Recent Advances in Multiuser MIMO Optimization

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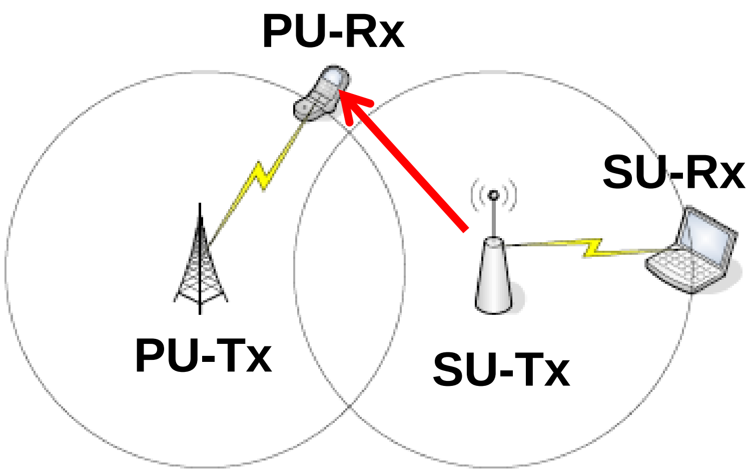
Agenda

- Overview of the talk
- Exploiting multi-antennas in
  - Cognitive Radio Networks
  - Cooperative Multi-Cell
  - Two-Way Relay Networks
  - Green Cellular Networks
  - Wireless Information and Power Transfer
- Concluding remarks

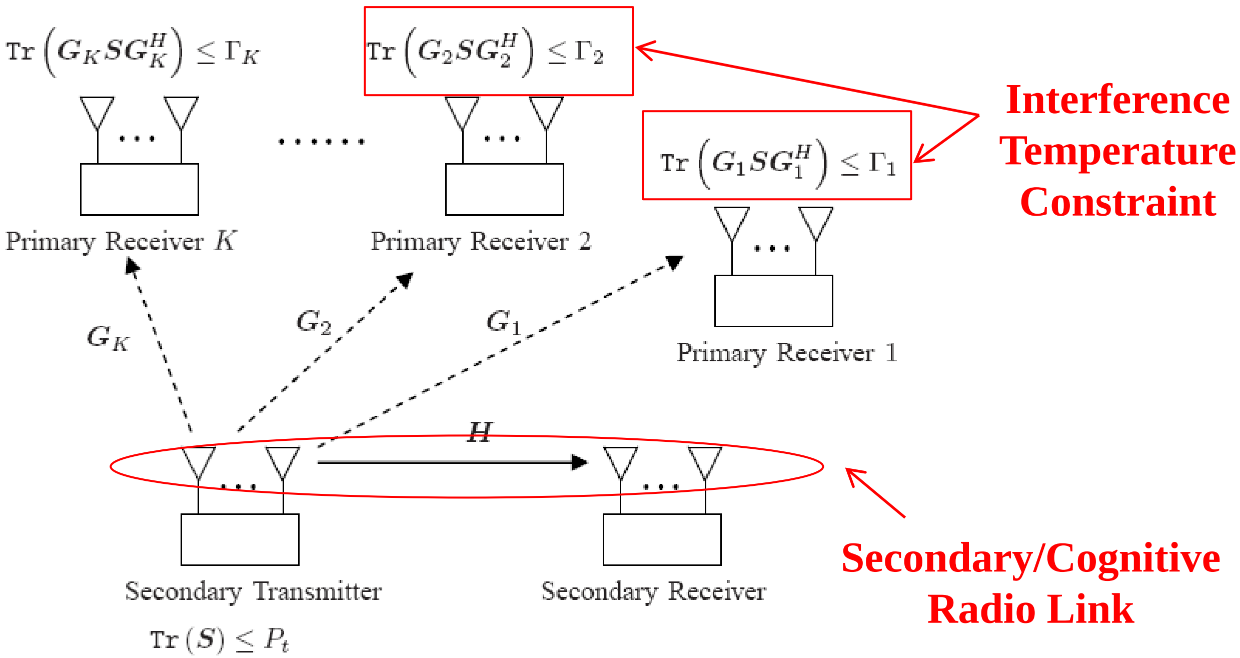
MIMO in Wireless Communication: A Brief Overview

- **Point-to-Point MIMO**
  - ✓ MIMO channel capacity, space-time code, MIMO precoding, MIMO detection, MIMO equalization, limited-rate MIMO feedback, MIMO-OFDM …
- **Multi-User MIMO (Single Cell)**
  - ✓ SDMA, MIMO-BC precoding, uplink-downlink duality, opportunistic beamforming, MIMO relay, distributed antenna, resource allocation …
- **Multi-User MIMO (Multi-Cell)**
  - ✓ network MIMO/CoMP, coordinated beamforming, MIMO-IC, interference management, interference alignment …
- **MIMO in emerging wireless systems/applications**
  - ✓ cognitive radio networks, ad hoc networks, secrecy communication, two-way communication, full-duplex communication, compressive sensing, MIMO radar, wireless power transfer …

Talk Overview (1): Cognitive MIMO

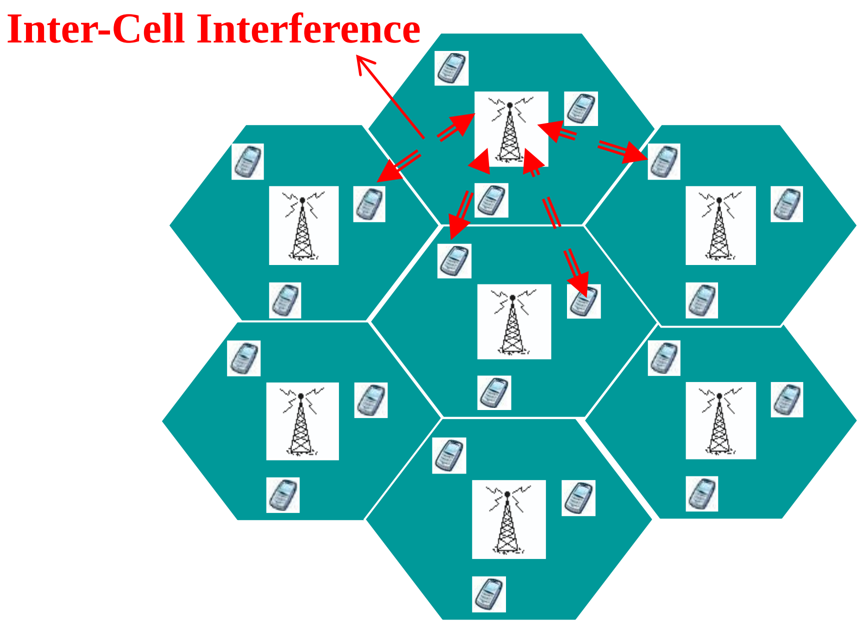


Spectrum Sharing Cognitive Radio

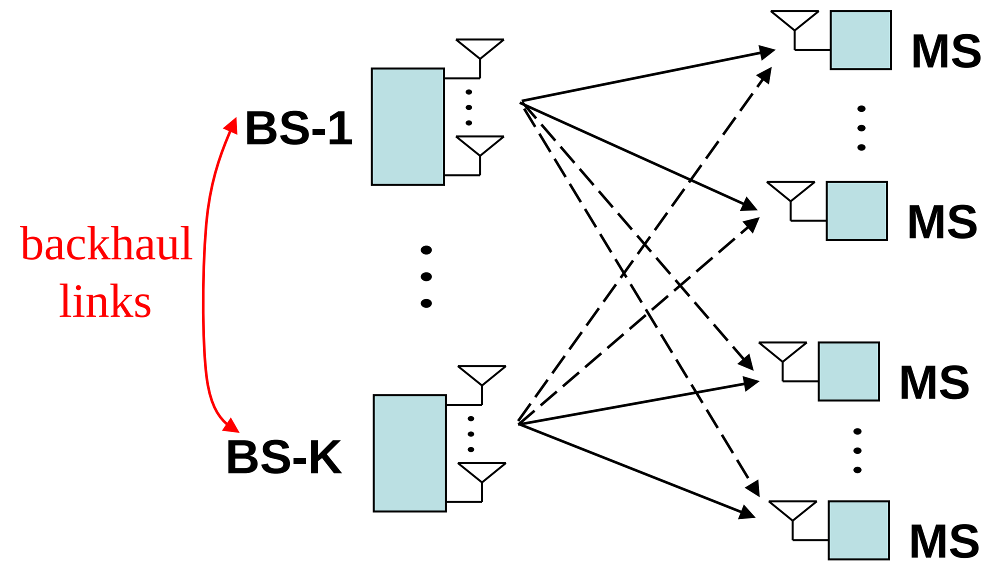


- **Exploiting MIMO in Cognitive Radio Networks**
  - ✓ How to optimize secondary MIMO transmissions subject to interference power constraints at all nearby primary receivers?
  - ✓ How to practically obtain the channel knowledge from secondary transmitter to primary receivers?
  - ✓ How to optimally set the interference power levels at different primary receivers?

Talk Overview (2): Multi-Cell MIMO



Universal Frequency Reuse in Cellular network

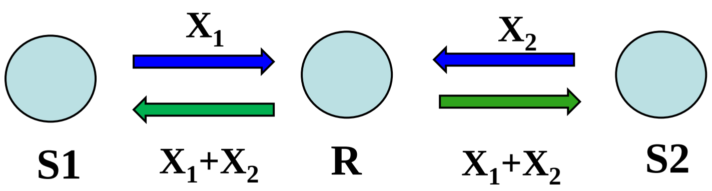


Multi-Cell Cooperative MIMO (Downlink)

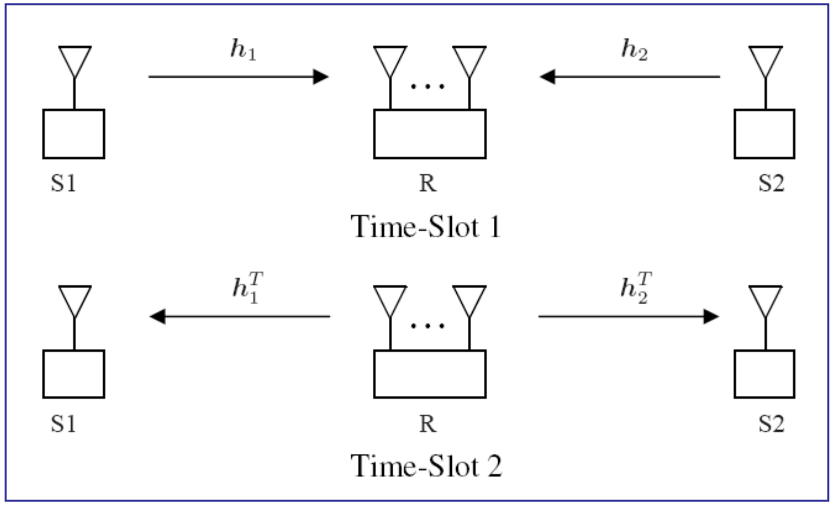
➢ Cooperative Interference Management in Multi-Cell MIMO

- Network MIMO (CoMP) with baseband signal-level coordination among BSs
  - ✓ How to design the optimal (linear/non-linear) joint downlink precoding with per-BS power constraints?
- Coordinated downlink beamforming for inter-cell interference control
  - ✓ How to jointly design beamforming and power control at all BSs to achieve optimal rate tradeoffs among different cells?
  - ✓ How to achieve optimal distributed beamforming with only local CSI at each BS?

Talk Overview (3): Two-Way Relay Beamforming for Wireless Network Coding



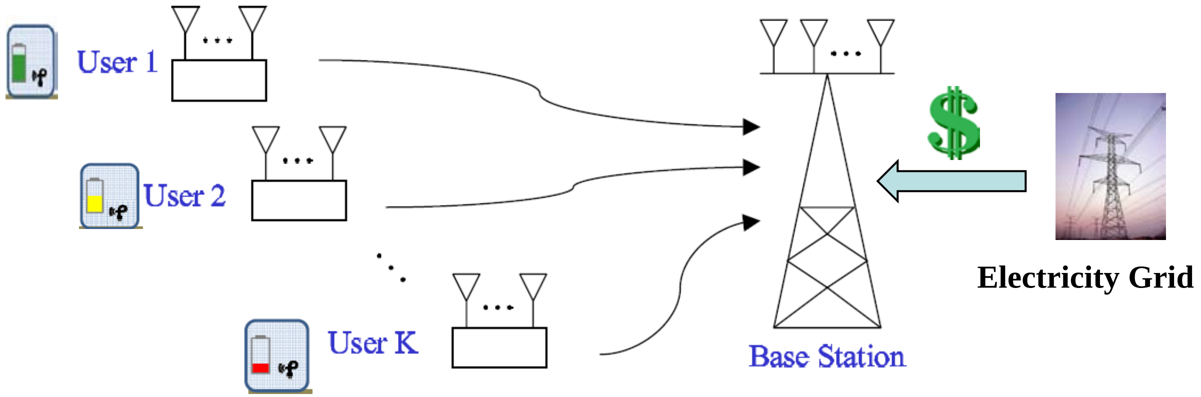
Two-Way Relay System
(with analogue network coding)



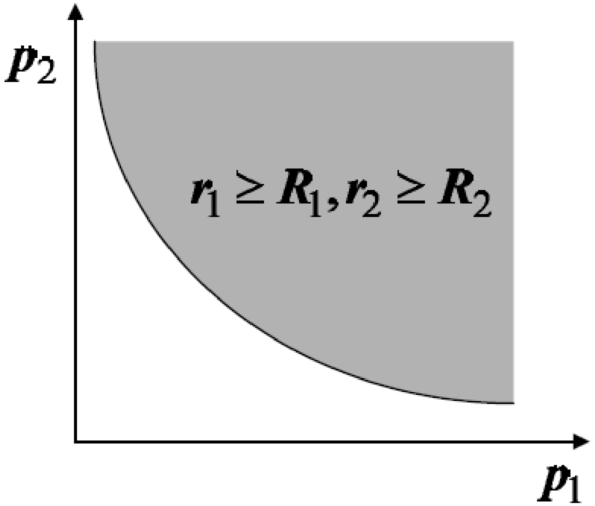
Two-Way Multi-Antenna Relay System

- **Exploiting Multi-Antenna Relay in Two-Way Communication**
  - ✓ How to optimally design the linear beamforming matrix at R to maximize two-way information exchange rates between S1 and S2?
  - ✓ How is the optimal design fundamentally different from traditional one-way relay beamforming (S1-R-S2 and S2-R-S1 alternatively)?

Talk Overview (4): Power Minimization in MIMO Cellular Networks



Power Minimization in Cellular Networks



Power Region

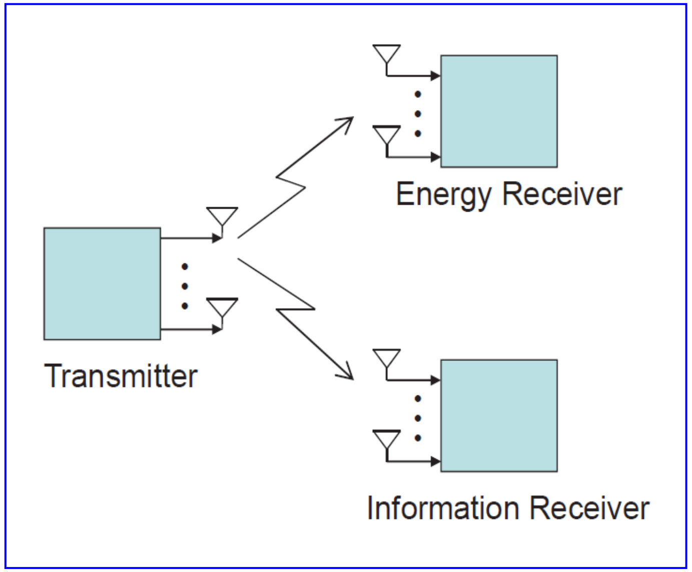
➢ Power Minimization in MU-MIMO given Rate Constraints

- ✓ How to characterize MU power region to achieve minimum power consumption tradeoffs in cellular uplink?
- ✓ How to achieve minimal BS power consumption in cellular downlink?
- ✓ What is the fundamental relationship between MU capacity region and power region?

Talk Overview (5): MIMO Broadcasting for Wireless Information and Power Transfer

Energy Transmitter Energy Receiver

RF-based Wireless Power Transfer



MIMO Broadcasting for Information and Power Transfer

➢ Exploiting MIMO in Wireless Information and Power Transfer

- ✓ How to optimally design MIMO transmissions to achieve simultaneously maximal information and power transfer?
- ✓ How to characterize the achievable rate-energy tradeoffs?
- ✓ What are practical design issues due to energy harvesting circuit limitations?

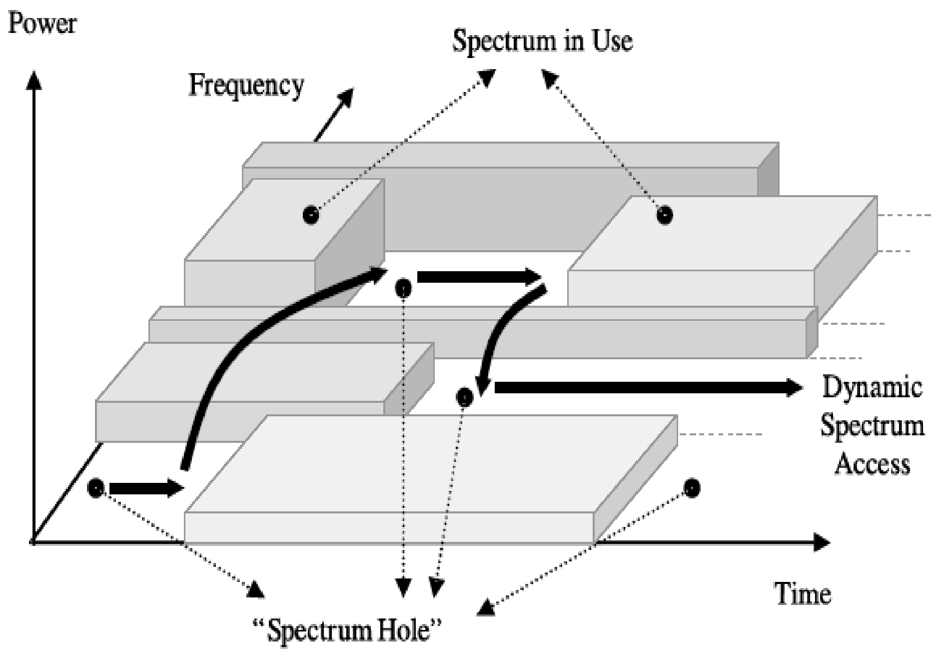
Agenda

- ❑ Overview of the talk
- ❑ Exploiting multi-antennas in
  - ➢ **Cognitive Radio Networks**
  - ➢ Cooperative Multi-Cell
  - ➢ Two-Way Relay Networks
  - ➢ Green Cellular Networks
  - ➢ Wireless Information and Power Transfer
- ❑ Concluding remarks

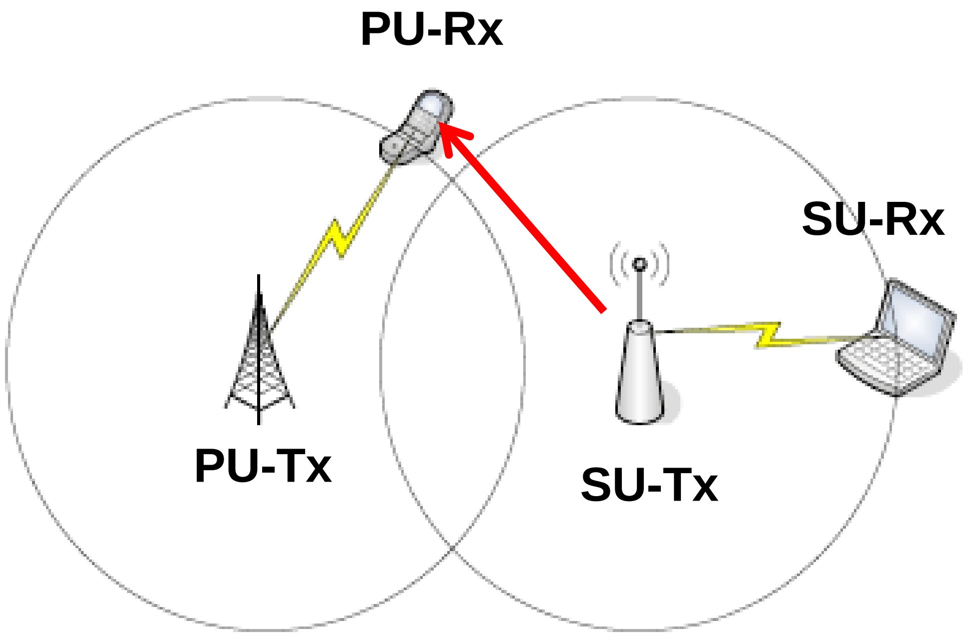
Topic #1: Cognitive MIMO Systems

Operation Models of Cognitive Radio

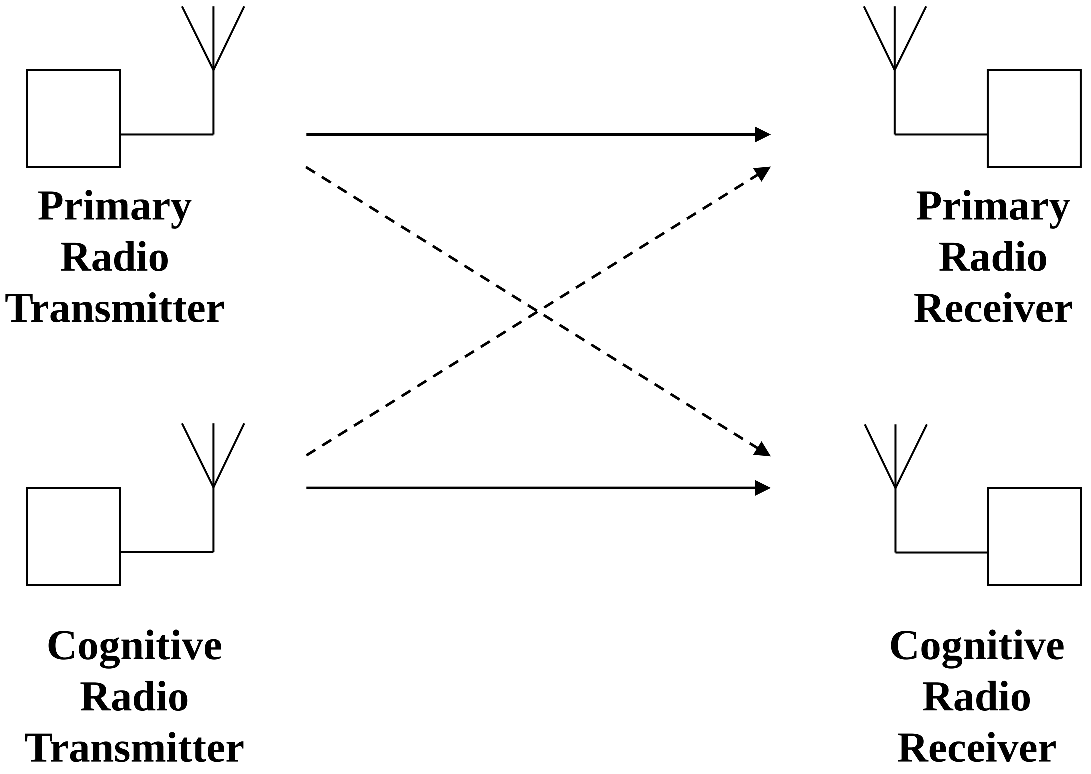
- **Dynamic Spectrum Access**
  - **Orthogonal transmissions: exploiting on-off activity of primary links**



