A method of incident angle estimation for high resolution spectral recovery in filter-array-based spectrometers

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ABSTRACT

In recent years, there has been an increasing interest in miniature spectrometers for research and development. Especially, filter-array-based spectrometers have advantages of low cost and portability, and can be applied in various fields such as biology, chemistry and food industry. Miniaturization in optical filters causes degradation of spectral resolution due to limitations on spectral responses and the number of filters. Nowadays, many studies have been reported that the filter-array-based spectrometers have achieved resolution improvements by using digital signal processing (DSP) techniques. The performance of the DSP-based spectral recovery highly depends on the prior information of transmission functions (TFs) of the filters. The TFs vary with respect to an incident angle of light onto the filter-array. Conventionally, it is assumed that the incident angle of light on the filters is fixed and the TFs are known to the DSP. However, the incident angle is inconstant according to various environments and applications, and thus TFs also vary, which leads to performance degradation of spectral recovery. In this paper, we propose a method of incident angle estimation (IAE) for high resolution spectral recovery in the filter-array-based spectrometers. By exploiting sparse signal reconstruction of the $L1$- norm minimization, IAE estimates an incident angle among all possible incident angles which minimizes the error of the reconstructed signal. Based on IAE, DSP effectively provides a high resolution spectral recovery in the filter-array-based spectrometers.

Keywords: miniature spectrometers, digital signal processing, incident angle estimation, high resolution

1. INTRODUCTION

Miniature spectrometers are a topic of interest for their potential role in research and development. Miniature spectrometers have advantages of low cost and portability, and can be applied in various fields such as biology, chemistry and food industry [1-3]. Optical filter-array-based spectrometers have a great potential to be minimized since the filter-array can be placed directly onto detector-array such as CCD and CMOS sensor. However, miniaturization causes degradation of spectral resolution due to the limitation of the number of filters and the difficulty of fabrication of an ideal filter.

Nowadays, a number of studies have been reported that the filter-array-based spectrometers equipped with digital signal processing (DSP) have achieved a high resolution in spectral recovery [4-8]. These studies were trying to take advantage of $L1$-norm minimization and recovery technique which can recover a spectrum in high resolution with smaller number of samples [9-11]. The filter-array-based spectrometers with non-negative $L1$-norm minimization algorithm was proposed in [6]. In addition, an advance which the resolution improvement is 7-fold over [6] has been made by using filters with random transmission functions (TFs) [7]. The random filter (RF) is implemented by using thin-film technology that multiple layers of high- and low-refractive index materials deposited on a substrate with varying the thickness of each layer. The practical design of RF is introduced in [8]. With this proposed design, the performance of DSP-based spectral recovery has been improved. However, DSP highly depends on the prior information of TFs of the filters. The TFs vary with respect to the incident angle onto the filter-array. In [4-8], the TFs of the filters were assumed to be known and the incident angle fixed as a normal incidence. In practice, the incident angle is inconstant and unstable according to unpredictable environments and applications. This problem can lead to degraded performance of spectral recovery.

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2. METHODOLOGY

2.1 System Description

In the filter-array-based spectrometer, a light source is incident onto a filter-array. We assume that the light rays are in parallel and the incident angle is an arbitrary constant. The filter-array consists of M filters, each of which is characterized as its own TF. The intensity of each filtered-light is measured by a photodetector. The measured signals are converted into M-measurements by an analog to digital converter. The M-measurements are fed into a DSP unit to recover the spectrum of the light source. A typical relation between the measurements $\mathbf{y} \in \mathbb{R}^{M\times 1}$ and the spectrum of input light source $\mathbf{x} \in \mathbb{R}^{N\times 1}$ can be written as a system of linear equations:

