

# Random Transmittance Based Filter Array Spectrometers: Sparse Spectrum Recovery and Resolution Improvement

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**Abstract**—In this paper, we introduce filters with random transmittance to improve the resolution of miniature spectrometers. We show that a sparse signal spectrum sensed by such filters when processed by an  $L_1$  algorithm aids in improving the resolution. We also show a way to design the filters with random transmittance using optical thin-film technology. We demonstrate that a resolution of 0.99nm, which is 7-fold compared to using filters in [3], can be achieved with only 40 random filters.

**Index Terms**—Compressive sensing,  $L_1$  norm minimization, random transmittance, resolution, spectrometers.

## I. INTRODUCTION AND MOTIVATION

Modern miniature spectrometers are equipped with digital signal processing (DSP) algorithms [1]. These algorithms process the raw spectrum obtained from a spectrometer to improve the resolution. In filter array based spectrometers, the resolution is limited by the number of filters and the shapes of the transmittance functions (TF) of the filters. Rather than increasing the number of filters (and the CCD elements), we design the TFs in order to improve the recovery performance of the DSP algorithms and hence the resolution.

In this paper, we assume to use the  $L_1$ -norm minimization based DSP algorithm in [2] and aim to address the following relevant questions: Is it possible to enhance the resolution by shaping the TFs in a particular way? Namely, is there a certain shape of the TF that is suitable with the  $L_1$  norm minimization algorithm we use at the back end? If yes, what is the shape and how to design and implement such a TF? From our study, we observe that TFs whose auto-covariance functions resemble that of a Dirac delta function acquire fine details of a signal spectrum. Filters with random TFs possess such auto-covariance. We show that it is possible to design filters with random TFs using thin-film technology and can improve the resolution up to 7-fold when compared to the filters with non-ideal TFs in [3].

## II. SYSTEM DESCRIPTION AND DESIGN APPROACH

We consider a filter array spectrometer with  $M$  filters each of which is specified in terms of a TF,  $T_i(\lambda), i = 1, 2, \dots, M$ . Each filter is attached to a CCD element whose output (after sampling) denotes a sample of the raw spectrum modeled as  $y_i = \int T_i(\lambda)x(\lambda) d\lambda + w_i$ , where  $x(\lambda)$  is a signal spectrum and  $w_i$  is a Gaussian noise sample. The model for the raw spectrum is  $\mathbf{y} = D\mathbf{G}\mathbf{s} + \mathbf{w}$ ,  $\mathbf{s} \geq 0$ , where  $D$  is an  $M \times N$  TF matrix with  $M < N$ . The  $i$ -th row of  $D$  contains the samples of  $T_i(\lambda)$ ,  $G$  is an  $N \times N$  Gaussian kernel matrix and  $\mathbf{s}$  is an  $N \times 1$   $K$ -sparse spectrum with  $K \ll N$  non-zero components. Since  $\mathbf{s}$  is  $K$ -sparse we use the  $L_1$  norm minimization algorithm derived in [2] to estimate  $\mathbf{s}$  from  $\mathbf{y}$ , given  $D$  and  $G$ . The matrix  $G$  is fixed depending on applications. Thus, it is important to design a good TF matrix  $D$  for the  $L_1$  algorithm to recover the sparse spectrum. Designing a good TF matrix  $D$  resembles that of designing a sensing matrix in compressive sensing (CS). In CS, random sensing matrices are shown to be good for sensing and reconstruction of sparse signals. Thus, in this paper, we propose random TFs for improving resolution. We note that TFs are analog functions and the central question now is how to design and implement filters with random TFs optically?

Filters with random TFs can be designed by using thin-film optical filter technology [4]. The usual quarter wavelength thicknesses of thin film filter layers provide band-pass like transmittance. We randomly vary the thicknesses of the thin film layers in each filter and generate  $M$  number of filters with random TFs. We found that these random TFs are white and uncorrelated with each other. Currently, the implementation of these random TFs is underway in our lab.

## III. RESULTS AND CONCLUSIONS

We consider a mercury lamp with 7 spectral lines (Fig. 1(a)). The least separation among these lines is 2.106nm (between the pair 576.959nm and 579.065nm). It is apparent from Fig. 1(c) that the random TFs clearly resolve these two closely spaced spectral lines. In fact, we found that random TFs are capable of resolving any two spectral lines which are more than 0.99nm apart. In contrast, the TFs in [3] cannot resolve the two closely spaced spectral lines (Fig. 1(b)) and we found that these TFs resolve spectral lines that are 6.5nm apart. Based on these results we conclude that random TFs outperform the TFs in [3] and improve the resolution by 7-fold.

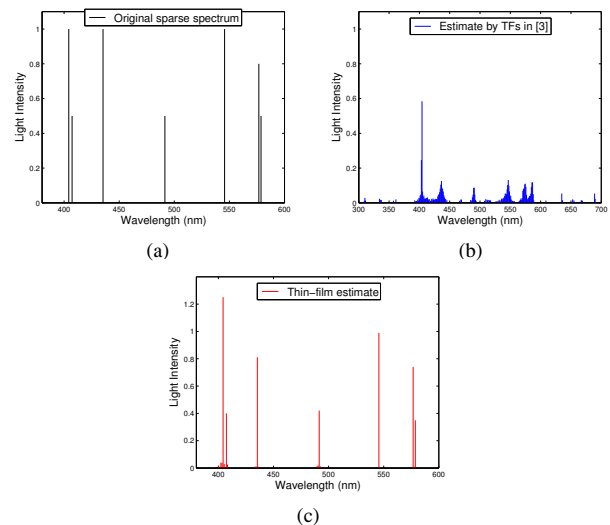


Fig. 1: (a) Original sparse spectrum (b) Estimate by TFs in [3] (c) Estimate by thin film based random TFs

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