- **Spectrum Sharing**
  - **Simultaneous transmissions: exploiting performance margin of primary links**

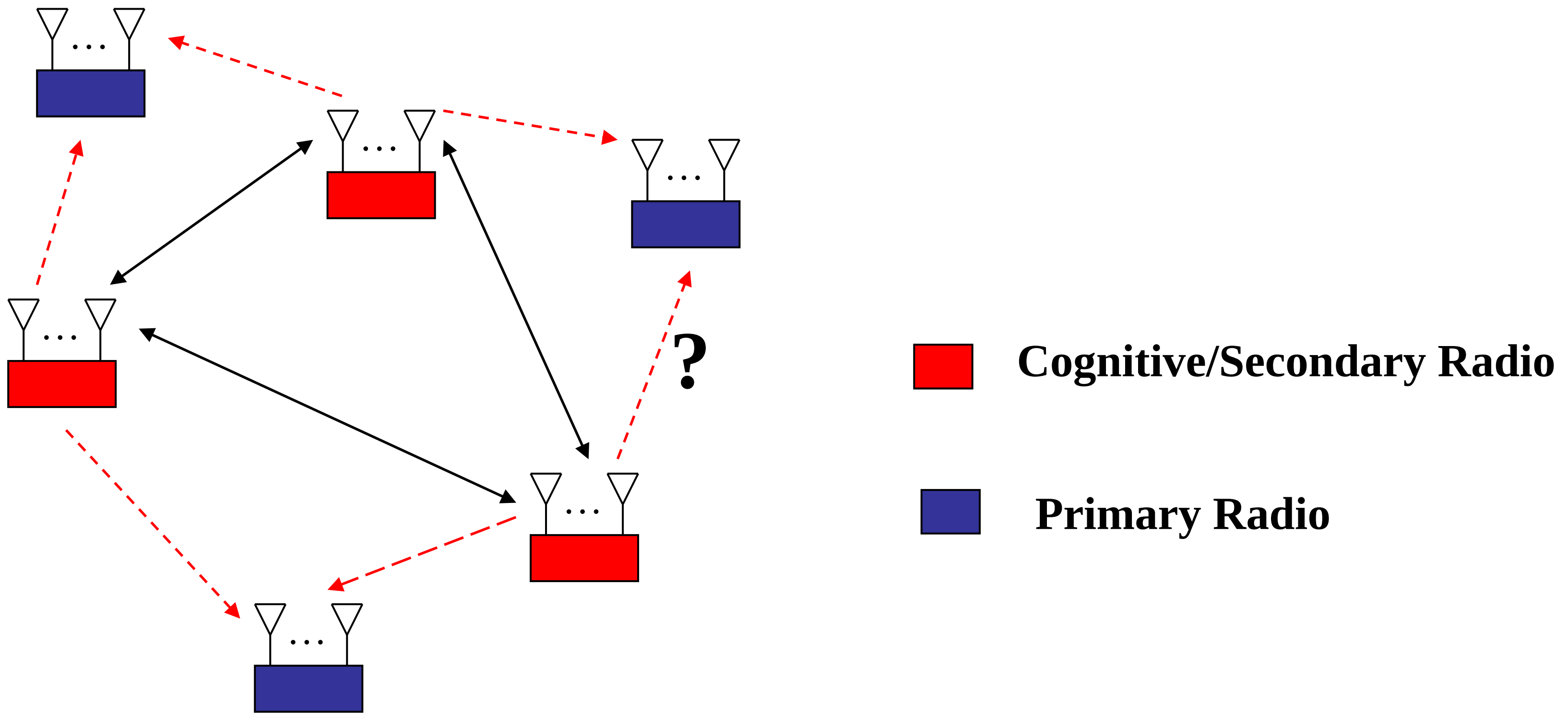


Spectrum Sharing Cognitive Radio



- **Information-theoretic approach:**
  - –**Cognitive Relay** [DevroyeMitranTarokh06] [JovicicViswanath06]
- **Pragmatic approach:**
  - –**Interference Temperature** [Gastpar07] [GhasemiSousa07]

Cognitive MIMO: Enabling Spatial Spectrum Sharing



Two main issues:

1. **How to optimally design secondary transmissions (precoding, power control) given interference temperature constraints?**
2. **How to practically obtain secondary-to-primary channels?**

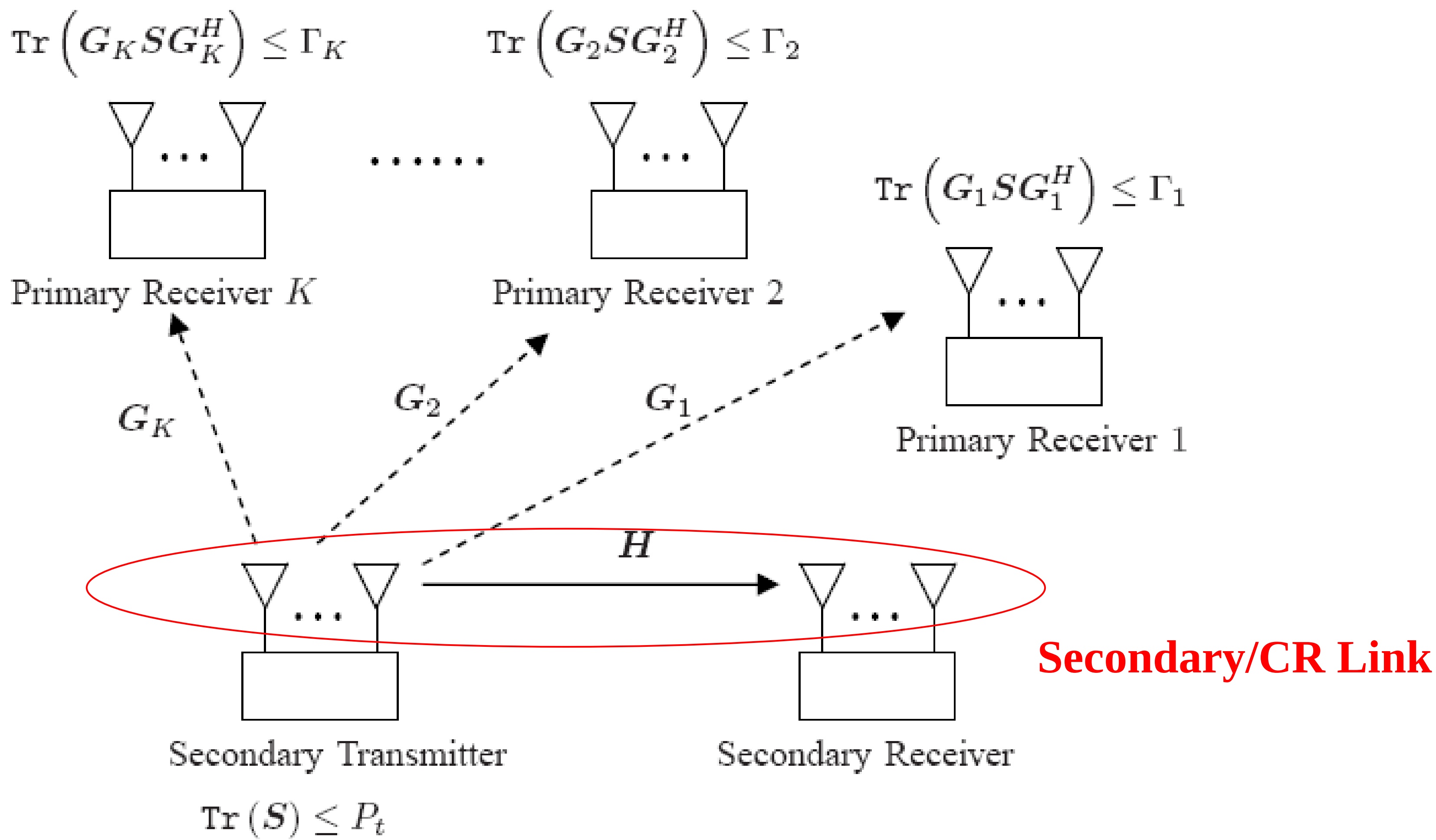
Outline for Cognitive MIMO

- **Part I: Fundamental Limits**
  - Assume **perfect** secondary-to-primary CSI
  - Characterize cognitive radio (CR) MIMO channel capacity subject to interference-temperature constraints in
    - **CR point-to-point MIMO channel**
    - **CR MIMO broadcast channel (BC)**

- **Part II: Practical Designs**
  - Assume **no prior** knowledge of secondary-to-primary CSI
  - Propose practical “cognitive beamforming” schemes via
    - **CR self-learning**
    - **Primary radio (PR) collaborative feedback**

Part I: Capacity Limits of Cognitive MIMO (with perfect CR-to-PR CSI)

CR Point-to-Point MIMO Channel



Problem Formulation [ZhangLiang08]

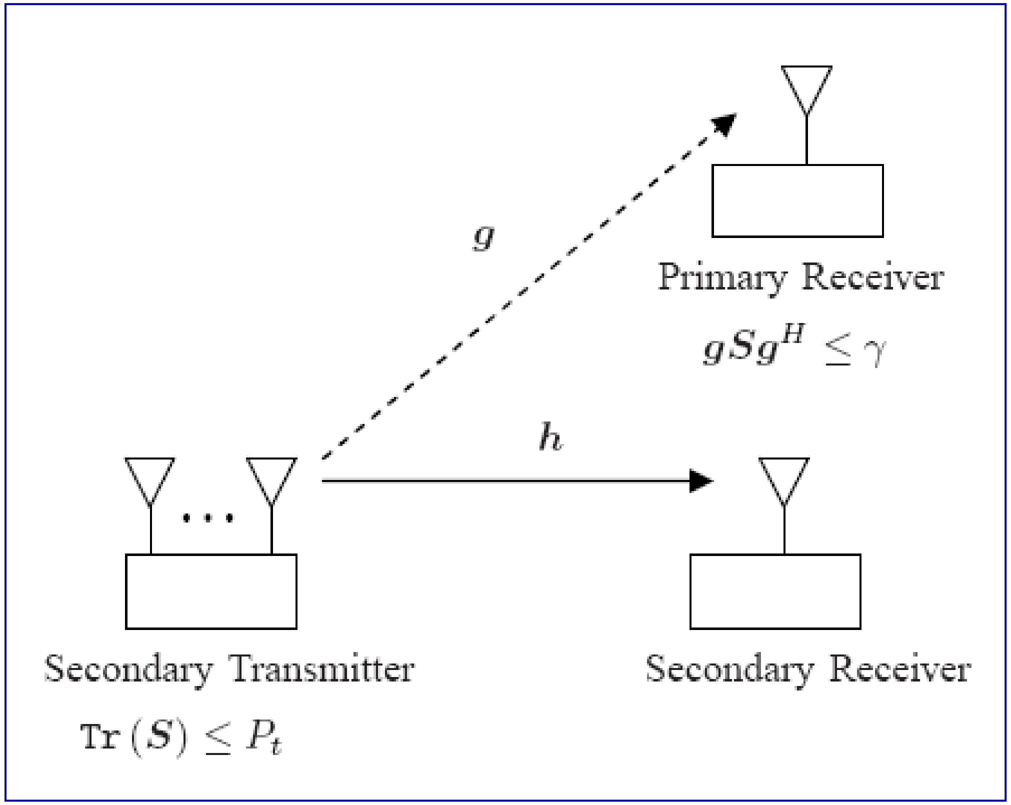
$$\begin{aligned}
\underset{\boldsymbol{S}}{\text{Maximize}} \quad & \log_2 \left| \boldsymbol{I} + \boldsymbol{H}\boldsymbol{S}\boldsymbol{H}^H \right| \\
\text{Subject to} \quad & \mathrm{Tr}(\boldsymbol{S}) \le P_t \\
& \mathrm{Tr}\left(\boldsymbol{G}_k \boldsymbol{S} \boldsymbol{G}_k^H \right) \le \Gamma_k, \;\; k = 1, \ldots, K \\
& \boldsymbol{S} \succeq 0
\end{aligned}$$

generalized linear transmit power constraint

- Problem is **convex**, and thus solvable by convex optimization techniques, e.g., the interior-point method, the Lagrange duality method (more details given later)
- Suboptimal low-complexity solution: "generalized" zero-forcing **(see [ZhangLiang08])**

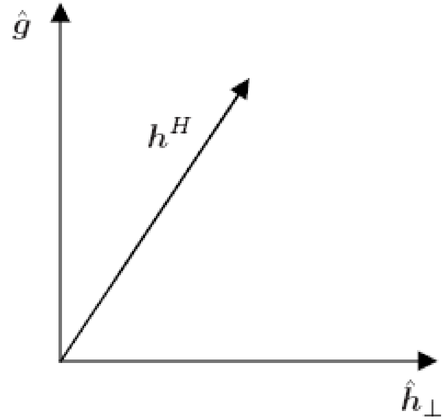
[ZhangLiang08]: R. Zhang and Y. C. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," ***IEEE Journal on Selected Topics in Signal Processing*****, Feb. 2008.**

Special Case: CR MISO Channel



$$
\begin{aligned}
\underset{S}{\text{Maximize}} \quad & \log_2\left(1 + hSh^H\right) \\
\text{Subject to} \quad & \mathrm{Tr}(S) \leq P_t \\
& gSg^H \leq \gamma \\
& S \succeq 0
\end{aligned}
$$

Optimal Solution



- Beamforming is optimal, i.e., $\mathrm{Rank}(\boldsymbol{S}) = 1$
- $\boldsymbol{S} = \boldsymbol{v}\boldsymbol{v}^H$, $\boldsymbol{v} = \alpha_v \hat{\boldsymbol{g}} + \beta_v \hat{\boldsymbol{h}}_\perp$
  - Case I (Interference Power Constraint Inactive): If $\gamma \geq \frac{||\boldsymbol{g}||^2 ||\alpha_h||^2}{||\alpha_h||^2 + ||\beta_h||^2} P_t$

$$\boxed{\alpha_v = \sqrt{\frac{P_t}{||\alpha_h||^2 + ||\beta_h||^2}}\alpha_h, \quad \beta_v = \sqrt{\frac{P_t}{||\alpha_h||^2 + ||\beta_h||^2}}\beta_h}$$

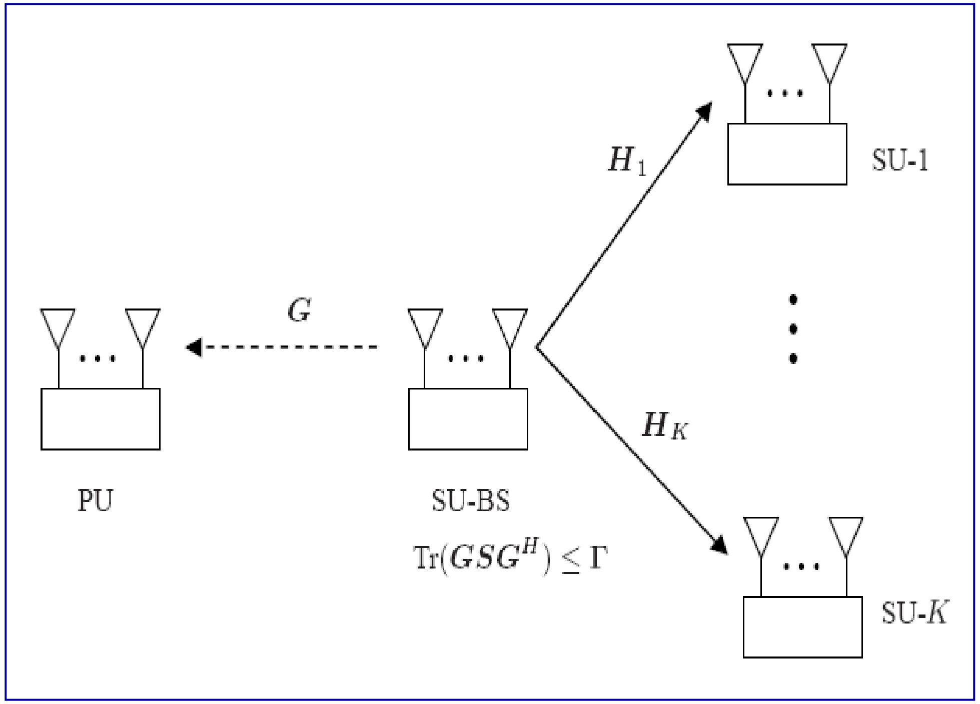
Conventional maximal-ratio transmission (MRT)

  - Case II (Interference Power Constraint Active): If $\gamma < \frac{||\boldsymbol{g}||^2 ||\alpha_h||^2}{||\alpha_h||^2 + ||\beta_h||^2} P_t$

$$\boxed{\alpha_v = \frac{\sqrt{\gamma}}{||\boldsymbol{g}||} \frac{\alpha_h}{||\alpha_h||}, \quad \beta_v = \sqrt{P_t - \frac{\gamma}{||\boldsymbol{g}||^2}} \frac{\beta_h}{||\beta_h||}}$$

"Cognitive beamforming (CB)"

CR MIMO-BC



$$\begin{aligned} \underset{S=\sum_{i=1}^K S_i}{\text{Maximize}} \quad & \sum_{i=1}^{K} \log_2 \left| \frac{I + \sum_{j=i}^{K} H_i S_j H_i^H}{I + \sum_{j=i+1}^{K} H_i S_j H_i^H} \right| \\ \text{Subject to} \quad & \mathrm{Tr}\left(GSG^H\right) \leq \Gamma \\ & S_i \succeq 0, \ i = 1, \ldots, K \end{aligned}$$

- **Problem is non-convex, thus not solvable by standard convex optimization techniques**
- **Optimal solution is obtained via generalized BC-MAC duality [Zhang et al. 12] (more details given later)**

[Zhang et al. 12]: L. Zhang, R. Zhang, Y. C. Liang, Y. Xin, and H. V. Poor, "On the Gaussian MIMO BC-MAC duality with multiple transmit covariance constraints," *IEEE Transactions on Information Theory,* April 2012.

Other Topics on Cognitive MIMO

- **Robust cognitive beamforming**
  - e.g., [ZhangLiangXinPoor09], [ZhengWongOttersten10]

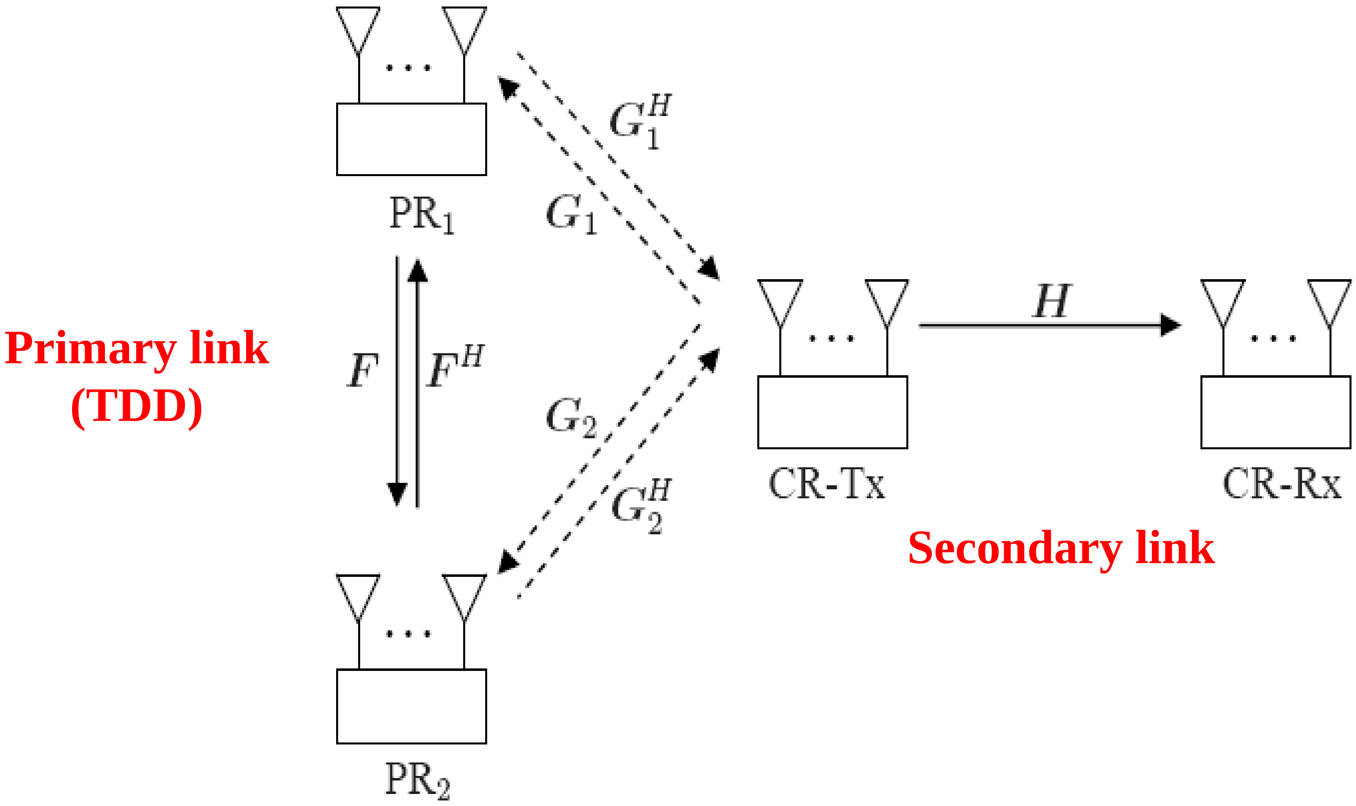
- **CR MIMO interference channel (MIMO-IC)**
  - e.g., [KimGiannakis08], [ScutariPalomarBarbarossa08], [TajerPrasadWang10]

- **A recent survey on related works available at**
  - [ZhangLiangCui10]

[ZhangLiangCui10]: R. Zhang, Y. C. Liang, and S. Cui, "Dynamic resource allocation in cognitive radio networks," *IEEE Signal Processing Magazine*, special issue on convex optimization for signal processing, June 2010.