$$\mathbf{y} = \mathbf{T}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{T} \in \mathbb{R}^{M\times N}$ is denoted as a sensing matrix and $\mathbf{n} \in \mathbb{R}^{M\times 1}$ is a noise vector. Each row of $\mathbf{T}$ represents a TF of each filter, which is obtained by uniformly N-sampling in the wavelength range of interest. For resolution improvements, the number of filters is set to be smaller than the length of input light source, i.e., $M < N$. Then, Eq. (1) becomes an ill-posed problem. This ill-posed problem can be solved by L-1 norm minimization if the input light spectrum is sparsely represented in a certain basis such as Fourier transform, discrete cosine transform, or wavelet transform. A sparse signal represents a vector which has few nonzero elements. That is, $\mathbf{x} = \mathbf{\Phi}\mathbf{s}$, where $\mathbf{\Phi} \in \mathbb{R}^{N\times N}$ is a sparsifying basis and $\mathbf{s} \in \mathbb{R}^{N\times 1}$ is a sparse vector. Then, Eq. (1) can be rewritten as:

$$\mathbf{y} = \mathbf{T}\mathbf{\Phi}\mathbf{s} + \mathbf{n}. \tag{2}$$

The sparse vector $\mathbf{s}$ can be recovered by solving the L-1 norm minimization as follows:

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \|\mathbf{s}\|_1 \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{T}\mathbf{\Phi}\mathbf{s}\|_2 \le \varepsilon \tag{3}$$

where $\varepsilon > 0$ is a constant. From the estimate $\hat{\mathbf{s}}$, the spectrum of input light source can be recovered by $\hat{\mathbf{x}} = \mathbf{\Phi}\hat{\mathbf{s}}$.

Because the incident angle is inconstant and unstable according to environments and applications, the information of sensing matrix is unknown. Figure 1 shows an example of the recovered spectrum of input light source. When the incident angle of light source coincides with the angle of $\mathbf{T}$, the input spectrum is perfectly recovered as seen in Fig. 1(a). However, when the incident angle of light does not coincide with the angle of $\mathbf{T}$, the spectrum recovery does not work well in Fig. 1(b).

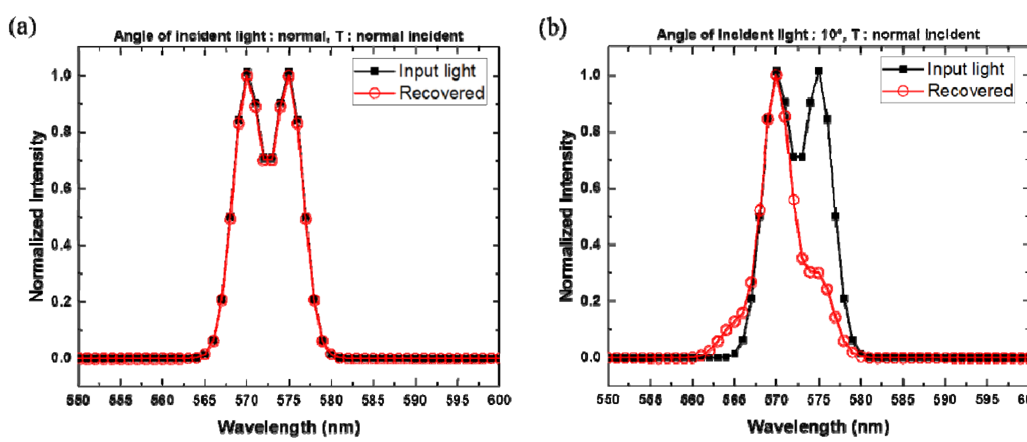

Figure 1. An example of spectral recovery: (a) Incident angle and angle of **T** are 0°, (b) Incident angle is 10° and angle of **T** is 0°.

2.2 Incident Angle Estimation (IAE)

In IAE process, we make sensing matrices with respect to various incident angles and horizontally concatenate them as a set **F**. **F** can be written as $\mathbf{F}=\left[\mathbf{T}_{\theta_1},\mathbf{T}_{\theta_2},\cdots,\mathbf{T}_{\theta_I}\right]\in\mathbb{R}^{M\times NI}$ where I is the number of sensing matrices. Due to the uniqueness of the sensing matrix **T** with respect to incident angles, the sensing matrices are uncorrelated with each other. Then, we can rewrite Eq. (1) as:

$$\begin{aligned}\mathbf{y}&=\left[\mathbf{T}_{\theta_1},\mathbf{T}_{\theta_2},\cdots,\mathbf{T}_{\theta_I}\right]\mathbf{X}+\mathbf{n}\\&=\mathbf{FX}+\mathbf{n}\end{aligned} \tag{4}$$

