRD	SNR
FD [11]	1	1.300000	164.2868	0.1704	27.6859
FD [11]	1	1.500000	88.7068	0.0920	30.3623
FD [11]	1	1.700000	165.6624	0.1718	27.6497
FD [11]	1	1.900000	386.6205	0.4010	23.9690
Proposed	1	1.370809	116.2129	0.1205	29.1894
Proposed	2	1.373603, 2.981385	81.9271	0.0850	30.7076

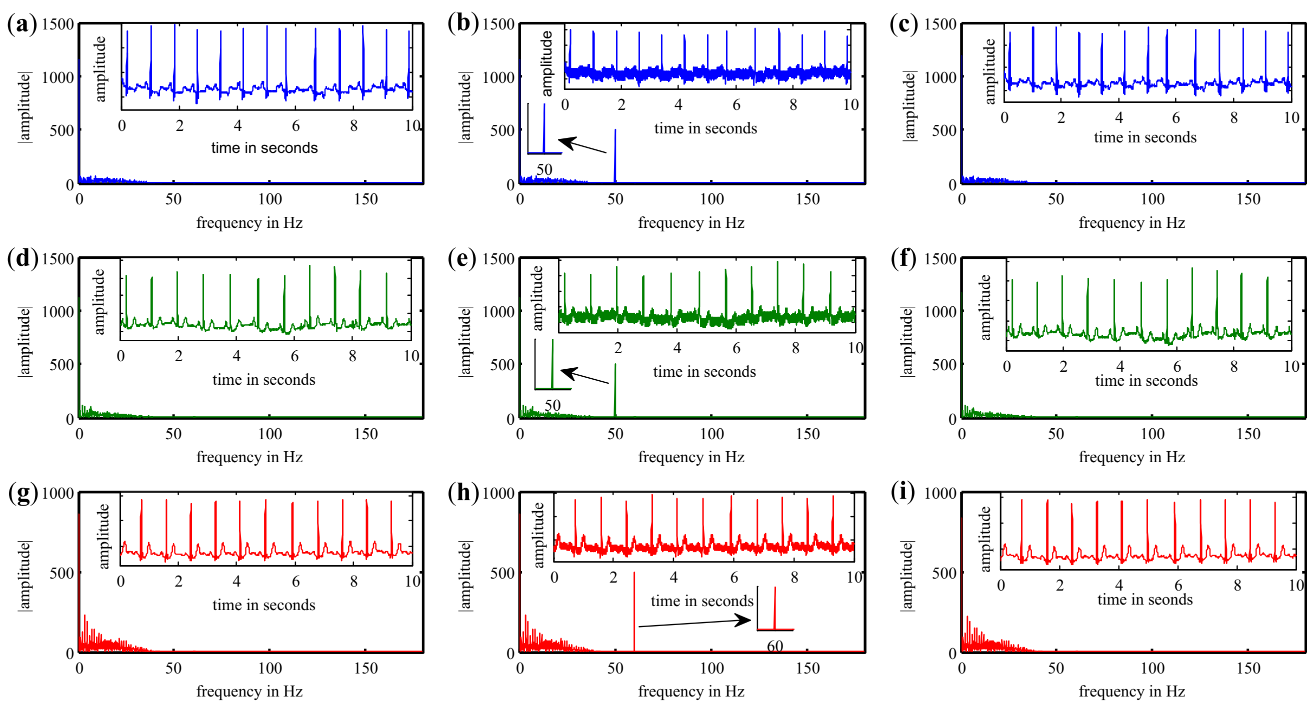

Figure 8. **(a)** ECG signal record MIT BIH 100, **(b)** contaminated ECG signal with 50 Hz interference, **(c)** filtered ECG signal, **(d)** ECG signal record MIT BIH 101, **(e)** contaminated ECG signal with 50 Hz interference, and **(f)** filtered ECG signal. **(d)** ECG signal record MIT BIH 103, **(e)** contaminated ECG signal with 60 Hz interference, and **(g)** filtered ECG signal. Filtering has been performed using filter designed using proposed methodology with $D = 2$.

Table 4. Performance evaluation of designed notch filter using real time recorded ECG signal take from ECG database from [30].

Signal	Sinusoid frequency	Technique	MSE	PRD	SNR
MIT-BIH 100	50 Hz	1-D using approach in [11]	0.00099	0.75484	21.22147
MIT-BIH 101	50 Hz		0.00086	0.62526	22.03940
MIT-BIH 106	50 Hz		0.00189	1.02824	19.87906
MIT-BIH 103	60 Hz		0.00049	0.32498	24.88146
MIT-BIH 104	60 Hz		0.00089	0.67221	21.72498
MIT-BIH 105	60 Hz		0.00032	0.21904	24.72498
MIT-BIH 100	50 Hz	2-D using proposed approach	0.00076	0.57848	22.37713
MIT-BIH 101	50 Hz		0.00066	0.47975	23.18982
MIT-BIH 106	50 Hz		0.00169	0.92277	20.34904
MIT-BIH 103	60 Hz		0.00045	0.29544	25.29529
MIT-BIH 104	60 Hz		0.00085	0.63760	21.95449
MIT-BIH 105	60 Hz		0.00085	0.63760	26.59480

contaminated by both 50 and 60 Hz interference and filtered as depicted in figure 8, and performance is summarized in table 4. The optimal value of u is found to be 1.389992 and 2.956899 in case of notch frequency equals to 0.2778, while it should be 1.390799 and 2.973813 for normalized notch frequency equals to 0.3333.

7. Conclusions

In this paper, a new design approach using fractional derivatives, which are explored using CFI-PSO, is presented. The exhaustive experimentation results have revealed that two fractional derivatives with second order derivative is sufficient for the design of optimal notch filter. There is reduction in passband error by 21%, however there is nominal increment of notch bandwidth by 2.1%, when compared with the double FD with single FD design approach. The thorough analysis made for analyzing the effect of swarm size reveals that swarm size consisting of ten solutions is the best, which also results in less computation time. On differentiating the mean of convergence w.r.t. iteration, it gives the reasonable iteration count for the convergence and found to be 13 for single and 66 for two FD based design. The designed filter is tested for power line interference removal, and was found to be very efficient.

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