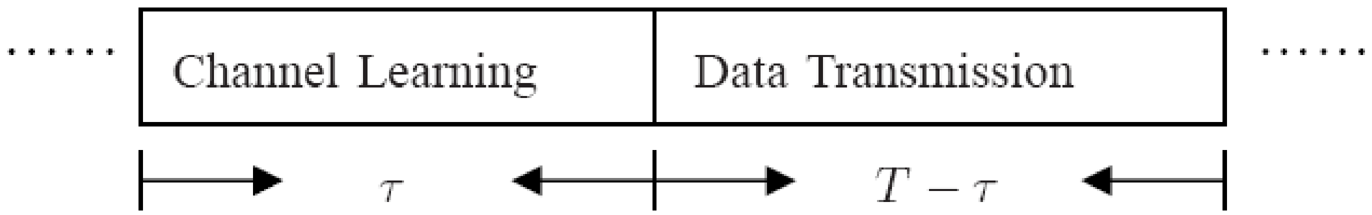
Part II: Practical Designs for Cognitive MIMO (without prior knowledge of CR-to-PR CSI)

Learning-Based MIMO CR [ZhangGaoLiang10]



[ZhangGaoLiang10]: R. Zhang, F. Gao, and Y. C. Liang, "Cognitive beamforming made practical: effective interference channel and learning-throughput tradeoff," *IEEE Transactions on Communications*, Feb. 2010.

Protocol for Learning-Based MIMO CR



- Two-phase protocol:
  - 1st phase: observe PR transmissions, compute PR signal sample covariance matrix, and then estimate CR-to-PR effective interference channel (EIC);
  - 2nd phase: transmit with (zero-forcing) precoding orthogonal to the EIC
- Joint design of learning time and precoding matrix to
  - Maximize CR link throughput
  - Minimize leakage interference to PR link

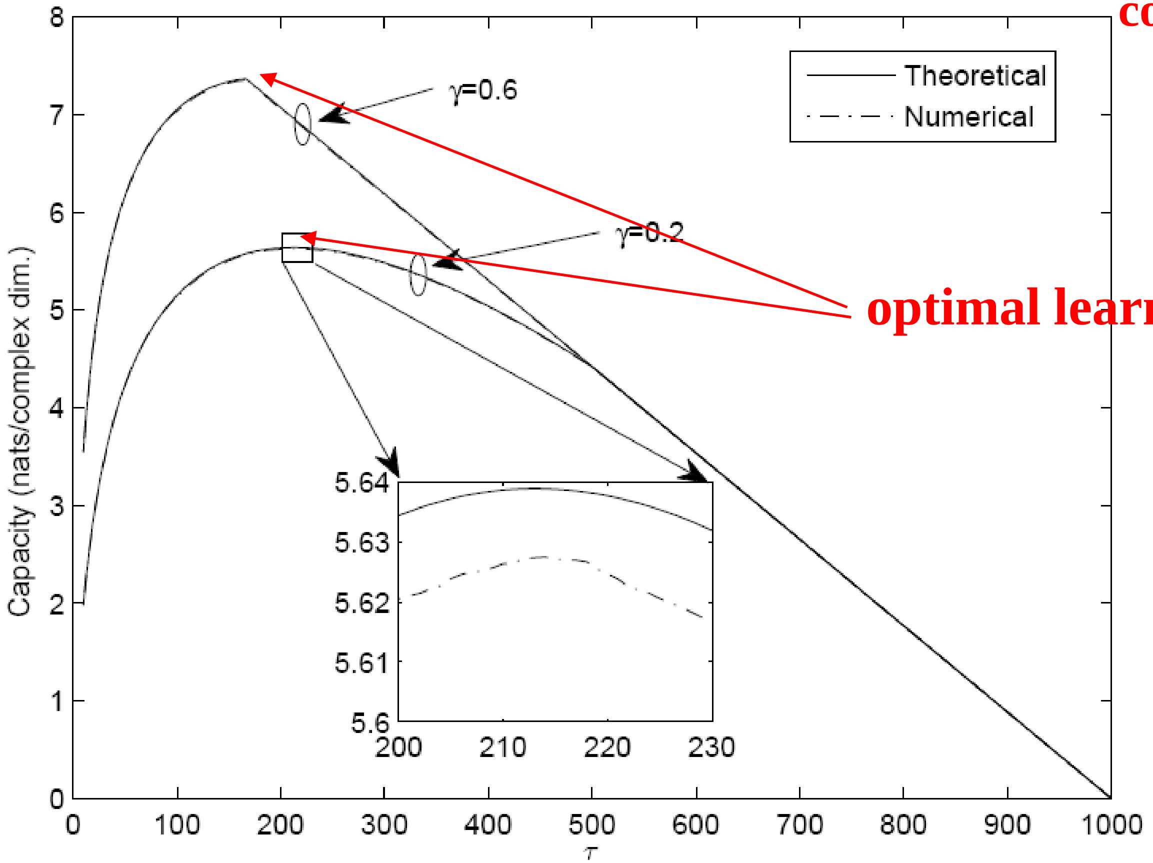
Learning-Throughput Tradeoff

$$\max_{\tau, \boldsymbol{C}_{CR}} \quad \frac{T-\tau}{T} \log \left| \boldsymbol{I} + \boldsymbol{H}\hat{\boldsymbol{U}}\boldsymbol{C}_{CR}\hat{\boldsymbol{U}}^H \boldsymbol{H}^H \right|$$

$$\text{s.t.} \quad \mathrm{Tr}(\boldsymbol{C}_{CR}) \leq \min(P_{CR}, \gamma\tau), \quad \boldsymbol{C}_{CR} \succcurlyeq \boldsymbol{0}, \quad 0 \leq \tau < T$$

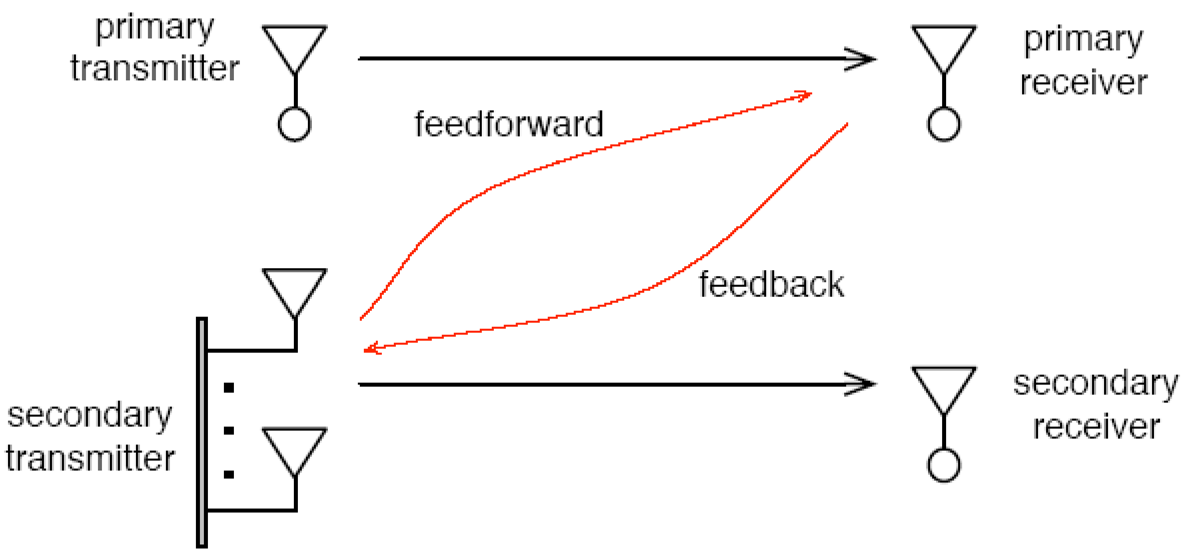
CR throughput loss proportional to learning time

Interference power constraint proportional to learning time



optimal learning time

Primary Radio Collaborative Feedback [HuangZhang11]



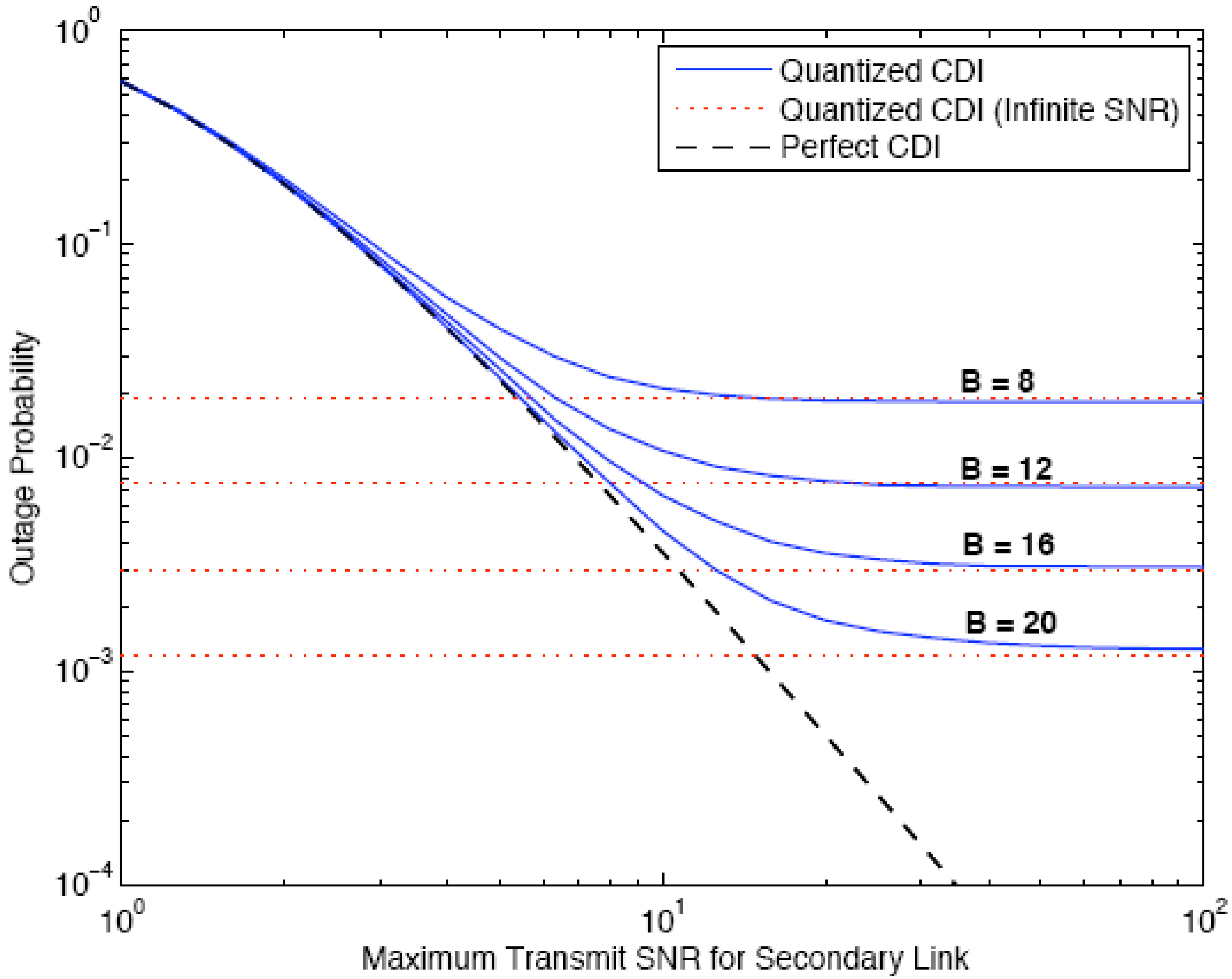
[HuangZhang11]: K.-B. Huang and R. Zhang, "Cooperative feedback for multi-antenna cognitive radio network", *IEEE Transactions on Signal Processing*, Feb. 2011.

Protocol for PR Collaborative Feedback

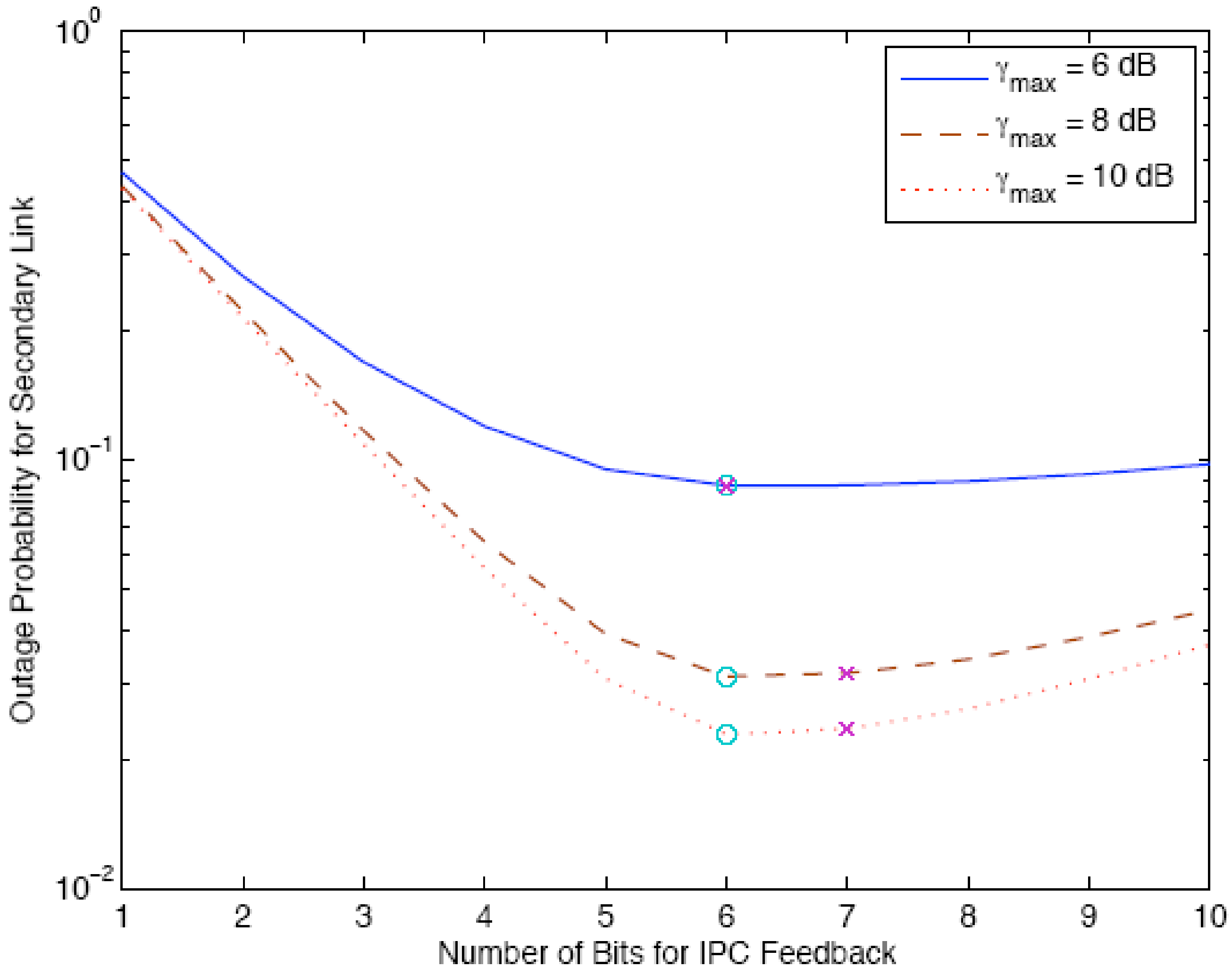
- P-Rx estimates the primary channel and determines the tolerable interference power from S-Tx, I_0;
- P-Rx estimates the channel from S-Tx to P-Rx, $\boldsymbol{h}_i = \sqrt{g_i}\boldsymbol{s}_i$;
- With I_0, g_i, and $\boldsymbol{s}_i$, P-Rx designs the feedback signal to S-Tx:
  - Quantized Interference Power Control (IPC), $\hat{\eta}$, to limit the transmit power of secondary beamforming, $\|\boldsymbol{v}\|^2 \leq \hat{\eta}$;
  - Quantized Channel Distribution Information (CDI), $\hat{\boldsymbol{s}}_i$, to constrain the transmit direction of secondary beamforming, $\boldsymbol{v}^H\hat{\boldsymbol{s}}_i = 0$;
  - Due to feedback quantization, $|\boldsymbol{v}^H\boldsymbol{s}_i| > 0$. Thus, $\hat{\eta}$ is designed to make $|\boldsymbol{v}^H\boldsymbol{h}_i|^2 \leq I_0$.
- With $\hat{\eta}$ and $\hat{\boldsymbol{s}}_i$ from P-Rx, and the secondary channel $\boldsymbol{s}_s$ from S-Rx, S-Tx designs cognitive beamforming:

$$\boldsymbol{f}_o = \arg\max_{\boldsymbol{v}\in\mathbb{C}^L} |\boldsymbol{v}^H\boldsymbol{s}_s|^2, \text{ s.t. } \boldsymbol{v}^H\hat{\boldsymbol{s}}_i = 0 \text{ and } \|\boldsymbol{v}\|^2 \leq \min(\hat{\eta}, P_s)$$

CR Link Outage Probability vs. Transmit Power Constraint (assuming perfect IPC feedback)



IPC and CDI Feedback Bit Allocation (assuming fixed sum feedback bits)



Concluding Remarks on Cognitive MIMO

- **Capacity limits of Cognitive MIMO channels**
  – Transmit covariance optimization under generalized linear transmit power constraints (more details given later)
- **Practical designs for Cognitive MIMO systems**
  – Learning-based cognitive radio
    - Learning-throughput tradeoff

  – Primary radio (PR) collaborative feedback
    - IPC vs. CDI feedback bit allocation
- **How to set Interference Temperature (IT) in practice?**
  – **Interference Diversity**: "Average" IT constraint (over time, frequency, space) better protects PR links than "Peak" counterpart **[Zhang09]**
  – **Active IT Control**: a new approach to optimal interference management in wireless networks, e.g.,
    - Cooperative multi-cell downlink beamforming (to be shown later)

[Zhang09]: R. Zhang "On peak versus average interference power constraints for protecting primary users in cognitive radio networks," *IEEE Transactions on Wireless Communications*, April 2009.

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- Exploiting multi-antennas in
  - Cognitive Radio Networks
  - **Multi-Cell Cooperation**
  - Two-Way Relay Networks
  - Green Cellular Networks
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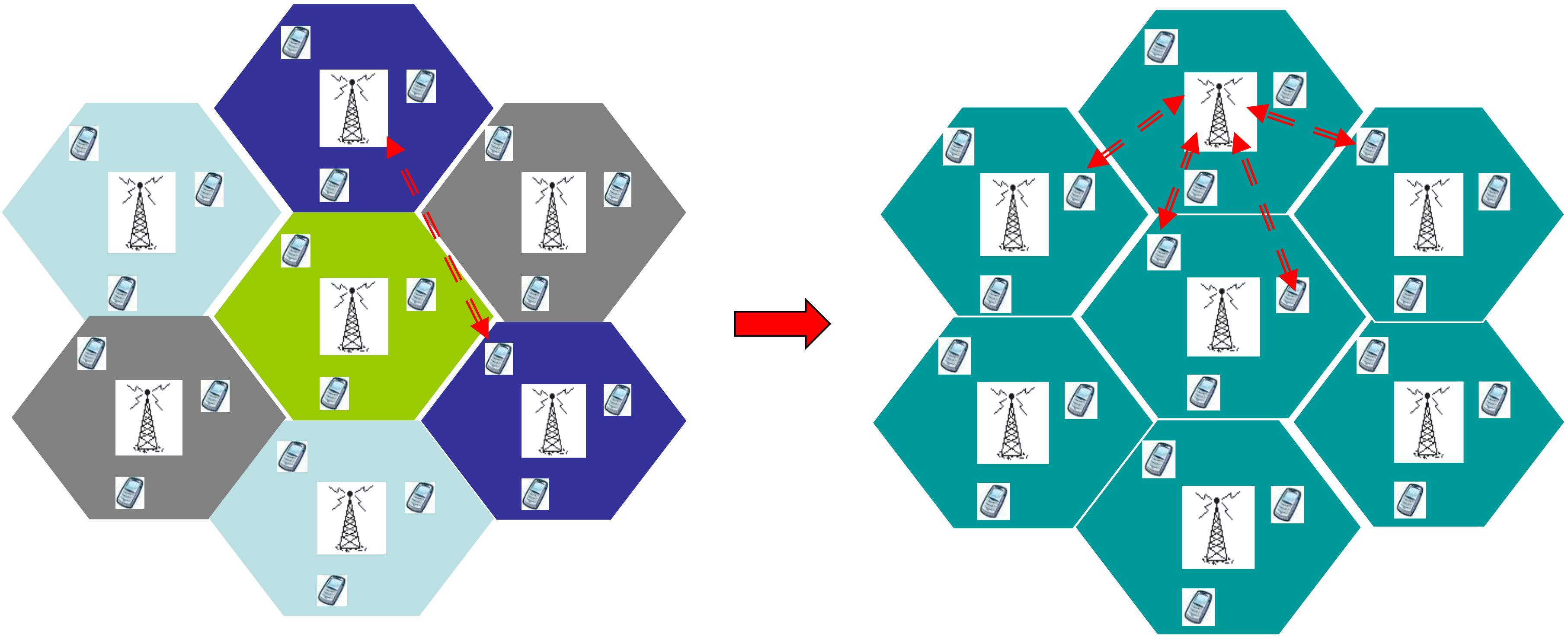
- Concluding remarks