where $\mathbf{X}=\left[\mathbf{x}_{\theta_1}{}^T,\mathbf{x}_{\theta_2}{}^T,\cdots,\mathbf{x}_{\theta_I}{}^T\right]^T\in\mathbb{R}^{NI\times 1}$ is a vector of spectrums, and $\mathbf{x}_{\theta_I}\in\mathbb{R}^{N\times 1}$ is a spectrum which is matched to sensing matrix $\mathbf{T}_{\theta_I}$. Similar to Eq. (2), by applying a sparse representation of **X**, i.e., $\mathbf{\Psi S}$, where $\mathbf{\Psi}=blkdiag\left[\mathbf{\Phi},\mathbf{\Phi},\cdots\mathbf{\Phi}\right]\in\mathbb{R}^{NI\times NI}$ and $\mathbf{S}=\left[\mathbf{s}_{\theta_1}{}^T,\mathbf{s}_{\theta_2}{}^T,\cdots,\mathbf{s}_{\theta_I}{}^T\right]^T\in\mathbb{R}^{NI\times 1}$, Eq. (4) becomes

$$\mathbf{y}=\mathbf{F\Psi S}+\mathbf{n}. \tag{5}$$

The $L1$-norm minimization problem for the recovery of the sparse vector **S** in Eq. (5) can be expressed as:

$$\hat{\mathbf{S}}=\arg\min_{\mathbf{S}}\|\mathbf{S}\|_1 \quad \text{subject to} \quad \|\mathbf{y}-\mathbf{F\Psi S}\|_2\le\varepsilon \tag{6}$$

where $\varepsilon>0$ is a constant. Then, the estimated vector $\hat{\mathbf{X}}$ can be expressed as $\mathbf{\Psi}\hat{\mathbf{S}}$.

Errors between measurements **y** and estimated measurements $\hat{\mathbf{y}}_{\theta_i}$ can be used to estimate the incident angle. The estimated measurements $\hat{\mathbf{y}}_{\theta_i}$ can be expressed as $\mathbf{F}\boldsymbol{\delta}_{\theta_i}$, where $\boldsymbol{\delta}_{\theta_i}=\left[0,\cdots,\hat{\mathbf{x}}_{\theta_i}{}^T,\cdots,0\right]^T\in\mathbb{R}^{NI\times 1}$ is a vector whose all estimated spectrums are zero except the estimated spectrum matched to sensing matrix $\mathbf{T}_{\theta_i}$. Then, the error can be written as follows:

$$\begin{aligned}e_{\theta_i}&=\left\|\mathbf{y}-\hat{\mathbf{y}}_{\theta_i}\right\|_2 \qquad i=1,\cdots,I\\&=\left\|\mathbf{y}-\mathbf{F}\boldsymbol{\delta}_{\theta_i}\right\|_2\end{aligned} \tag{7}$$

The incident angle can be estimated as by selecting an angle which has the minimum error.

From the estimated incident angle $\hat{\theta}$, we can find the matched sensing matrix $\mathbf{T}_{\hat{\theta}}$. Eqs. (1-3) can be rewritten as:

$$\begin{aligned}\mathbf{y}&=\mathbf{T}_{\hat{\theta}}\mathbf{x}_{\theta}+\mathbf{n}\\\mathbf{y}&=\mathbf{T}_{\hat{\theta}}\mathbf{\Phi}\mathbf{s}_{\theta}+\mathbf{n}\end{aligned} \tag{8}$$

$$\hat{\mathbf{s}}_{\theta}=\arg\min_{\mathbf{s}_{\theta}}\|\mathbf{s}_{\theta}\|_1 \quad \text{subject to} \quad \left\|\mathbf{y}-\mathbf{T}_{\hat{\theta}}\mathbf{\Phi}\mathbf{s}_{\theta}\right\|_2\le\varepsilon \tag{9}$$

where $\varepsilon>0$ is a constant. Then, the spectrum of input light source at θ can be recovered by $\hat{\mathbf{x}}_{\theta}=\mathbf{\Phi}\hat{\mathbf{s}}_{\theta}$.