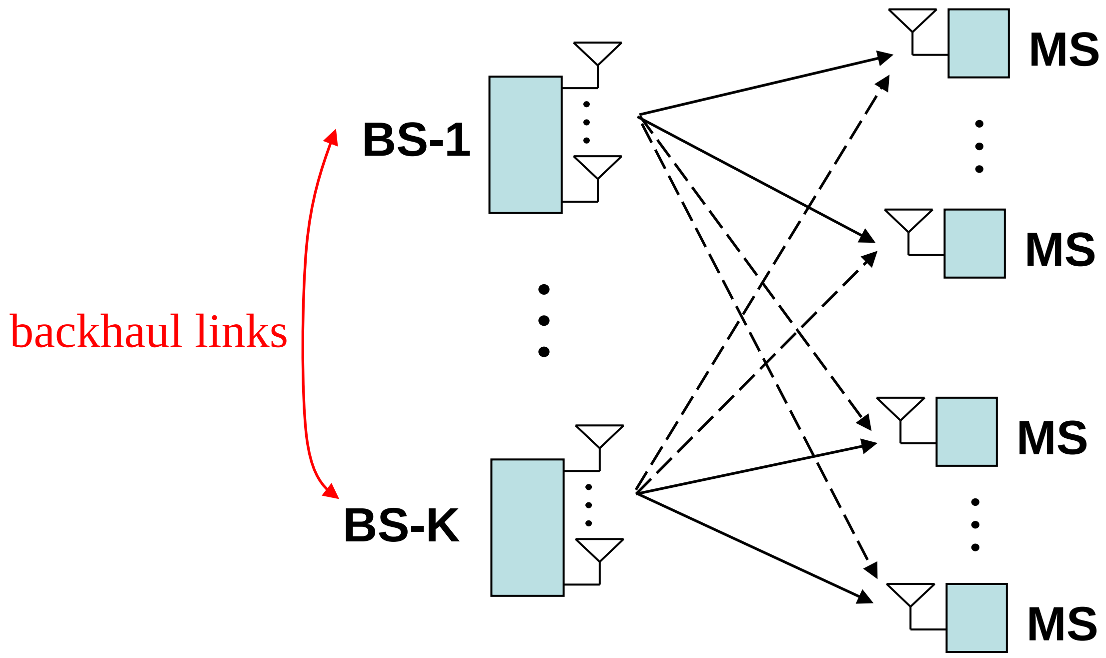
Topic #2: Multi-Cell Cooperative MIMO

A New Look at Cellular Networks



- **Future trends: universal/opportunistic frequency reuse**
  - ❑ Pros: more abundant/flexible bandwidth allocation
  - ❑ Cons: more severe/dynamic inter-cell interference (ICI)
  - ❑ Need more advanced cooperative interference management among BSs

Multi-Cell Cooperative MIMO (Downlink)



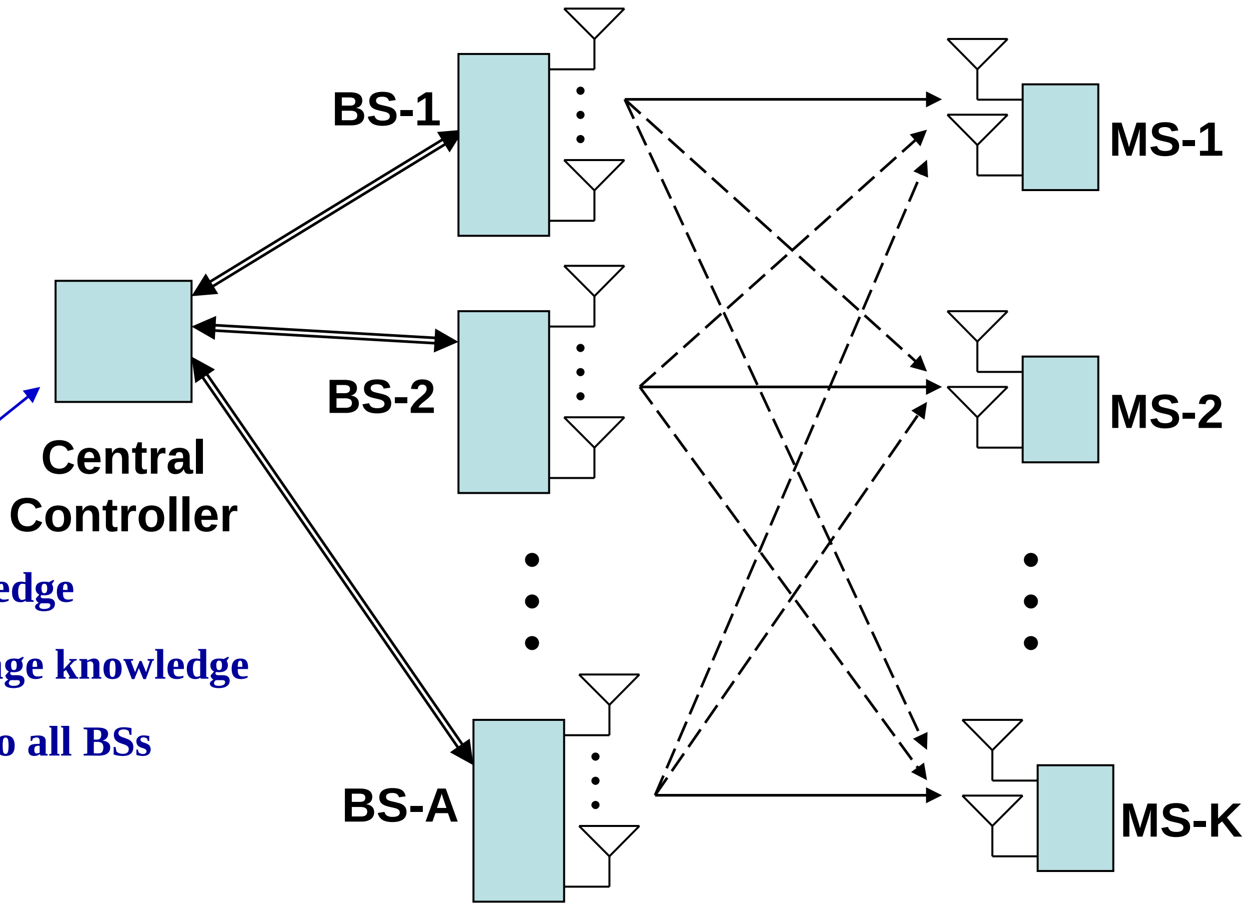
- **Network MIMO/CoMP**
  - ❑ Global transmit message sharing across all BSs
  - ❑ ICI utilized for coherent transmissions: baseband signal-level coordination (high complexity)
  - ❑ MIMO Broadcast Channel (MIMO-BC) with per-BS power constraints
- **Interference Coordination**
  - ❑ Local transmit message known at each BS
  - ❑ ICI controlled to the best effort: interference management (relatively lower complexity)
  - ❑ MIMO Interference Channel (MIMO-IC) *or* partially interfering MIMO-BC
- Hybrid Models: *clustered network MIMO, MIMO X channel…*

Outline for Multi-Cell MIMO

- **Part I: Network MIMO Optimization**
  - –MIMO BC with per-BS power constraints
  - –Weighted sum-rate maximization (WSRMax)
    - Optimal non-linear precoding with “dirty-paper coding (DPC)”
    - Optimal linear precoding with “block diagonalization (BD)”
- **Part II: Optimal Coordinated Downlink Beamforming**
  - –MISO Interference Channel (MISO-IC)
  - –Characterization of Pareto-optimal rates
    - Centralized algorithms with global CSI at all BSs
    - Distributed algorithms with local CSI at each BS

Part I: Network MIMO Optimization

System Model of Network MIMO



Assumptions:

- ❑ global channel knowledge
- ❑ global transmit message knowledge
- ❑ ideal backhaul links to all BSs

Equivalent to a MIMO-BC with per-BS power constraints

Network MIMO: Capacity Upper Bound

MIMO-BC with per-BS power constraints

- Nonlinear dirty-paper precoding (DPC)
  - Optimality of DPC [CaireShamai03] [ViswanathTse03] [YuCioffi04] [WeingartenSteinbergShamai06]
  - DPC region characterization (via WSRMax)
    - BC-MAC duality for sum-power constraint [VishwanathJindalGoldsimith03]
    - Min-Max duality for sum-/per-antenna power constraints [YuLan07]
    - Generalized BC-MAC duality for arbitrary linear power constraints: **[Zhang et al. 12]**
- Linear zero-forcing (ZF) *or* BD precoding
  - Sum-power constraint (MIMO-BC): [WongMurchLetaief03], [SpencerSwindlehurstHaardt04]
  - Per-antenna power constraint (MISO-BC): [WieselEldarShamai08] [HuhPapadopoulosCaire09]
  - Arbitrary linear transmit power constraints (MISO-/MIMO-BC): **[Zhang10]**

[Zhang et al. 12]: L. Zhang, R. Zhang, Y. C. Liang, Y. Xin, and H. V. Poor, "On the Gaussian MIMO BC-MAC duality with multiple transmit covariance constraints," *IEEE Transactions on Information Theory*, April 2012.

[Zhang10]: R. Zhang, "Cooperative multi-cell block diagonalization with per-base-station power constraints," *IEEE Journal on Selected Areas in Communications*, Dec. 2010.

Channel Model (1)

- **MIMO-BC baseband signal model:**

$$\boxed{\boldsymbol{y}_k = \boldsymbol{H}_k \boldsymbol{x}_k + \sum_{j \neq k} \boldsymbol{H}_k \boldsymbol{x}_j + \boldsymbol{z}_k, \quad k = 1, \cdots, K}$$

- $\boldsymbol{y}_k \in \mathbb{C}^{N \times 1}$: received signal at the kth MS
- $\boldsymbol{x}_k \in \mathbb{C}^{M \times 1}$: transmitted signal for the kth MS, $M = M_B A$
- $\boldsymbol{H}_k \in \mathbb{C}^{N \times M}$: downlink channel to the kth MS
- $\boldsymbol{z}_k \in \mathbb{C}^{N \times 1}$: receiver noise at the kth MS, $\boldsymbol{z}_k \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}), \forall k$

Channel Model (2)

- **Precoding (linear/nonlinear) matrix:**

$$\boxed{\boldsymbol{x}_k = \boldsymbol{T}_k \boldsymbol{s}_k, \quad k = 1, \ldots, K}$$

- $\boldsymbol{T}_k \in \mathbb{C}^{M \times D_k}$: precoding matrix for the kth MS, $D_k \leq \min(M, N)$
- $\boldsymbol{s}_k \in \mathbb{C}^{D_k \times 1}$: information-bearing signal for the kth MS, $\boldsymbol{s}_k \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{I})$
- $\boldsymbol{S}_k \triangleq \mathbb{E}[\boldsymbol{x}_k \boldsymbol{x}_k^H]$: transmit covariance matrix for the kth MS, $\boldsymbol{S}_k = \boldsymbol{T}_k \boldsymbol{T}_k^H$

- **Per-BS power constraints:**

$$\boxed{\sum_{k=1}^{K} \mathrm{Tr}\left(\boldsymbol{B}_a \boldsymbol{S}_k\right) \leq P, \quad a = 1, \cdots, A}$$

$$\boxed{\boldsymbol{B}_a \triangleq \mathrm{Diag}\Big(\underbrace{0, \cdots, 0}_{(a-1)M_B}, \underbrace{1, \cdots, 1}_{M_B}, \underbrace{0, \cdots, 0}_{(A-a)M_B}\Big)}$$

WSRMax in Network MIMO

- **Nonlinear DPC precoding:**

$$
\begin{aligned}
\text{(PA)}: \quad \max_{S_1,\cdots,S_K} \quad & \sum_{k=1}^{K} w_k \log \frac{\left| I + H_k \left(\sum_{i=k}^{K} S_i \right) H_k^H \right|}{\left| I + H_k \left(\sum_{i=k+1}^{K} S_i \right) H_k^H \right|} \\
\text{s.t.} \quad & \sum_{k=1}^{K} \mathrm{Tr}\left(B_a S_k \right) \leq P, \ \forall a \\
& S_k \succeq \mathbf{0}, \ \forall k
\end{aligned}
$$

non-convex problem, with the same structure as CR MIMO-BC optimization

- **Linear BD precoding:**

$$
\begin{aligned}
\text{(PB)}: \quad \max_{S_1,\cdots,S_K} \quad & \sum_{k=1}^{K} w_k \log \left| I + H_k S_k H_k^H \right| \\
\text{s.t.} \quad & H_j S_k H_j^H = 0, \ \forall j \neq k \\
& \sum_{k=1}^{K} \mathrm{Tr}\left(B_a S_k \right) \leq P, \ \forall a \\
& S_k \succeq \mathbf{0}, \ \forall k
\end{aligned}
$$

convex problem

Nonlinear DPC Precoding Optimization with Per-BS Power Constraints

- **WSRMax problem (PA):**

$$
\begin{aligned}
J^{(\mathrm{PA})} := \max_{\boldsymbol{S}_1,\cdots,\boldsymbol{S}_K}. \quad & \sum_{k=1}^{K} w_k \log \frac{\left|\boldsymbol{I}+\boldsymbol{H}_k\left(\sum_{i=k}^{K}\boldsymbol{S}_i\right)\boldsymbol{H}_k^H\right|}{\left|\boldsymbol{I}+\boldsymbol{H}_k\left(\sum_{i=k+1}^{K}\boldsymbol{S}_i\right)\boldsymbol{H}_k^H\right|} \\
\text{s.t.} \quad & \mathrm{Tr}\left(\boldsymbol{B}_a\sum_{k=1}^{K}\boldsymbol{S}_k\right) \le P, \ \forall a \\
& \boldsymbol{S}_k \succeq \boldsymbol{0}, \ \forall k
\end{aligned}
$$

per-BS power constraints

- **Auxiliary problem (PA-1):**

$$
\begin{aligned}
F(\lambda_1,\ldots,\lambda_A) := \max_{\boldsymbol{S}_1,\cdots,\boldsymbol{S}_K}. \quad & \sum_{k=1}^{K} w_k \log \frac{\left|\boldsymbol{I}+\boldsymbol{H}_k\left(\sum_{i=k}^{K}\boldsymbol{S}_i\right)\boldsymbol{H}_k^H\right|}{\left|\boldsymbol{I}+\boldsymbol{H}_k\left(\sum_{i=k+1}^{K}\boldsymbol{S}_i\right)\boldsymbol{H}_k^H\right|} \\
\text{s.t.} \quad & \mathrm{Tr}\left(\boldsymbol{B}_\lambda\sum_{k=1}^{K}\boldsymbol{S}_k\right) \le P_\lambda \\
& \boldsymbol{S}_k \succeq \boldsymbol{0}, \ \forall k
\end{aligned}
$$

$$
\begin{aligned}
&\boldsymbol{B}_\lambda \triangleq \textstyle\sum_{a=1}^{A}\lambda_a\boldsymbol{B}_a \\
&P_\lambda \triangleq (\textstyle\sum_{a=1}^{A}\lambda_a)P \\
&\lambda_a \ge 0, a=1,\ldots,A
\end{aligned}
$$

Algorithm for Solving (PA)

- Easy to verify the upper bound

$$\boxed{F(\lambda_1, \ldots, \lambda_A) \geq J^{(\mathrm{PA})}, \forall \boldsymbol{\lambda} \triangleq [\lambda_1, \ldots, \lambda_A]^T \succeq 0}$$

- Interestingly, the upper bound is also tight **(see [Zhang et al. 12])**

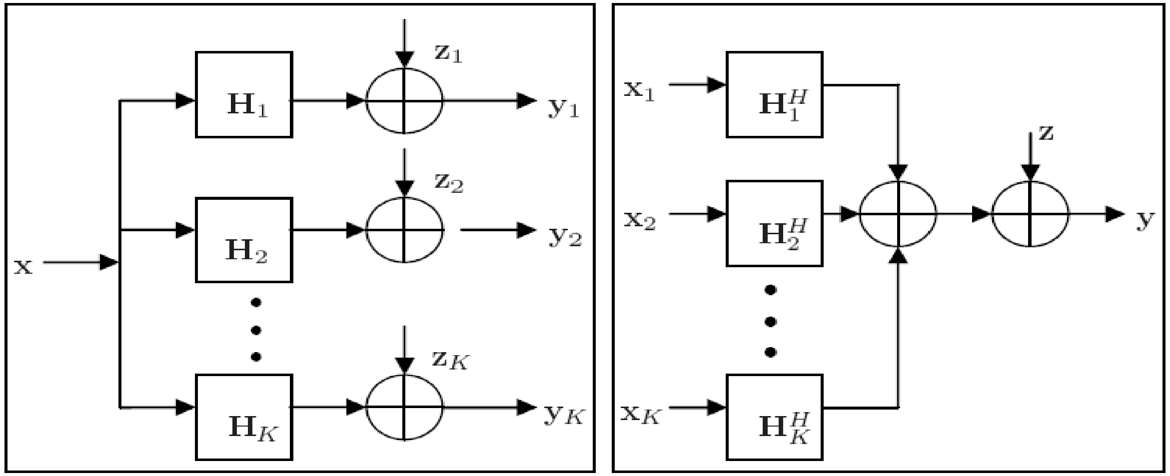
$$\boxed{J^{(\mathrm{PA})} = \min_{\boldsymbol{\lambda} \succeq 0} F(\lambda_1, \ldots, \lambda_A)}$$

- (PA) is solved by an iterative inner-outer-loop algorithm:
  - ❖ **Outer loop**: Solve the above minimization problem via sub-gradient based methods, e.g., the ellipsoid method
  - ❖ **Inner loop:** Solve the maximization problem (PA-1) via the generalized MIMO BC-MAC duality (shown in next slide).

Generalized BC-MAC Duality

Primal MIMO-BC
$\mathbf{z}_k \sim \mathcal{CN}(\mathbf{0},\mathbf{I})$, $\mathrm{Tr}(\mathbf{A}\sum_{k=1}^{K}\mathbf{S}_k) \le P$

Dual MIMO-MAC
$\mathbf{z} \sim \mathcal{CN}(\mathbf{0},\mathbf{A})$, $\sum_{k=1}^{K}\mathrm{Tr}(\mathbf{Q}_k) \le P$



(a) (b)

convex problem, solvable by e.g. the interior-point method

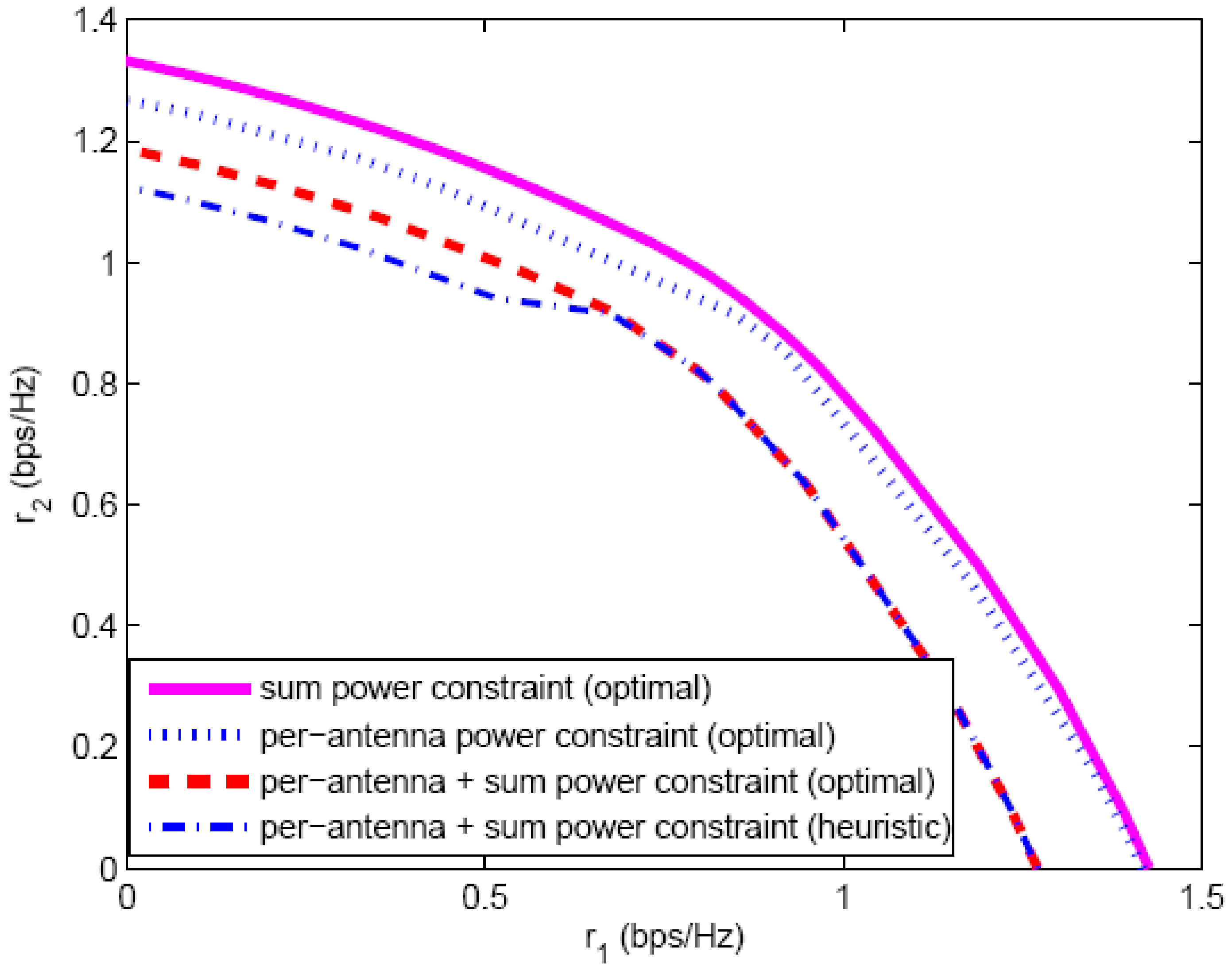
- **(PA-1) is equivalent to WSRMax in dual MIMO-MAC:**

$$\max_{\boldsymbol{Q}_1,\cdots,\boldsymbol{Q}_K} \quad \sum_{k=1}^{K-1}(w_k - w_{k+1})\log\left|\boldsymbol{B}_\lambda + \sum_{i=1}^{k}\boldsymbol{H}_i^H\boldsymbol{Q}_i\boldsymbol{H}_i\right| + w_K\log\left|\boldsymbol{B}_\lambda + \sum_{i=1}^{K}\boldsymbol{H}_i^H\boldsymbol{Q}_i\boldsymbol{H}_i\right|$$

$$\text{s.t.} \quad \sum_{k=1}^{K}\mathrm{Tr}\left(\boldsymbol{Q}_k\right) \le P_\lambda$$

$$\boldsymbol{Q}_k \succeq \mathbf{0}, \ \forall k$$

Two-User MISO-BC with Per-Antenna Power Constraints (DPC Precoding)



Linear BD Precoding Optimization with Per-BS Power Constraints

- **WSRMax problem (PB):**

$$
\begin{aligned}
(\mathrm{PB}): \quad \max_{\boldsymbol{S}_1,\cdots,\boldsymbol{S}_K} \quad & \sum_{k=1}^{K} w_k \log\left|\boldsymbol{I} + \boldsymbol{H}_k \boldsymbol{S}_k \boldsymbol{H}_k^H\right| \\
\texttt{s.t.} \quad & \boldsymbol{H}_j \boldsymbol{S}_k \boldsymbol{H}_j^H = 0, \ \forall j \neq k \\
& \sum_{k=1}^{K} \mathrm{Tr}\left(\boldsymbol{B}_a \boldsymbol{S}_k\right) \leq P, \ \forall a \\
& \boldsymbol{S}_k \succeq \boldsymbol{0}, \ \forall k
\end{aligned}
$$

ZF constraints

per-BS power constraints

(PB) is convex, thus solvable by convex optimization techniques

Remove ZF Constraints (1)

Assume $M \geq NK$

- Denote $\boldsymbol{G}_k = [\boldsymbol{H}_1^T, \cdots, \boldsymbol{H}_{k-1}^T, \boldsymbol{H}_{k+1}^T, \cdots, \boldsymbol{H}_k^T]^T, k = 1, \cdots, K, \boldsymbol{G}_k \in \mathbb{C}^{L \times M}$ with $L = N(K-1)$.
- Denote the (reduced) singular value decomposition (SVD) of $\boldsymbol{G}_k$ as $\boldsymbol{G}_k = \boldsymbol{U}_k \boldsymbol{\Sigma}_k \boldsymbol{V}_k^H$.
- Define the projection matrix: $\boldsymbol{P}_k = (\boldsymbol{I} - \boldsymbol{V}_k \boldsymbol{V}_k^H)$.
- Rewrite $\boldsymbol{P}_k$ as $\boldsymbol{P}_k = \tilde{\boldsymbol{V}}_k \tilde{\boldsymbol{V}}_k^H$, $\tilde{\boldsymbol{V}}_k \in \mathbb{C}^{M \times (M-L)}$ with $\boldsymbol{V}_k^H \tilde{\boldsymbol{V}}_k = \boldsymbol{0}$.