3. SIMULATION RESULTS

A filter-array-based spectrometer with M =36 is considered. The spectrum of light source **x** and TFs of filters are sampled with 1 nm spacing in the range of 500 nm to 1000 nm (N=500). We consider 4 cases of incident angles such as θ= 0°, 10°, 20° and 30° (I=4). The RFs can be implemented by depositing multiple layers of SiO_2 and SiN_x with different thicknesses. The TFs of RFs are calculated based on thin-film theory [12]. Figure 2(a) shows an example of TFs with respect to the incident angles. As the incident angle increases, the TF is shifted to the left. Figure 2(b) shows the TFs of RFs at θ=0°. As illustrated in Fig. 2(b), the TFs are fluctuating sharply in the range of wavelength and highly uncorrelated with each other.

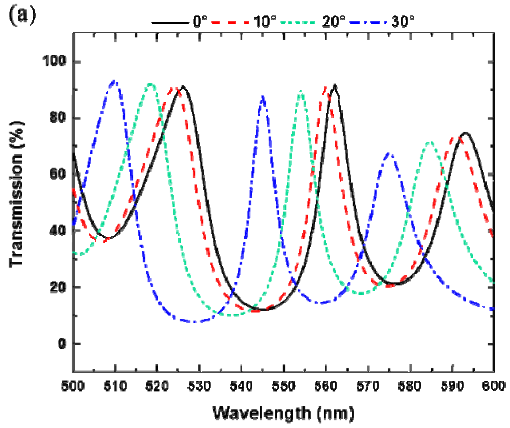

Figure 2. (a) TFs of a RF with respect to incident angles, (b) TFs of RF #10, #20, and #30 at $\theta = 0°$.

As an input light source, a synthetic spectrum having two neighboring peaks at 570 nm and 575 nm is used to analyze the performance of recovery. The measurements are acquired by Eq. (1). The Gaussian kernels are used as a sparsifying basis [13]. The results are obtained by using the Non-Negative $L1$-minimization algorithm [6].

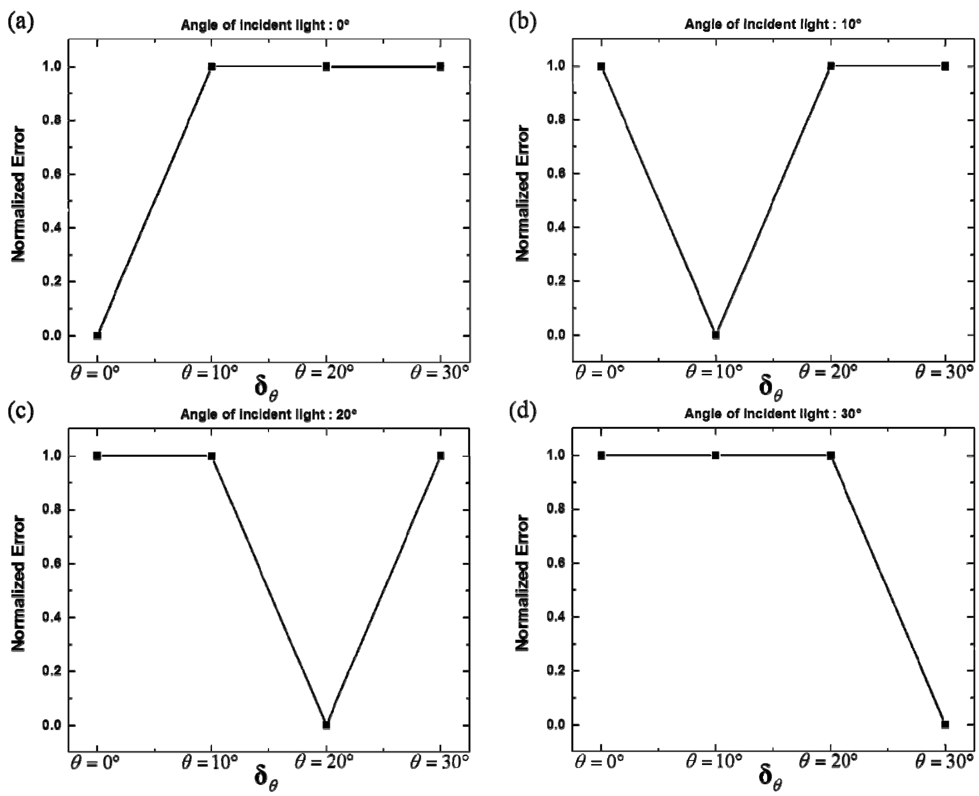

Figure 3. The normalized errors with respect to incident angles: (a) 0°, (b) 10°, (c) 20°, (d) 30°.