Lemma 1: The optimal solution of (PB) has the following structure:

$$\boldsymbol{S}_k = \tilde{\boldsymbol{V}}_k \boldsymbol{Q}_k \tilde{\boldsymbol{V}}_k^H, \quad k = 1, \cdots, K$$

where $\boldsymbol{Q}_k \in \mathbb{C}^{(M-L) \times (M-L)}$ and $\boldsymbol{Q}_k \succeq \boldsymbol{0}$.

Remove ZF Constraints (2)

- **Using Lemma 1, (PB) is reduced to**

$$
\begin{aligned}
(\mathrm{PB}-1): \quad \max_{\boldsymbol{Q}_1,\cdots,\boldsymbol{Q}_K}. \quad & \sum_{k=1}^{K} w_k \log \left| \boldsymbol{I} + \boldsymbol{H}_k \tilde{\boldsymbol{V}}_k \boldsymbol{Q}_k \tilde{\boldsymbol{V}}_k^H \boldsymbol{H}_k^H \right| \\
\text{s.t.} \quad & \sum_{k=1}^{K} \mathrm{Tr}\left(\boldsymbol{B}_a \tilde{\boldsymbol{V}}_k \boldsymbol{Q}_k \tilde{\boldsymbol{V}}_k^H \right) \le P, \ \forall a \\
& \boldsymbol{Q}_k \succeq \boldsymbol{0}, \ \forall k
\end{aligned}
$$

generalized linear transmit power constraint

(PB-1) is convex, thus solvable by Lagrange duality method

✓ (PB-1) has the same structure as CR point-to-point MIMO optimization if *K*=1

Algorithm for Solving (PB-1)

- Introduce a set of dual variables for (PB-1), $\mu_1, \ldots, \mu_A$, corresponding to individual per-BS power constraints.
- Denote $\boldsymbol{B}_\mu = \sum_{a=1}^{A} \mu_a \boldsymbol{B}_a$.
- Apply the following SVD: $\boldsymbol{H}_k \tilde{\boldsymbol{V}}_k (\tilde{\boldsymbol{V}}_k^H \boldsymbol{B}_\mu \tilde{\boldsymbol{V}}_k)^{-1/2} = \hat{\boldsymbol{U}}_k \hat{\boldsymbol{\Sigma}}_k \hat{\boldsymbol{V}}_k^H$.
- Denote $\hat{\boldsymbol{\Sigma}}_k = \mathrm{Diag}(\hat{\sigma}_{k,1}, \cdots, \hat{\sigma}_{k,N})$.
- Obtain $\boldsymbol{\Lambda}_k = \mathrm{Diag}(\lambda_{k,1}, \cdots, \lambda_{k,N})$, $\lambda_{k,i} = \left(w_k - \frac{1}{\hat{\sigma}_{k,i}^2}\right)^+, i = 1, \ldots, N$, with $(x)^+ \triangleq \max(0, x)$.

Lemma 2: The optimal solution of (PB-1) is give by

$$\boldsymbol{Q}_k^\star = (\tilde{\boldsymbol{V}}_k^H \boldsymbol{B}_\mu \tilde{\boldsymbol{V}}_k)^{-1/2} \hat{\boldsymbol{V}}_k \boldsymbol{\Lambda}_k \hat{\boldsymbol{V}}_k^H (\tilde{\boldsymbol{V}}_k^H \boldsymbol{B}_\mu \tilde{\boldsymbol{V}}_k)^{-1/2}, \ k = 1, \cdots, K.$$

(PB-1) is solvable by an iterative inter-outer loop algorithm, similarly as (PA)

Optimal BD Precoding Matrix

- **Combining Lemmas 1 & 2 yields**

Theorem: The optimal solution of (PB) is given by

$$\boldsymbol{S}_k^\star = \tilde{\boldsymbol{V}}_k(\tilde{\boldsymbol{V}}_k^H \boldsymbol{B}_\mu^\star \tilde{\boldsymbol{V}}_k)^{-1/2}\hat{\boldsymbol{V}}_k\boldsymbol{\Lambda}_k\hat{\boldsymbol{V}}_k^H(\tilde{\boldsymbol{V}}_k^H \boldsymbol{B}_\mu^\star \tilde{\boldsymbol{V}}_k)^{-1/2}\tilde{\boldsymbol{V}}_k^H, \;\; k = 1, \cdots, K$$

where $\boldsymbol{B}_\mu^\star = \sum_{a=1}^{A} \mu_a^\star \boldsymbol{B}_a$.

Corollary: The optimal BD precoding matrix is given by

$$\boldsymbol{T}_k^\star = \tilde{\boldsymbol{V}}_k(\tilde{\boldsymbol{V}}_k^H \boldsymbol{B}_\mu^\star \tilde{\boldsymbol{V}}_k)^{-1/2}\hat{\boldsymbol{V}}_k\boldsymbol{\Lambda}_k^{1/2}, \;\; k = 1, \ldots, K.$$

Optimal precoding vectors for each user are non-orthogonal

Properties of Optimal BD Precoding

- **Channel diagonalization:**

$$\hat{\boldsymbol{U}}_k^H \boldsymbol{H}_k \boldsymbol{T}_k^\star = \hat{\boldsymbol{\Sigma}}_k \boldsymbol{\Lambda}_k^{1/2}$$

Linear (non-orthogonal) precoders achieve per-user MIMO capacity

- **Precoding matrix in traditional sum-power constraint case:**

$$\boldsymbol{T}_k^{\star\star} = \frac{1}{\sqrt{\mu^\star}} \tilde{\boldsymbol{V}}_k \hat{\boldsymbol{V}}_k \boldsymbol{\Lambda}_k^{1/2}$$

Linear (orthogonal) precoding vectors for each user are optimal

Special Case: MISO-BC with Per-Antenna Power Constraints

- **Optimal ZF precoding vector:**

$$\boxed{\boldsymbol{t}_k^\star = \lambda_k^{1/2}\hat{\sigma}_k^{-1}\tilde{\boldsymbol{V}}_k(\tilde{\boldsymbol{V}}_k^H\boldsymbol{B}_\mu^\star\tilde{\boldsymbol{V}}_k)^{-1}\tilde{\boldsymbol{V}}_k^H\boldsymbol{h}_k,\ \ k=1,\ldots,K}$$

✓ can be shown equivalent to generalized channel inverse [WieselEldarShamai08]

- **Sum-power constraint case:**

$$\boxed{\boldsymbol{t}_k^{\star\star} = \lambda_k^{1/2}\hat{\sigma}_k^{-1}(\mu^\star)^{-1}\tilde{\boldsymbol{V}}_k\tilde{\boldsymbol{V}}_k^H\boldsymbol{h}_k,\ \ k=1,\ldots,K}$$

✓ can be shown equivalent to channel pseudo inverse

Separation Approach (suboptimal)

- **First, apply "orthogonal" BD precoders for the sum-power constraint case:**

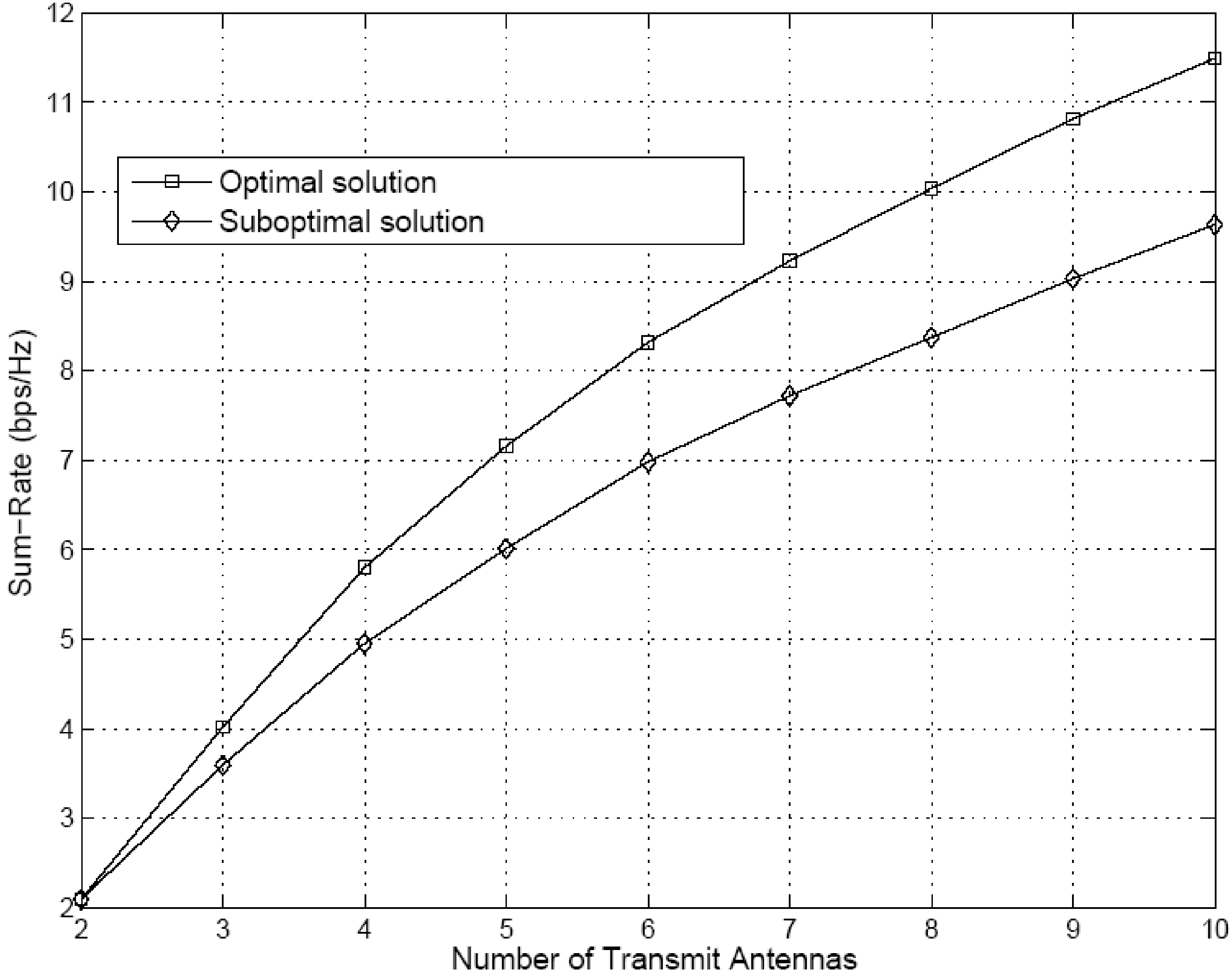
$$\boxed{\bar{\boldsymbol{S}}_k = \boldsymbol{V}_k^{\perp} \bar{\boldsymbol{\Lambda}}_k (\boldsymbol{V}_k^{\perp})^H}$$

with $\boldsymbol{H}_k \boldsymbol{P}_k = \boldsymbol{U}_k^{\perp} \boldsymbol{\Sigma}_k^{\perp} (\boldsymbol{V}_k^{\perp})^H, k = 1, \ldots, K.$

- **Second, optimize power allocation for WSRMax under per-BS power constraints:**

$$\boxed{\bar{\lambda}_{k,i} = \left(\frac{w_k}{\sum_{a=1}^{A} \mu_a \|\boldsymbol{v}_k^{\perp}[a,i]\|^2} - \frac{1}{(\sigma_{k,i}^{\perp})^2} \right)^+}$$

Two-User MISO-BC with Per-Antenna Power Constraints (ZF Precoding)



Summary of Part I

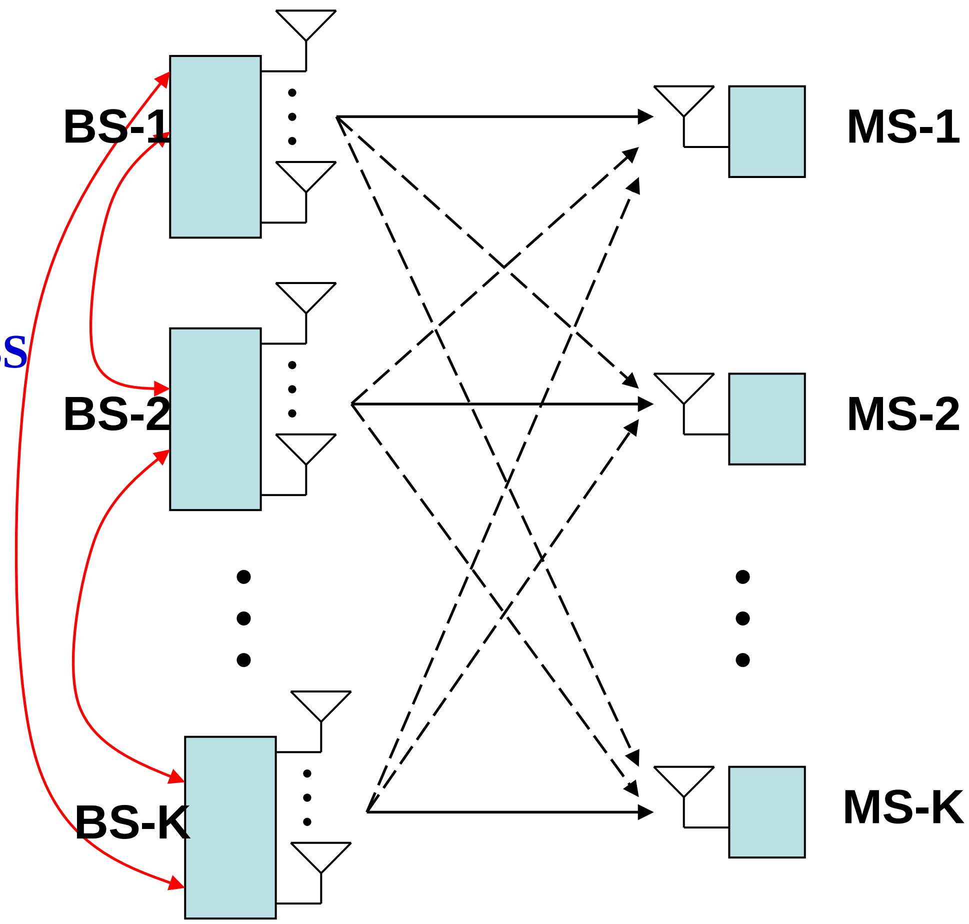
- **Network MIMO Optimization**
  - –WSRMax for MIMO-/MISO-BC with **linear** (per-BS, per-antenna, sum-antenna) power constraints
  - –Nonlinear DPC precoding
    - **Generalized MAC-BC duality**
  - –Linear ZF/BD precoding
    - **Joint precoder and power optimization**

Part II: Optimal Coordinated Downlink Beamforming

System Model of Coordinated Downlink Beamforming

<u>Assumptions:</u>

- limited-rate backhaul links
- local transmit message at each BS
- one active user per cell (w.l.o.g.)
- ICI treated as Gaussian noise



MISO-IC with partial transmitter-side cooperation

Related Work on Gaussian Interference Channel (selected)

➢ Information-Theoretic Approach

- ❑ Capacity region unknown in general
- ❑ Best known achievability scheme: [HanKobayashi81]
- ❑ Capacity within 1-bit: [EtkinTseWang08]

➢ Pragmatic Approach (interference treated as Gaussian noise)

- ❑ Interference alignment [Jafar *et al.*]
  - DoF optimality at asymptotically high SNR
  - New ingredients: *improper complex Gaussian signaling, time symbol extension, non-separability of parallel Gaussian ICs*
- ❑ MISO-IC (finite-SNR regime, proper complex Gaussian signaling, no time symbol extension)
  - Achievable rate region characterization [JorswieckLarssonDanev08], [ZakhourGesbert09]
  - Power minimization with SINR constraints [DahroujYu10]
  - Optimality of beamforming (rank-one transmit covariance matrix) [ShangChenPoor11]
- ❑ WSRMax via "Monotonic Optimization"
  - SISO-IC ("Mapel" algorithm) [QianZhangHuang09]
  - MISO-IC [JorswieckLarsson10], [UtschickBrehmer12], [BjornsonZhengBengtssonOttersten12]
- ❑ WSRMax for MIMO-IC
  - [PetersHeath10], [RazaviyaynSanjabiLuo12]….

Channel Model

- **MISO-IC baseband signal model:**

$$\boxed{y_k = \boldsymbol{h}_{kk}^H \boldsymbol{x}_k + \sum_{j \neq k}^{K} \boldsymbol{h}_{jk}^H \boldsymbol{x}_j + z_k, \quad k = 1, \ldots, K}$$

- y_k: received signal at the kth MS
- $\boldsymbol{x}_k \in \mathbb{C}^{M_k \times 1}$: transmitted signal from the kth BS, $M_k \geq 1$
- $\boldsymbol{h}_{kk}^H \in \mathbb{C}^{1 \times M_k}$: direct-link channel for the kth BS-MS pair
- $\boldsymbol{h}_{jk}^H \in \mathbb{C}^{1 \times M_j}$: cross-link channel from the jth BS to kth MS, $j \neq k$
- z_k: receiver noise at the the kth MS, $z_k \sim \mathcal{CN}(0, \sigma_k^2)$
- $\boldsymbol{x}_k$'s are independent over k: no message sharing among BSs
- $\boxed{\boldsymbol{S}_k \triangleq \mathbb{E}[\boldsymbol{x}_k \boldsymbol{x}_k^H]}$: transmit covariance matrix for the kth BS, $\boldsymbol{S}_k \succeq 0$

Assumed proper/circularly-symmetric complex Gaussian signaling (for the time being)

Pareto Optimal Rates in MISO-IC

- Achievable user rate (with interference treated as noise):

$$R_k(\boldsymbol{S}_1, \ldots, \boldsymbol{S}_K) = \log\left(1 + \frac{\boldsymbol{h}_{kk}^H \boldsymbol{S}_k \boldsymbol{h}_{kk}}{\sum_{j \neq k} \boldsymbol{h}_{jk}^H \boldsymbol{S}_j \boldsymbol{h}_{jk} + \sigma_k^2}\right), \quad k = 1, \ldots, K$$

- Achievable rate region (without time sharing):

$$\mathcal{R} \triangleq \bigcup_{\{\boldsymbol{S}_k\}:\mathrm{Tr}(\boldsymbol{S}_k)\leq P_k, \forall k} \left\{(r_1, \ldots, r_K) : 0 \leq r_k \leq R_k(\boldsymbol{S}_1, \ldots, \boldsymbol{S}_K), k = 1, \ldots, K\right\}$$

- Pareto rate optimality:

Definition: A rate-tuple $(r_1, \ldots, r_K)$ is *Pareto optimal* if there is no other rate-tuple $(r'_1, \ldots, r'_K)$ with $(r'_1, \ldots, r'_K) \geq (r_1, \ldots, r_K)$ and $(r'_1, \ldots, r'_K) \neq (r_1, \ldots, r_K)$ (the inequality is component-wise).

WSRMax for MISO-IC

$$
\begin{aligned}
(\text{WSRMax}):\ \max_{\boldsymbol{S}_1,\cdots,\boldsymbol{S}_K}\quad & \sum_{k=1}^{K} w_k \log\left(1+\frac{\boldsymbol{h}_{kk}^H \boldsymbol{S}_k \boldsymbol{h}_{kk}}{\sum_{j\neq k}\boldsymbol{h}_{jk}^H \boldsymbol{S}_j \boldsymbol{h}_{jk}+\sigma_k^2}\right)\\
\text{s.t.}\quad & \operatorname{Tr}(\boldsymbol{S}_k)\le P_k,\ \forall k\\
& \boldsymbol{S}_k \succeq \boldsymbol{0},\ \forall k
\end{aligned}
$$

Non-convex problem, thus cannot be solved directly by convex optimization techniques

SINR Feasibility Problem

Assuming transmit beamforming *i.e.* $\boldsymbol{S}_k = \boldsymbol{v}_k \boldsymbol{v}_k^H, \forall k$

$$
\begin{aligned}
(\mathrm{SINR-Feas.}): \quad & \mathtt{find} \quad \{\boldsymbol{v}_k\} \\
& \mathtt{s.t.} \quad \frac{1}{\bar{\gamma}_k} \|\boldsymbol{h}_{kk}^H \boldsymbol{v}_k\|^2 \geq \sum_{j \neq k} \|\boldsymbol{h}_{jk}^H \boldsymbol{v}_j\|^2 + \sigma_k^2, \quad \forall k \\
& \|\boldsymbol{v}_k\|^2 \leq P_k, \forall k
\end{aligned}
$$

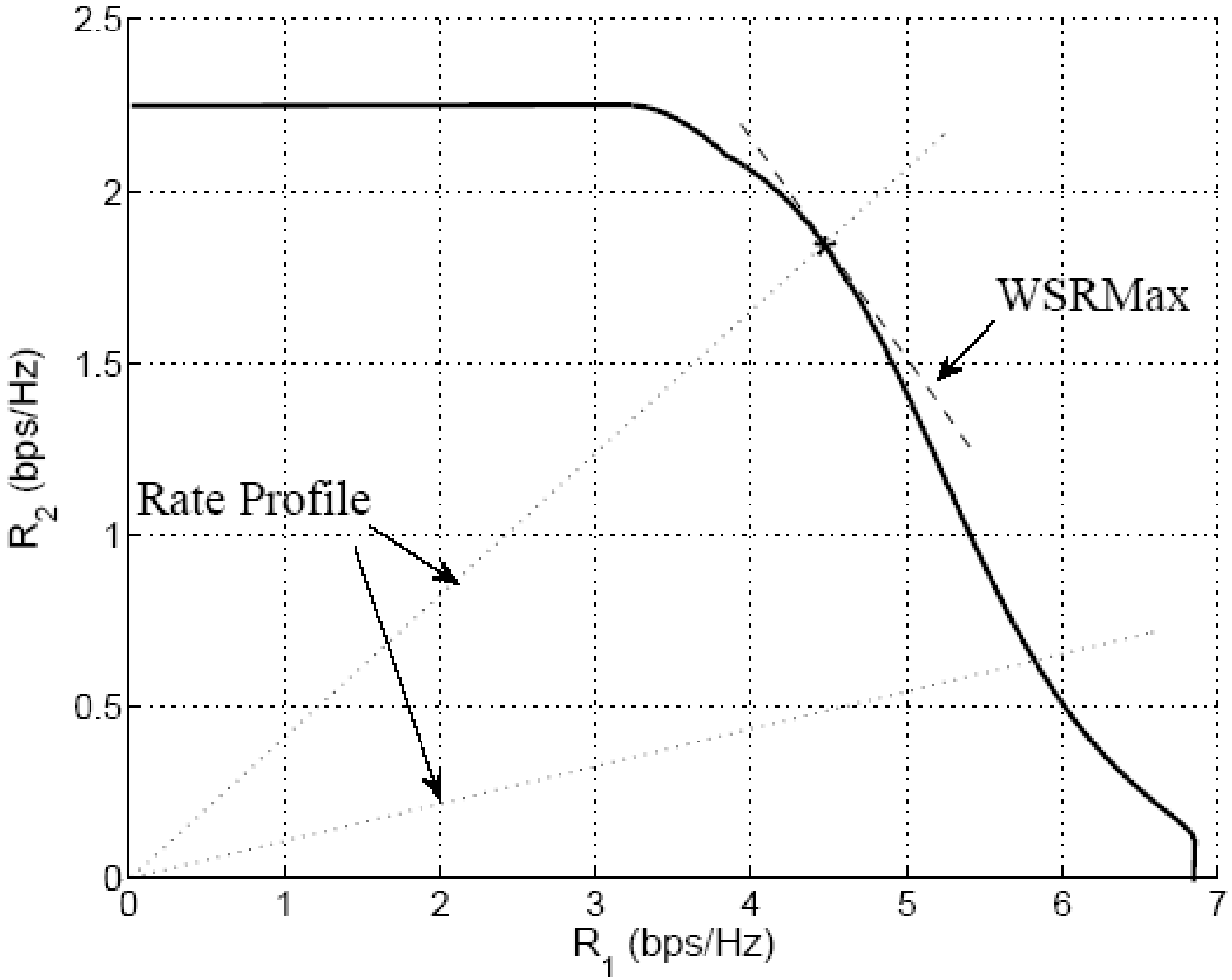
Convex problem, can be solved efficiently via convex Second Order Cone Programming (SOCP) feasibility problem

Question: Can we solve WSRMax via SINR-Feas. problem for ICs?

[LiuZhangChua12]: L. Liu, R. Zhang, and K. C. Chua, "Achieving global optimality for weighted sum-rate maximization in the K-user Gaussian interference channel with multiple antennas," *IEEE Transactions on Wireless Communications*, May 2012. (also see [UtschickBrehmer12], [BjornsonZhengBengtssonOttersten12])

Rate-Profile Approach

Two-User MISO-IC



Sum-Rate Maximization with Rate-Profile Constraints [ZhangCui10]

Given a rate-profile vector $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_K] \succeq 0$, $\sum_{k=1}^{K} \alpha_k = 1$

$$\begin{aligned} \max_{R_{\text{sum}}, \{\boldsymbol{w}_k\}} \quad & R_{\text{sum}} \\ \text{s.t.} \quad & \log\left(1 + \gamma_k(\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K)\right) \geq \alpha_k R_{\text{sum}}, \quad \forall k \\ & \|\boldsymbol{w}_k\|^2 \leq P_k, \quad \forall k \end{aligned}$$

$$\begin{aligned} \text{find} \quad & \{\boldsymbol{w}_k\} \\ \text{s.t.} \quad & \log\left(1 + \gamma_k(\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K)\right) \geq \alpha_k r_{\text{sum}}, \quad \forall k \\ & \|\boldsymbol{w}_k\|^2 \leq P_k, \quad \forall k \end{aligned}$$

SINR-Feas. Problem

Non-convex problem, but efficiently solvable via a sequence of convex SINR-Feas. problems

[ZhangCui10]: R. Zhang and S. Cui, "Cooperative interference management with MISO beamforming," *IEEE Transactions on Signal Processing*, Oct. 2010.

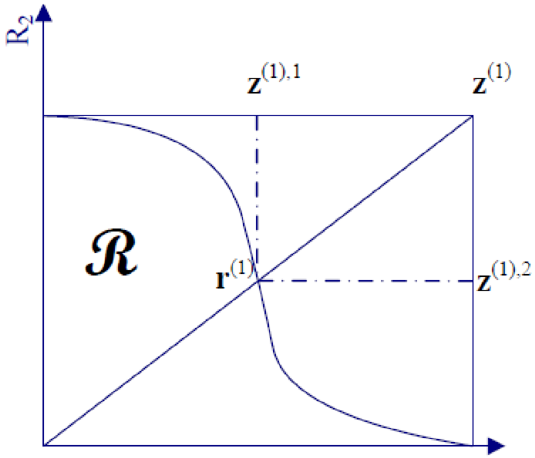
Monotonic Optimization

Key observation: Maximize WSR in MISO-IC rate region directly!

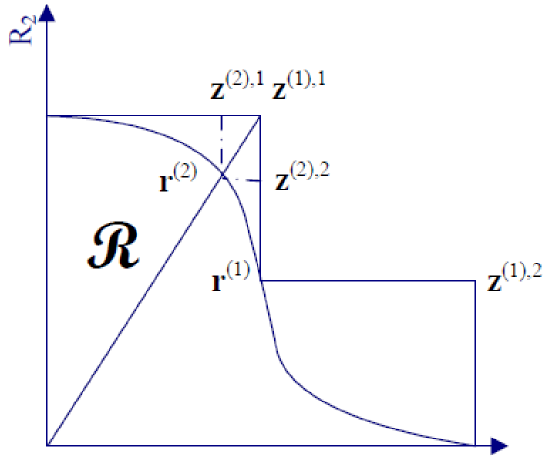
$$\text{(WSRMax)}: \quad \max_{\boldsymbol{r}=[R_1,\ldots,R_K]} \; U(\boldsymbol{r}) := \sum_{k=1}^{K} \mu_k R_k \qquad \text{s.t.} \quad \boldsymbol{r} \in \mathcal{R}$$

Monotonic optimization problem (maximizing a strictly increasing function over a "normal" set), thus solvable by *e.g.* the "outer polyblock approximation" algorithm (shown in next slide)

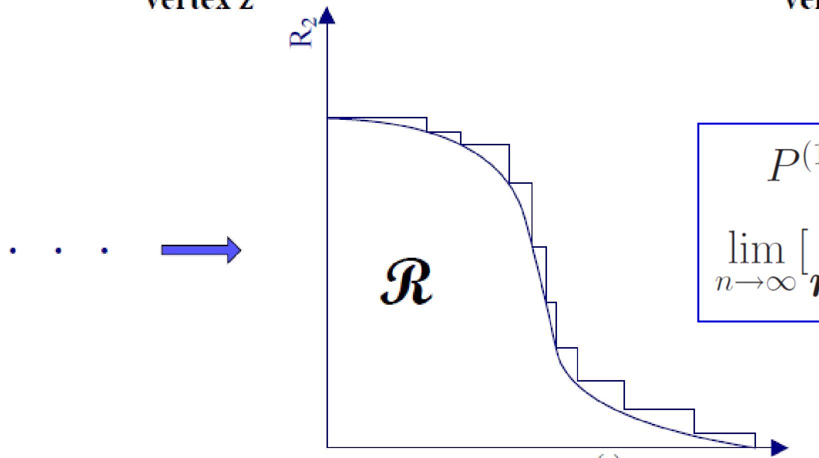
Outer Polyblock Approximation



The polyblock $P^{(1)}$ with vertex $z^{(1)}$



The polyblock $P^{(2)}$ with vertices $z^{(1),1}$, $z^{(1),2}$



The polyblock $P^{(n)}$ which approaches the rate region with increasing n

$$P^{(1)} \supset P^{(2)} \supset \cdots \supset \mathcal{R},$$

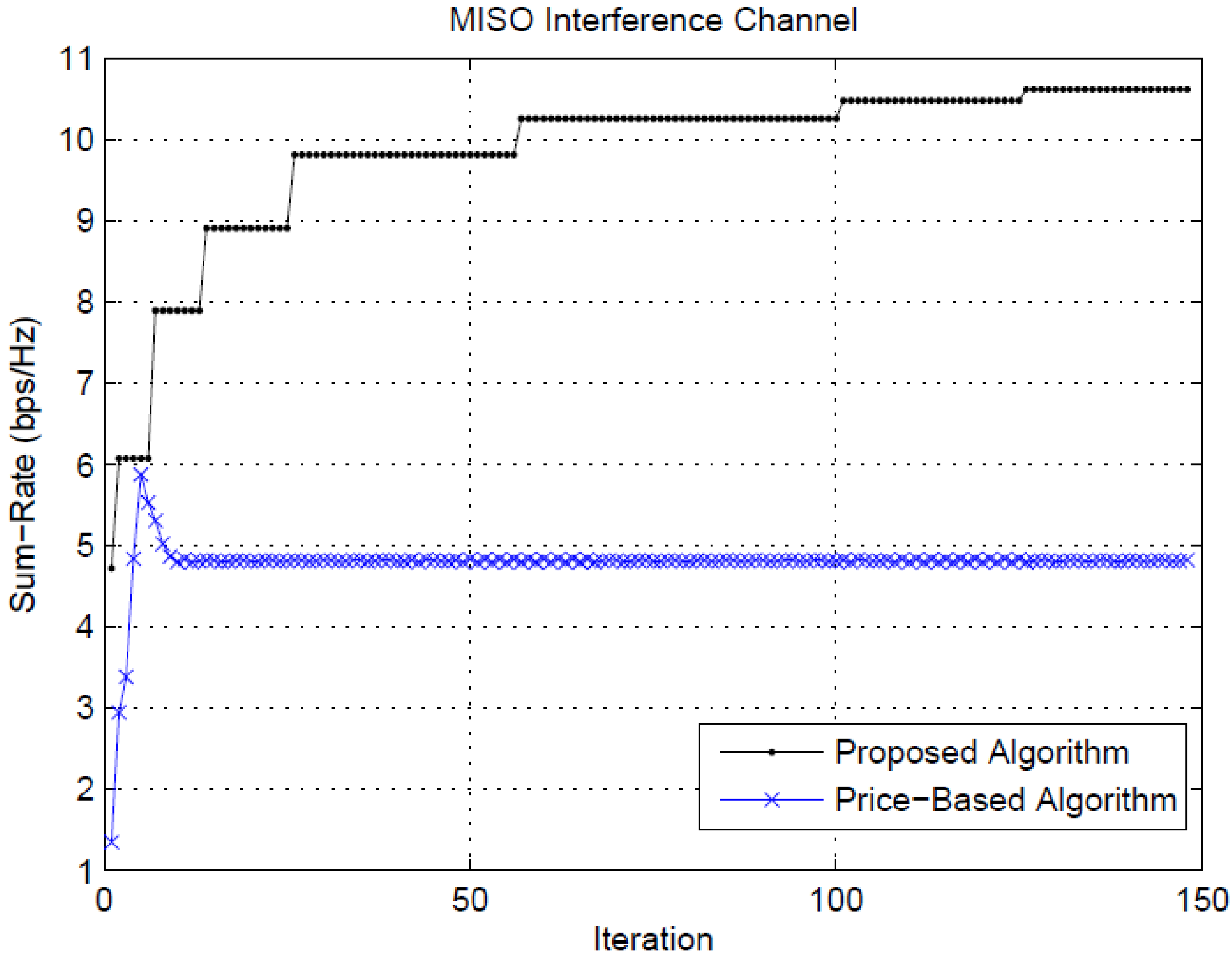
$$\lim_{n\to\infty}\left[\max_{\boldsymbol{r}\in P^{(n)}} U(\boldsymbol{r})\right] = \max_{\boldsymbol{r}\in\mathcal{R}} U(\boldsymbol{r})$$

- Guaranteed convergence
- Controllable accuracy
- Complexity: ???
- **Key step** in each iteration: Find intersection point with Pareto boundary given a rate profile, which is solved by Sum-Rate Maximization with Rate-Profile Constraints

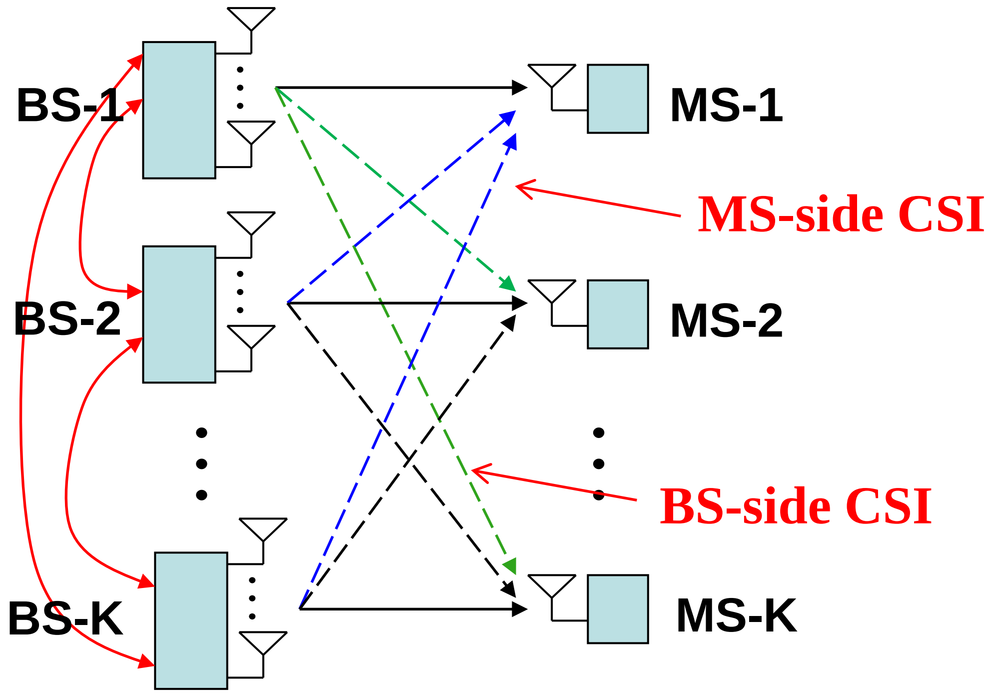
❖ Rate Profile + Monotonic Optimization solves WSRMax for MISO-IC

Numerical Example

- Benchmark scheme: “price-based” algorithm [Schmidt *et al.*09]
- MISO-IC: $M_k=2$, $K=4$, *i.i.d. Rayleigh fading*, $SNR_k=3$, $w_k=1$



Distributed Beamforming for MISO-IC



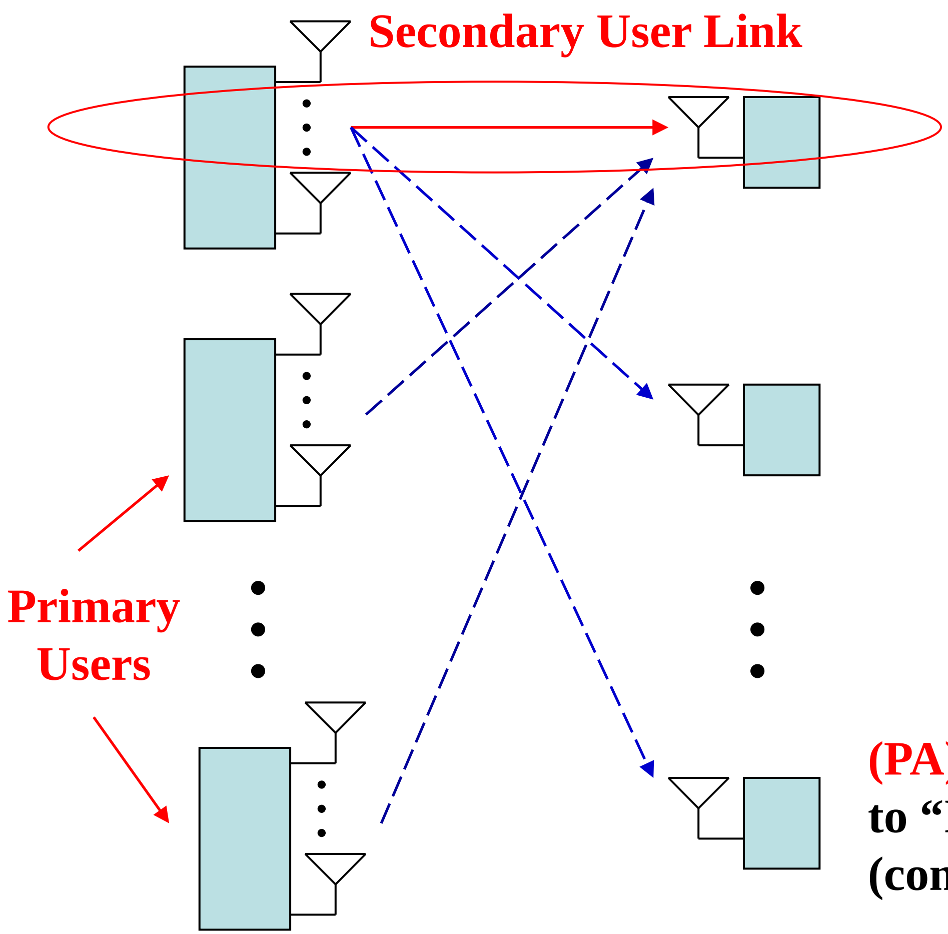
- **Distributed Algorithms for Coordinated Downlink Beamforming**
  - low-rate information exchange across BSs
  - only "local" (BS-side or MS-side) channel knowledge available at each BS

Question: Can we archive distributed (Pareto-rate) optimal beamforming?

[ZhangCui10]: R. Zhang and S. Cui, "Cooperative interference management with MISO beamforming," *IEEE Transactions on Signal Processing*, Oct. 2010. (with BS-side CSI)

[QiuZhangLuoCui11]: J. Qiu, R. Zhang, Z.-Q. Luo, and S. Cui, "Optimal distributed beamforming for MISO interference channels," *IEEE Transactions on Signal Processing*, Nov. 2011. (with MS-side CSI)

Exploiting Relationship between MISO-IC and MISO CR Channel [ZhangCui10]



- **For the *kth* MISO CR link:**

$$C_k(\Gamma_k) := \max_{S_k} . \ \log\left(1 + \frac{h_{kk}^H S_k h_{kk}}{\sum_{j\neq k}\Gamma_{jk} + \sigma_k^2}\right)$$

$$\text{s.t.} \ h_{kj}^H S_k h_{kj} \leq \Gamma_{kj}, \ \forall j \neq k$$

$$\mathrm{Tr}(S_k) \leq P_k, \ S_k \succeq 0$$

Interference power to other MSs

Interference power from other BSs

(PA): "Cognitive beamforming (CB)" problem subject to "Interference Temperature (IT)" constraints (considered in Topic #1)

[ZhangCui10]: R. Zhang and S. Cui, "Cooperative interference management with MISO beamforming," *IEEE Transactions on Signal Processing*, Oct. 2010.

Optimal Cognitive Beamforming (CB)

Theorem: The optimal solution for $\boldsymbol{S}_k$ in (PA) is $\boxed{\textit{rank-one}}$, i.e., $\boldsymbol{S}_k = \boldsymbol{w}_k \boldsymbol{w}_k^H$, and

$$\boldsymbol{w}_k = \left(\sum_{j \neq k} \lambda_{kj} \boldsymbol{h}_{kj} \boldsymbol{h}_{kj}^H + \lambda_{kk} \boldsymbol{I} \right)^{-1} \boldsymbol{h}_{kk} \sqrt{p_k}$$

where λ_{kj}, $j \neq k$, and λ_{kk} are non-negative constants (solutions for the dual problem of (PA)); and p_k is given by

$$p_k = \left(\frac{1}{\ln 2} - \frac{\sum_{j \neq k} \Gamma_{jk} + \sigma_k^2}{\|\boldsymbol{A}_k \boldsymbol{h}_{kk}\|^2} \right)^+ \frac{1}{\|\boldsymbol{A}_k \boldsymbol{h}_{kk}\|^2}$$

where $\boldsymbol{A}_k \triangleq \left(\sum_{j \neq k} \lambda_{kj} \boldsymbol{h}_{kj} \boldsymbol{h}_{kj}^H + \lambda_{kk} \boldsymbol{I} \right)^{-1/2}$ and $(x)^+ \triangleq \max(0, x)$.

A semi-closed-form solution, which is efficiently solvable by an iterative inner-outer-loop algorithm

Interference Temperature (IT) Approach to Characterize MISO-IC Pareto Boundary

Proposition: For any rate-tuple $(R_1, \ldots, R_K)$ on the Pareto boundary of the MISO-IC rate region, which is achievable with a set of transmit covariance matrices, $\boldsymbol{S}_1, \ldots, \boldsymbol{S}_K$, there is a corresponding interference-power/interferecne-temperature constraint vector, $\boldsymbol{\Gamma} \geq 0$, with $\Gamma_{kj} = \boldsymbol{h}_{kj}^H \boldsymbol{S}_k \boldsymbol{h}_{kj}, \forall j \neq k, j \in \{1, \ldots, K\}$, and $k \in \{1, \ldots, K\}$, such that $R_k = C_k(\boldsymbol{\Gamma}_k), \forall k$, and $\boldsymbol{S}_k$ is the optimal solution of (PA) for the given k.