The normalized errors with respect to the incident angles are shown in Fig. 3. As illustrated in Figs. 3(a-d), The normalized error has the minimum value when the incident angle is matched to the angle of $\boldsymbol{\delta}_\theta$. By finding the minimum error, we can estimate the incident angle. From the estimated incident angle $\hat{\theta}$, we can choose the sensing matrix $\mathbf{T}_{\hat{\theta}}$. Then, DSP recovers the spectrum of input light source as $\hat{\mathbf{x}}$ using the sensing matrix $\mathbf{T}_{\hat{\theta}}$ and measurement vector **y**. The recovery results are shown in Fig. 4. DSP recovers a spectrum of input light source in high resolution with sensing matrix $\mathbf{T}_{10^\circ}$ and measurement vector **y** as shown Fig. 4(a). Two neighboring peaks which are located in 5 nm apart are resolved. Also, the high resolution recovery is possible with sensing matrix $\mathbf{T}_{30^\circ}$ and measurement vector **y** as shown Fig. 4(b).

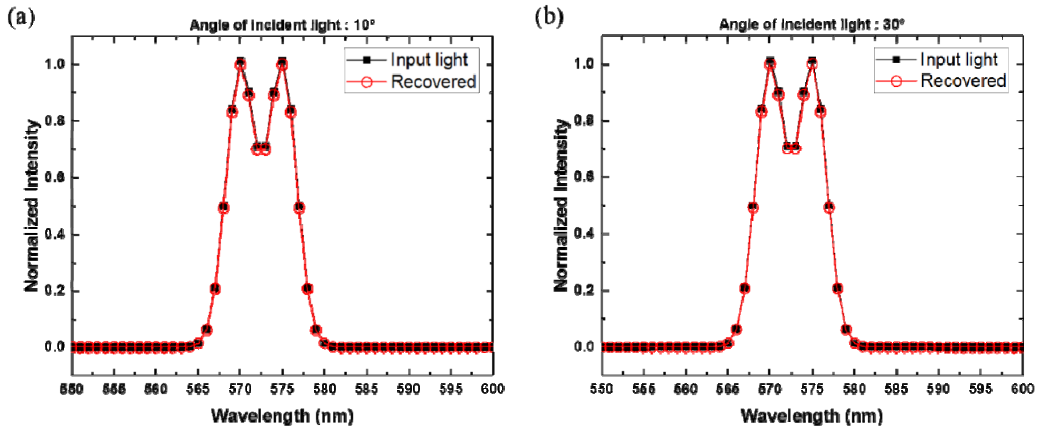

Figure 4. (a) Spectral recovery of input light source at $\hat{\theta}$ = 10°, (b) Spectral recovery of input light source at $\hat{\theta}$ = 30°.

4. CONCLUSIONS

In this paper, we have proposed a method of incident angle estimation in the filter-array-based spectrometers equipped with DSP technique. For high resolution spectral recovery, the spectrometer requires correct incident angle of light onto the filter-array and corresponding measurement matrix. By exploiting uniqueness of measurement matrix with respect to the incident angle, the proposed method estimates the incident angle which minimizes error between measurements and estimated measurements. Given the estimated incident angle and corresponding measurement matrix, the spectrometer equipped with DSP is able to recover the spectrum of input light source in high resolution. Simulation results illustrate that the proposed method recovers the spectrum of input light source in high resolution without the prior knowledge of the incident angle. In this study, we assume the incident light go into filter-array in parallel and consider only a few cases of incident angles. In future works, we will extend cases of incident angles and conduct real experiments.

5. ACKNOWLEDGEMENT

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