- ❑A new parametrical characterization of MISO-IC Pareto boundary in terms of BSs' mutual IT levels, which constitute a lower-dimensional manifold than original transmit covariance matrices

- ❑ Optimality of beamforming for MISO-IC is proved
  (see an alternative proof given by [ShangChenPoor11])

Necessary Condition of Pareto Optimality

Theorem: For an arbitrarily chosen $\boldsymbol{\Gamma} = [\boldsymbol{\Gamma}_1, \ldots, \boldsymbol{\Gamma}_K] \geq 0$, if the optimal rate values for all k's, $C_k(\boldsymbol{\Gamma}_k)$'s, are Pareto-optimal on the boundary of the MISO-IC rate region, then for any pair of $(i, j), i \in \{1, \ldots, K\}, j \in \{1, \ldots, K\}$, and $i \neq j$, it must hold that $|\boldsymbol{D}_{ij}| = 0$, where

$$\boldsymbol{D}_{ij} = \begin{bmatrix} \frac{\partial C_i(\boldsymbol{\Gamma}_i)}{\partial \Gamma_{ij}} & \frac{\partial C_i(\boldsymbol{\Gamma}_i)}{\partial \Gamma_{ji}} \\ \frac{\partial C_j(\boldsymbol{\Gamma}_j)}{\partial \Gamma_{ij}} & \frac{\partial C_j(\boldsymbol{\Gamma}_j)}{\partial \Gamma_{ji}} \end{bmatrix} := \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

where

$$\frac{\partial C_i(\boldsymbol{\Gamma}_i)}{\partial \Gamma_{ij}} = \lambda_{ij}$$

$$\frac{\partial C_i(\boldsymbol{\Gamma}_i)}{\partial \Gamma_{ji}} = \frac{-\boldsymbol{h}_{ii}^H \boldsymbol{S}_i^\star \boldsymbol{h}_{ii}}{\ln 2(\sum_{l \neq i} \Gamma_{li} + \sigma_i^2)(\sum_{l \neq i} \Gamma_{li} + \sigma_i^2 + \boldsymbol{h}_{ii}^H \boldsymbol{S}_i^\star \boldsymbol{h}_{ii})}$$

Optimal Distributed Beamforming based on CB and "Active IT Control"

- **BS pair-wise IT update:**

$$\boxed{[\Gamma_{ij}, \Gamma_{ji}]^T \leftarrow [\Gamma_{ij}, \Gamma_{ji}]^T + \delta_{ij} \cdot \boldsymbol{d}_{ij}}$$

step size

fairness control

where $\boldsymbol{d}_{ij} = \mathrm{sign}(ad - bc) \cdot [\alpha_{ij} d - b, a - \alpha_{ij} c]^T$

- **Distributed algorithm for coordinated downlink beamforming:**

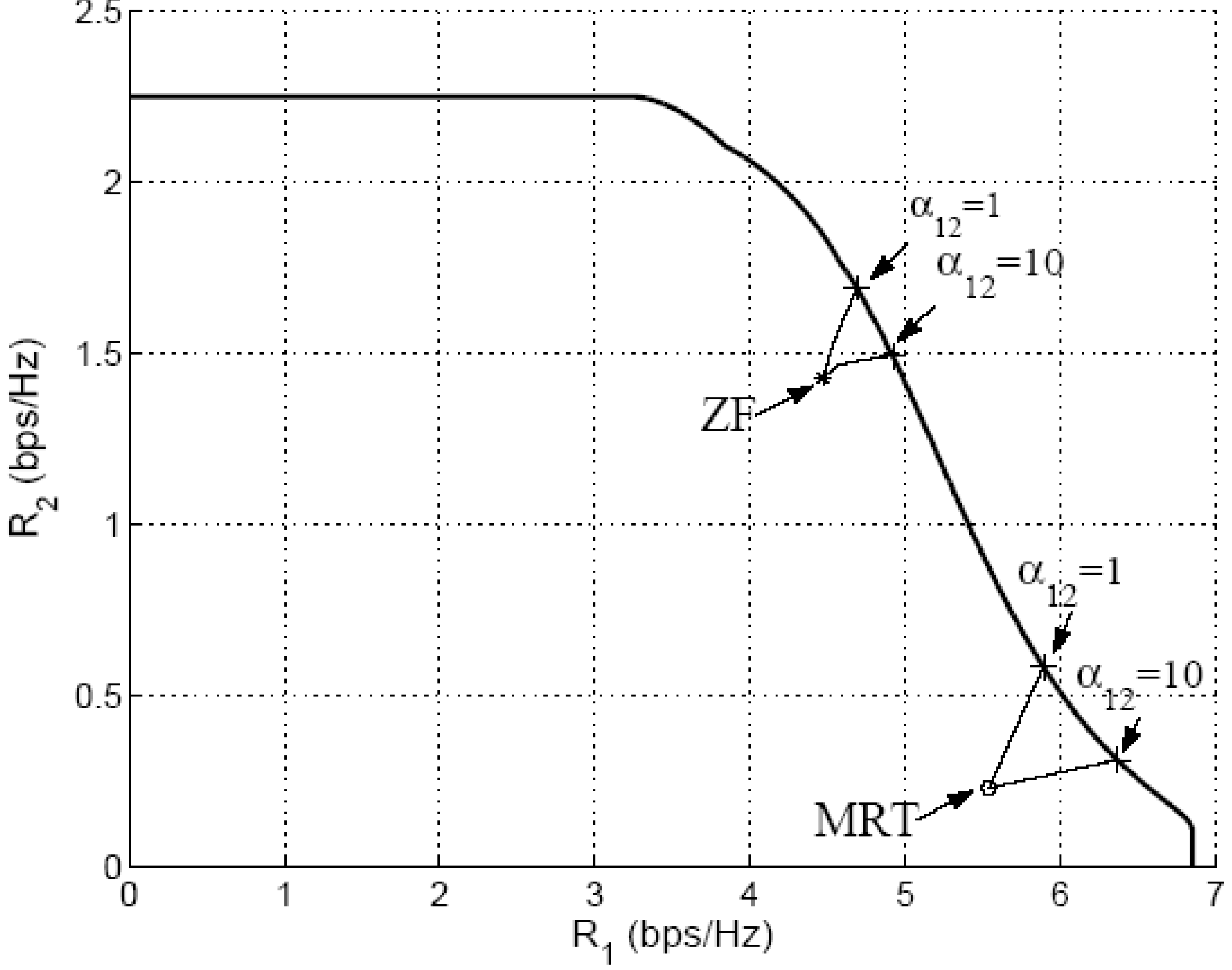
exchange two scalars per update

Initialize $\boldsymbol{\Gamma} \geq 0$ in the network
BS k sets $\boldsymbol{w}_k$ by solving (PA) with the given $\boldsymbol{\Gamma}_k$, $k = 1, \ldots, K$
Repeat
For $i = 1, \ldots, K$, $j = 1, \ldots, K$, $j \neq i$,
BS i computes a and b in $\boldsymbol{D}_{ij}$ with the given $\boldsymbol{\Gamma}_i$
BS j computes d and c in $\boldsymbol{D}_{ij}$ with the given $\boldsymbol{\Gamma}_j$
BS i sends a and b to BS j
BS j sends c and d to BS i
BS i (j) computes $\boldsymbol{d}_{ij}$, and updates Γ_{ij} and Γ_{ji}
BS i (j) resets $\boldsymbol{w}_i$ ($\boldsymbol{w}_j$) by solving (PA) with the updated $\boldsymbol{\Gamma}_i$ ($\boldsymbol{\Gamma}_j$)
End For
Until $|\boldsymbol{D}_{ij}| = 0, \forall i \neq j$.

Numerical Example

- MISO-IC: $M_1 = M_2 = 3$, $K=2$, *i.i.d. Rayleigh fading*, $SNR_1 = 5$, $SNR_2 = 1$

Two-User MISO-IC



Distributed Beamforming via Alternating or Cyclic Projection [QiuZhangLuoCui11]

- **Recall SINR feasibility problem:**

SINR target of MS i

$$\begin{aligned} \max_{\{\omega_i\}} \quad & 0 \\ s.t. \quad & \left\|\boldsymbol{h}_{ii}^H \boldsymbol{\omega}_i\right\|^2 \geq \beta_i \left(\sum_{j=1, j\neq i}^{M} \left|\boldsymbol{h}_{ji}^H \boldsymbol{\omega}_j\right|^2 + \sigma_i^2 \right), \ i = 1, \ldots, M, \\ & \|\boldsymbol{\omega}_j\| \leq \sqrt{P_j}, \ j = 1, \ldots, M. \end{aligned}$$

- **Problem reformulated as (solvable by centralized SOCP):**

$$\begin{aligned} \max_{\boldsymbol{x}} \quad & 0 \\ s.t. \quad & \sqrt{\beta_i} \left\|\boldsymbol{A}_i \boldsymbol{x} + \boldsymbol{n}_i\right\| \leq \sqrt{1+\beta_i} \left(\boldsymbol{h}_{ii}^H \boldsymbol{S}_i \boldsymbol{x} \right), \ i = 1, \ldots, M, \\ & \boldsymbol{p}^T \boldsymbol{x} = 0, \\ & \|\boldsymbol{S}_j \boldsymbol{x}\| \leq \sqrt{P_j}, \ j = 1, \ldots, M. \end{aligned}$$

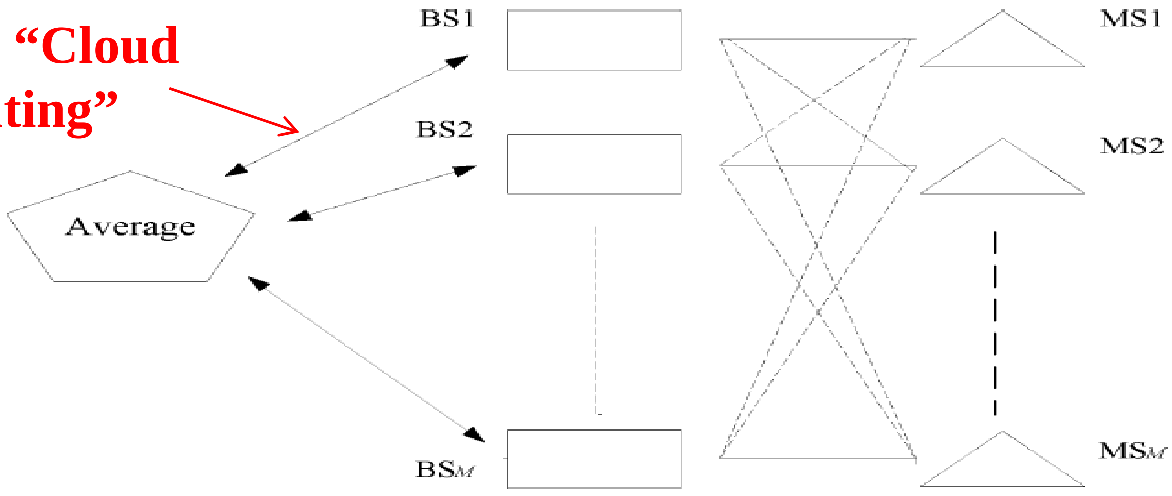
$\boldsymbol{x} = [\boldsymbol{\omega}_1; \boldsymbol{\omega}_2; \cdots; \boldsymbol{\omega}_M; 0]$

Question: Can we solve SINR feasibility problem in a distributed way?

[QiuZhangLuoCui11]: J. Qiu, R. Zhang, Z.-Q. Luo, and S. Cui, "Optimal distributed beamforming for MISO interference channels," *IEEE Transactions on Signal Processing*, Nov. 2011.

Alternating Projection

Similar to "Cloud Computing"



- **Distributed beamforming computation at each BS (via SOCP):**

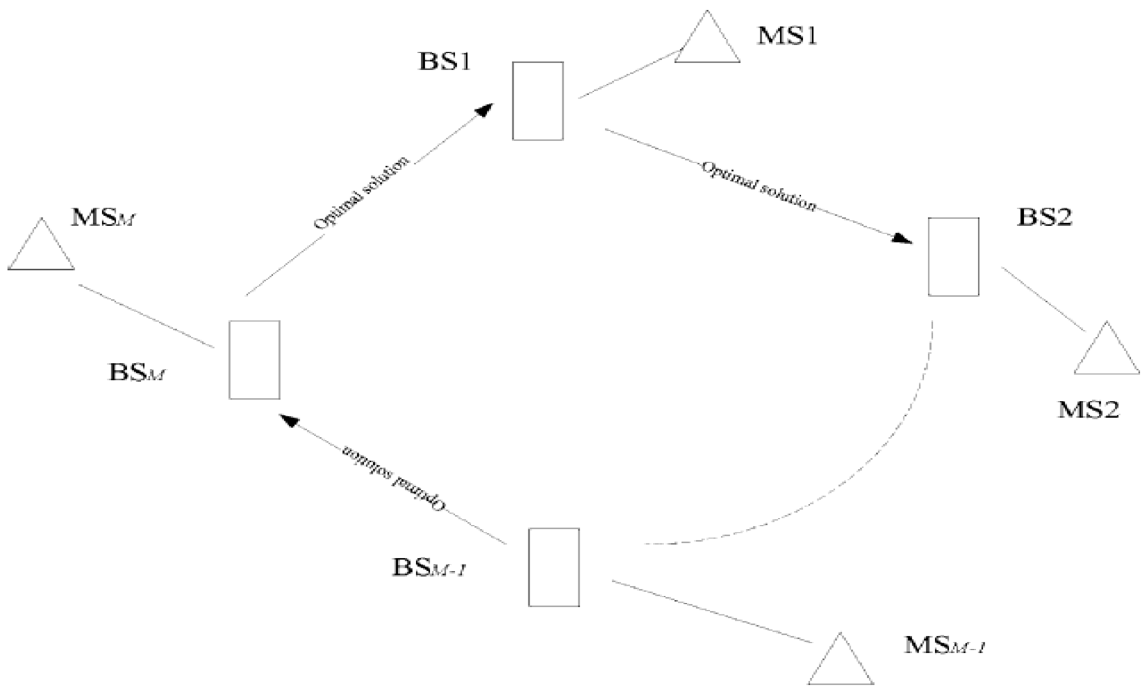
$$\boxed{\begin{aligned} \min_{\boldsymbol{x}} \quad & \|\boldsymbol{x} - \tilde{\boldsymbol{x}}_{n-1}\| \\ s.t. \quad & \sqrt{\beta_i}\|\boldsymbol{A}_i\boldsymbol{x} + \boldsymbol{n}_i\| \leq \sqrt{1+\beta_i}(\boldsymbol{h}_{ii}^H\boldsymbol{S}_i\boldsymbol{x}), \\ & \boldsymbol{p}^T\boldsymbol{x} = 0, \\ & \|\boldsymbol{S}_j\boldsymbol{x}\| \leq \sqrt{P_j}, j = 1, \ldots, M. \end{aligned}}$$

local SINR constraint for MS i only

- **Average operation at a central computer:**

$$\boxed{\tilde{\boldsymbol{x}}_{n-1} = \frac{1}{M}\sum_{i=1}^{M}\boldsymbol{x}_{n-1}^{(i)}}$$

Cyclic Projection



- **Cyclic beamforming computation at each BS (via SOCP):**

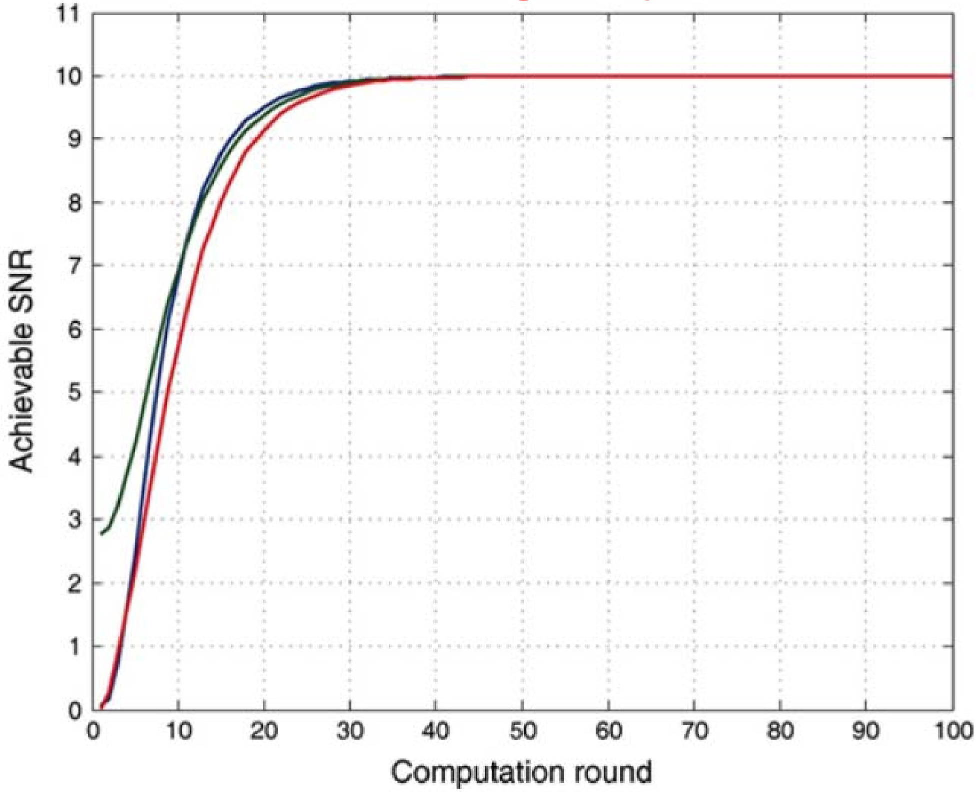
$$
\begin{aligned}
\min_{\boldsymbol{x}} \quad & \left\| \boldsymbol{x} - \boldsymbol{x}_n^{(i-1)} \right\| \\
s.t. \quad & \sqrt{\beta_i} \|\boldsymbol{A}_i \boldsymbol{x} + \boldsymbol{n}_i\| \leq \sqrt{1+\beta_i} (\boldsymbol{h}_{ii}^H \boldsymbol{S}_i \boldsymbol{x}), \\
& \boldsymbol{p}^T \boldsymbol{x} = 0, \\
& \|\boldsymbol{S}_j \boldsymbol{x}\| \leq \sqrt{P_j}, \ \ j = 1, \ldots, M.
\end{aligned}
$$

local SINR constraint for MS i only

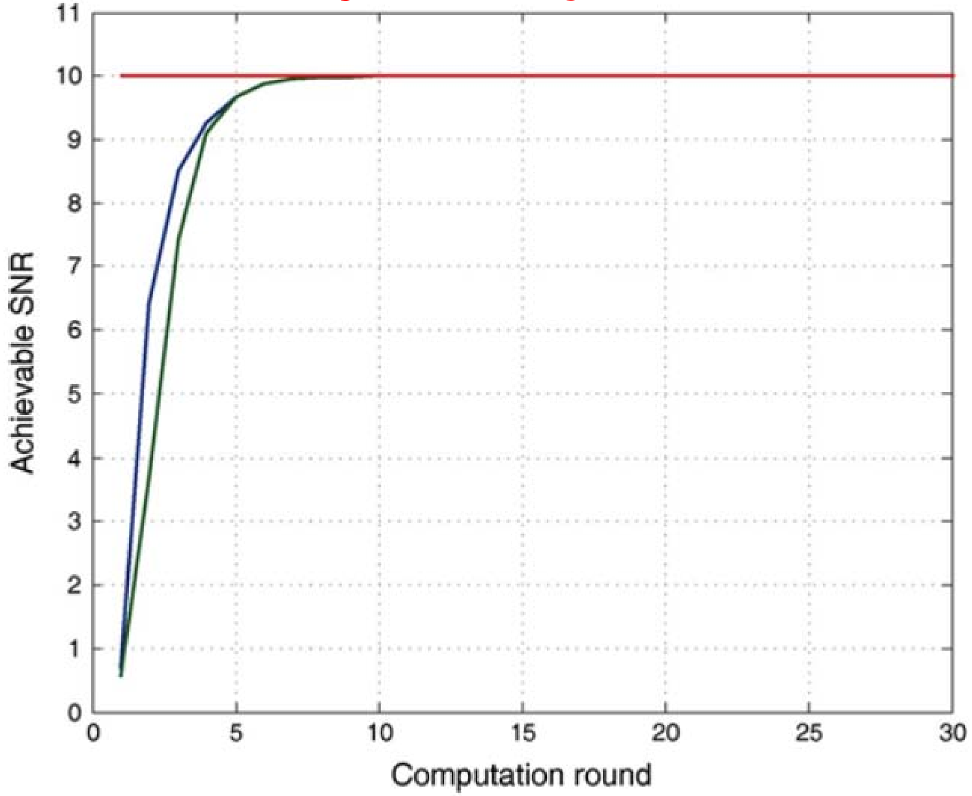
Numerical Example

3-user MISO-IC, SNR target = 10 dB (feasible)

Alternating Projection



Cyclic Projection



Recap of Part II

- ➢ Pareto rate characterization for MISO-IC (with interference treated as noise)
  - non-convex problems in general
  - rate profile *vs.* WSRMax
    - ✓ rate-profile: polynomial complexity, scalable with # of users
    - ✓ WSRMax: unknown complexity, non-scalable with # of users
  - similar results hold for SISO-IC *or* SIMO-IC (see [LiuZhangChua12])

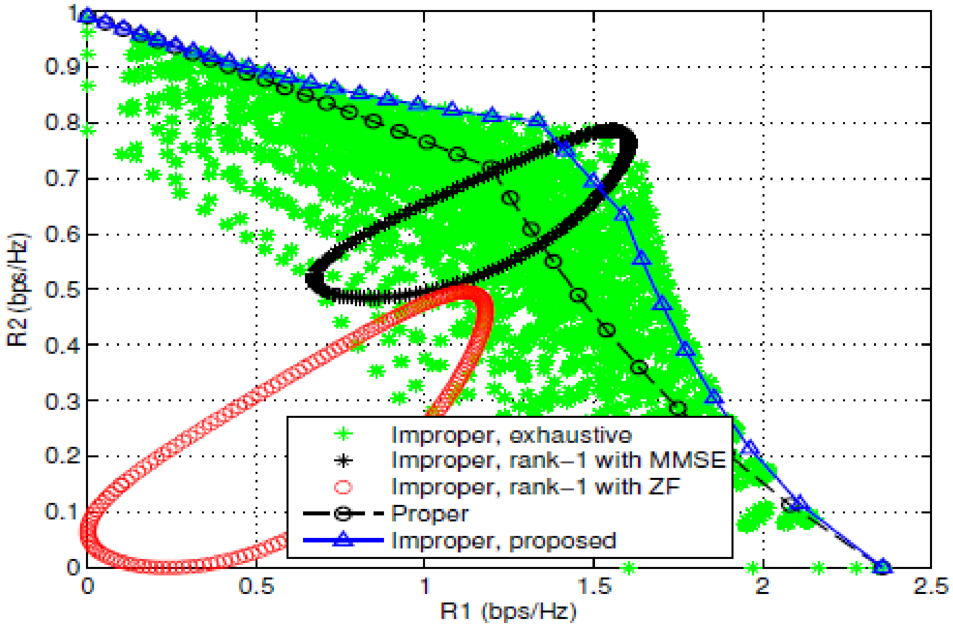
- ➢ A new general framework for ***non-convex*** utility optimization in multiuser systems via rate profile + monotonic optimization, provided
  - utility region is a normal set
  - problem size is not so large
  - finding intersection points with Pareto boundary is efficiently solvable

- ➢ Optimal distributed beamforming for MISO-IC
  - Approach 1: cognitive beamforming + active IT control
  - Approach 2: (reduced) SOCP + alternating/cyclic projection

Extension: Improper Gaussian Signaling

- Joint covariance and pseudo-covariance optimization
  - Two-User SISO-IC [Zeng et al. 12]

$$R_r = \underbrace{\log\left(1 + \frac{|h_{rr}|^2 C_{x_r}}{\sigma^2 + |h_{r\bar{r}}|^2 C_{x_{\bar{r}}}}\right)}_{R_r^{\text{proper}}(C_{x_1}, C_{x_2})} + \frac{1}{2}\log\frac{1 - C_{y_r}^{-2}|\widetilde{C}_{y_r}|^2}{1 - C_{s_r}^{-2}|\widetilde{C}_{s_r}|^2}.$$



[Zeng et al. 12]: Y. Zeng, C. M. Yetis, E. Gunawan, Y. L. Guan, and R. Zhang, "Improving achievable rate for the two-user SISO interference channel with improper Gaussian signaling," *IEEE Asilomar Conference on Signals, Systems and Computers,* 2012. (Invited Paper, Available Online at http://arxiv.org/abs/1205.0281)

Concluding Remarks on Cooperative Multi-Cell MIMO

- **Fundamental limits**
  - Capacity region characterization for interfering MIMO-MAC (uplink), and interfering MIMO-BC (downlink)
  - In general, very difficult (non-convex) optimization problems
- **Interference alignment (IA) techniques**
  - Provide optimal signal dimension sharing at high-SNR: DoF optimality
  - Reveal new design principles for K-user Gaussian ICs at finite-SNR, e.g.,
    - ✓ improper complex Gaussian signaling
    - ✓ symbol extension
    - ✓ non-separability of parallel Gaussian channels
  - open challenge: How to optimally exploit IA gains in practical wireless systems?
- **Other issues**
  - imperfect backhaul/feedback links
  - channel estimation error
  - interference cancelation (not treating interference as noise?)
  - cooperation in heterogeneous networks

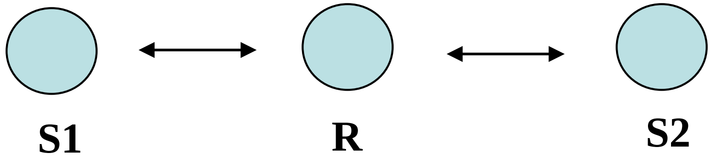
Agenda

- Overview of the talk
- Exploiting multi-antennas in
  - Cognitive Radio Networks
  - Cooperative Multi-Cell
  - **Two-Way Relay Networks**
  - Green Cellular Networks
  - Wireless Information and Power Transfer
- Concluding remarks

Topic #3: Two-Way Relay Beamforming

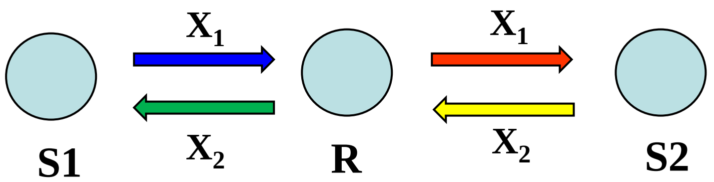
Two-Way Relay System (1)

- Two source nodes (S1 and S2) exchange information via a relay node (R)
  - ✓ all nodes operate half-duplex
  - ✓ no direct channel between S1 and S2

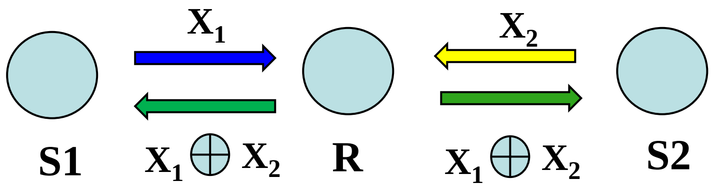


- Question: How many time slots needed for one round of information exchange between S1 and S2?

- Traditional orthogonal approach (4 slots needed)



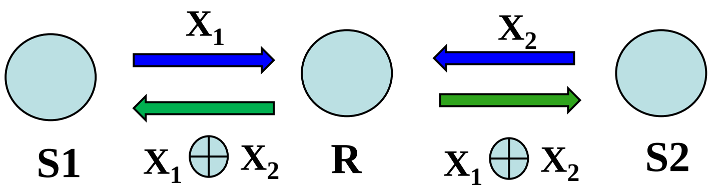
- Wireless network coding (3 slots needed) **[WuChouKung05]**



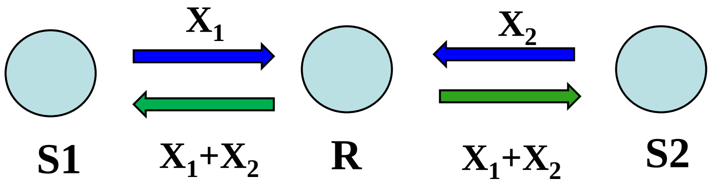
Two-Way Relay System (2)

- Question: Can we do better?

- Physical-Layer Network Coding (2 slots needed) **[ZhangLiewLam06]**



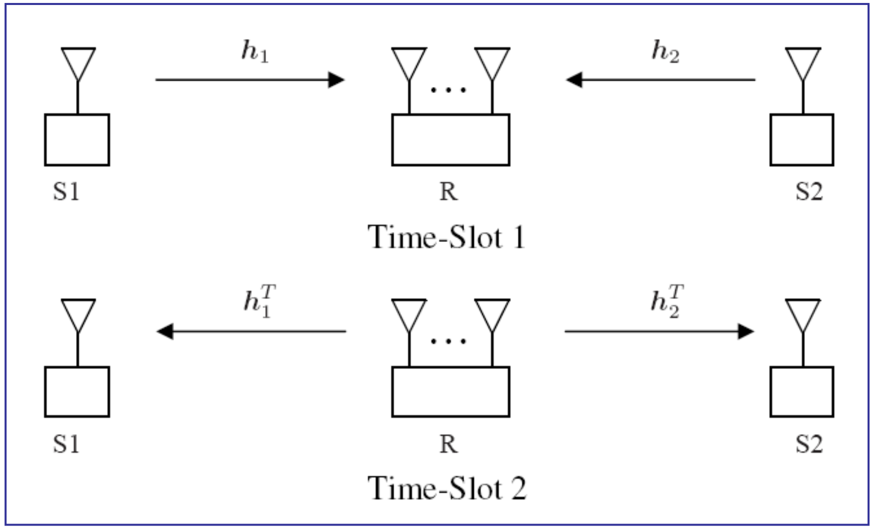
- Analogue Network Coding (2 slots needed) **[KattiGollakota Katabi07]**



- Other related work
  - ✓ information-theoretic study **[OechteringSchnurrBjelakovicBoche08]**
  - ✓ two-way amplify-and-forward (AF) relaying **[RankovWittneben05]**

Two-Way Relay Beamforming [Zhang et al. 09]

- Consider analogue network coding (*or* two-way AF relaying)
- Assume single-antenna source, multi-antenna relay, channel reciprocity



- Related work
  - ✓ one-way AF MIMO relay **[TangHua07], [MunozVidalAgustin07]**
  - ✓ two-way *distributed* relay beamforming **[HavaryShahGrami10], [ZengZhangCui11]**

[Zhang et al. 09]: R. Zhang, Y. C. Liang, C. C. Chai, and S. Cui, "Optimal beamforming for two-way multi-antenna relay channel with analogue network coding," *IEEE Journal on Selected Areas in Communications*, June 2009.

Signal Model of Two-Way Relay BF

- At 1st time-slot, **R** receives

$$\boxed{\boldsymbol{y}_R(n) = \boldsymbol{h}_1\sqrt{p_1}s_1(n) + \boldsymbol{h}_2\sqrt{p_2}s_2(n) + \boldsymbol{z}_R(n)}$$

- **R** linearly processes (AF relaying) received signal as

$$\boxed{\boldsymbol{x}_R(n) = \boldsymbol{A}\boldsymbol{y}_R(n)}$$

$$\boxed{\boldsymbol{A} \in \mathbb{C}^{M\times M}}$$

- At 2nd time-slot, **S1** (similarly as for **S2**) receives

$$\begin{aligned} y_1(n) &= \boldsymbol{h}_1^T\boldsymbol{x}_R(n) + z_1(n) \\ &= \boxed{\boldsymbol{h}_1^T\boldsymbol{A}\boldsymbol{h}_1\sqrt{p_1}s_1(n)} + \boldsymbol{h}_1^T\boldsymbol{A}\boldsymbol{h}_2\sqrt{p_2}s_2(n) + \boldsymbol{h}_1^T\boldsymbol{A}\boldsymbol{z}_R(n) + z_1(n) \end{aligned}$$

Assumed perfect "self-interference" cancellation

Achievable Rate Region

- Achievable rates at **S1** and **S2:**

$$\boxed{r_{21} \leq \frac{1}{2}\log_2\left(1 + \frac{|\boldsymbol{h}_1^T \boldsymbol{A}\boldsymbol{h}_2|^2 p_2}{\|\boldsymbol{A}^H \boldsymbol{h}_1^*\|^2 + 1}\right)} \quad (1) \qquad \boxed{r_{12} \leq \frac{1}{2}\log_2\left(1 + \frac{|\boldsymbol{h}_2^T \boldsymbol{A}\boldsymbol{h}_1|^2 p_1}{\|\boldsymbol{A}^H \boldsymbol{h}_2^*\|^2 + 1}\right)} \quad (2)$$

- Relay power consumption:

$$\boxed{p_R(\boldsymbol{A}) = \|\boldsymbol{A}\boldsymbol{h}_1\|^2 p_1 + \|\boldsymbol{A}\boldsymbol{h}_2\|^2 p_2 + \mathtt{tr}(\boldsymbol{A}\boldsymbol{A}^H)}$$

- Achievable rate region given p_1, p_2, and P_R :

$$\boxed{\mathcal{R}(p_1, p_2, P_R) \triangleq \bigcup_{\boldsymbol{A}:\ p_R(\boldsymbol{A}) \leq P_R} \{(r_{21}, r_{12}) : (1), (2)\}}$$

- "Capacity region" (assuming AF relaying)

$$\boxed{\mathcal{C}(P_1, P_2, P_R) \triangleq \bigcup_{(p_1,p_2):p_1 \leq P_1, p_2 \leq P_2} \mathcal{R}(p_1, p_2, P_R)}$$

Dimension Reduction on Optimal BF Matrix *A*

Theorem 1: The optimal relay beamforming matrix, $\boldsymbol{A}$, that attains a boundary rate-pair of $\mathcal{R}(p_1, p_2, P_R)$ has the following structure:

$$\boldsymbol{A} = \boldsymbol{U}^* \boldsymbol{B} \boldsymbol{U}^H$$

where $\boxed{\boldsymbol{B} \in \mathbb{C}^{2\times 2}}$ is an unknown matrix, $\boldsymbol{U} \in \mathbb{C}^{M\times 2}$ is obtained from SVD of $\boldsymbol{H}_{\mathrm{UL}} = [\boldsymbol{h}_1, \boldsymbol{h}_2] \in \mathbb{C}^{M\times 2}$, i.e., $\boldsymbol{H}_{\mathrm{UL}} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^H$.

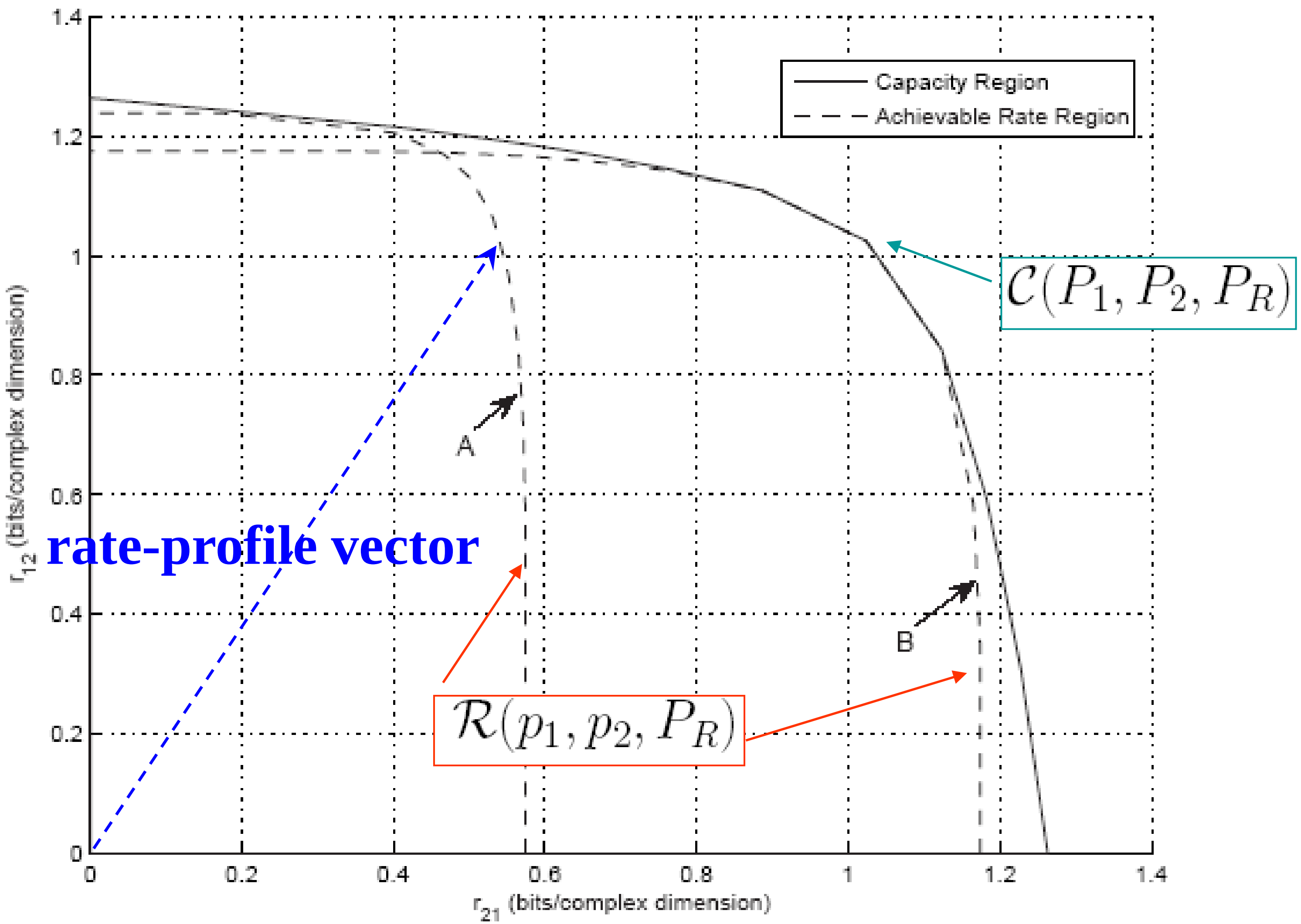
Corollary 1: $\mathcal{R}(p_1, p_2, P_R)$ can be equivalently expressed as

$$\bigcup_{\boldsymbol{B}:\ p_R(\boldsymbol{B}) \le P_R} \left\{ (r_{21}, r_{12}) : r_{21} \le \frac{1}{2}\log_2\left(1 + \frac{|\boldsymbol{g}_1^T \boldsymbol{B} \boldsymbol{g}_2|^2 p_2}{\|\boldsymbol{B}^H \boldsymbol{g}_1^*\|^2 + 1}\right), \right.$$
$$\left. r_{12} \le \frac{1}{2}\log_2\left(1 + \frac{|\boldsymbol{g}_2^T \boldsymbol{B} \boldsymbol{g}_1|^2 p_1}{\|\boldsymbol{B}^H \boldsymbol{g}_2^*\|^2 + 1}\right) \right\}$$

where $p_R(\boldsymbol{B}) = \|\boldsymbol{B}\boldsymbol{g}_1\|^2 p_1 + \|\boldsymbol{B}\boldsymbol{g}_2\|^2 p_2 + \mathtt{tr}(\boldsymbol{B}\boldsymbol{B}^H)$.

non-convex rate region

Rate Profile Approach



Problem Reformulation

Sum-Rate Max. with Rate-Profile Constraints

$$\begin{aligned}\underset{R_{\text{sum}},\boldsymbol{B}}{\texttt{Max.}} \quad & R_{\text{sum}} \\ \texttt{s.t.} \quad & \frac{1}{2}\log_2\left(1+\frac{|\boldsymbol{g}_1^T\boldsymbol{B}\boldsymbol{g}_2|^2 p_2}{\|\boldsymbol{B}^H\boldsymbol{g}_1^*\|^2+1}\right) \geq \alpha_{21}R_{\text{sum}} \\ & \frac{1}{2}\log_2\left(1+\frac{|\boldsymbol{g}_2^T\boldsymbol{B}\boldsymbol{g}_1|^2 p_1}{\|\boldsymbol{B}^H\boldsymbol{g}_2^*\|^2+1}\right) \geq \alpha_{12}R_{\text{sum}} \\ & \|\boldsymbol{B}\boldsymbol{g}_1\|^2 p_1 + \|\boldsymbol{B}\boldsymbol{g}_2\|^2 p_2 + \texttt{tr}(\boldsymbol{B}\boldsymbol{B}^H) \leq P_R.\end{aligned}$$

$$r^* \rightarrow R^*_{\text{sum}}$$

$$p_R^* > P_R, r\downarrow$$

$$p_R^* < P_R, r\uparrow$$

$$\begin{aligned}\underset{\boldsymbol{B}}{\texttt{Min.}} \quad & p_R := \|\boldsymbol{B}\boldsymbol{g}_1\|^2 p_1 + \|\boldsymbol{B}\boldsymbol{g}_2\|^2 p_2 + \texttt{tr}(\boldsymbol{B}\boldsymbol{B}^H) \\ \texttt{s.t.} \quad & \frac{1}{2}\log_2\left(1+\frac{|\boldsymbol{g}_1^T\boldsymbol{B}\boldsymbol{g}_2|^2 p_2}{\|\boldsymbol{B}^H\boldsymbol{g}_1^*\|^2+1}\right) \geq \alpha_{21}r \\ & \frac{1}{2}\log_2\left(1+\frac{|\boldsymbol{g}_2^T\boldsymbol{B}\boldsymbol{g}_1|^2 p_1}{\|\boldsymbol{B}^H\boldsymbol{g}_2^*\|^2+1}\right) \geq \alpha_{12}r.\end{aligned}$$

$$\bar{\gamma}_1 = 2^{2\alpha_{21}r} - 1$$

$$\bar{\gamma}_2 = 2^{2\alpha_{12}r} - 1$$

SNR targets

PMin-SNR (power minimization with SNR constraints)

$$\begin{aligned}\underset{\boldsymbol{B}}{\texttt{Min.}} \quad & p_R := \|\boldsymbol{B}\boldsymbol{g}_1\|^2 p_1 + \|\boldsymbol{B}\boldsymbol{g}_2\|^2 p_2 + \texttt{tr}(\boldsymbol{B}\boldsymbol{B}^H) \\ \texttt{s.t.} \quad & |\boldsymbol{g}_1^T\boldsymbol{B}\boldsymbol{g}_2|^2 \geq \frac{\bar{\gamma}_1}{p_2}\|\boldsymbol{B}^H\boldsymbol{g}_1^*\|^2 + \frac{\bar{\gamma}_1}{p_2} \\ & |\boldsymbol{g}_2^T\boldsymbol{B}\boldsymbol{g}_1|^2 \geq \frac{\bar{\gamma}_2}{p_1}\|\boldsymbol{B}^H\boldsymbol{g}_2^*\|^2 + \frac{\bar{\gamma}_2}{p_1}.\end{aligned}$$

Solve PMin-SNR by SDP

$\boldsymbol{b} = \texttt{Vec}(\boldsymbol{B})$

$$\begin{array}{ll} \underset{\boldsymbol{b}}{\texttt{Min.}} & p_R := \|\boldsymbol{\Phi b}\|^2 \\ \texttt{s.t.} & |\boldsymbol{f}_1^T \boldsymbol{b}|^2 \geq \frac{\bar{\gamma}_1}{p_2}\|\boldsymbol{G}_1\boldsymbol{b}\|^2 + \frac{\bar{\gamma}_1}{p_2} \\ & |\boldsymbol{f}_2^T \boldsymbol{b}|^2 \geq \frac{\bar{\gamma}_2}{p_1}\|\boldsymbol{G}_2\boldsymbol{b}\|^2 + \frac{\bar{\gamma}_2}{p_1}. \end{array}$$

$\boldsymbol{X} = [\boldsymbol{b}_R; \boldsymbol{b}_I] \times [\boldsymbol{b}_R; \boldsymbol{b}_I]^T$

$$\begin{array}{ll} \underset{\boldsymbol{X}}{\texttt{Min.}} & p_R := \texttt{tr}(\boldsymbol{F}_0\boldsymbol{X}) \\ \texttt{s.t.} & \texttt{tr}(\boldsymbol{F}_1\boldsymbol{X}) \geq 1,\ \texttt{tr}(\boldsymbol{F}_2\boldsymbol{X}) \geq 1,\ \boldsymbol{X} \succeq 0, \\ & \boxed{\texttt{rank}(\boldsymbol{X}) = 1.} \end{array}$$

Semi-Definite Programming (SDP) with rank-one constraint: **non-convex!**

SDP Relaxation

$$\min_{\boldsymbol{X}} \quad p_R := \mathtt{tr}(\boldsymbol{F}_0\boldsymbol{X})$$
$$\text{s.t.} \quad \mathtt{tr}(\boldsymbol{F}_1\boldsymbol{X}) \geq 1, \ \mathtt{tr}(\boldsymbol{F}_2\boldsymbol{X}) \geq 1, \ \boldsymbol{X} \succeq 0,$$
$$\boxed{\mathtt{rank}(\boldsymbol{X}) = 1.}$$

$\boldsymbol{X}^{\star\star}$ of rank one

removing rank-one constraint

[Ye&Zhang03]

$$\min_{\boldsymbol{X}} \quad p_R := \mathtt{tr}(\boldsymbol{F}_0\boldsymbol{X})$$
$$\text{s.t.} \quad \mathtt{tr}(\boldsymbol{F}_1\boldsymbol{X}) \geq 1, \ \mathtt{tr}(\boldsymbol{F}_2\boldsymbol{X}) \geq 1, \ \boldsymbol{X} \succeq 0.$$

$\boldsymbol{X}^{\star}$ of rank $r > 1$

SDP in standard form, solvable by e.g. CVX

Low-Complexity Suboptimal Schemes

- **Maximal-Ratio** (MR) Relay Beamforming

$$\boxed{\boldsymbol{A}_{\mathrm{MR}} = \boldsymbol{H}_{\mathrm{DL}}^{H} \begin{bmatrix} a_{\mathrm{MR}} & 0 \\ 0 & b_{\mathrm{MR}} \end{bmatrix} \boldsymbol{H}_{\mathrm{UL}}^{H}}$$

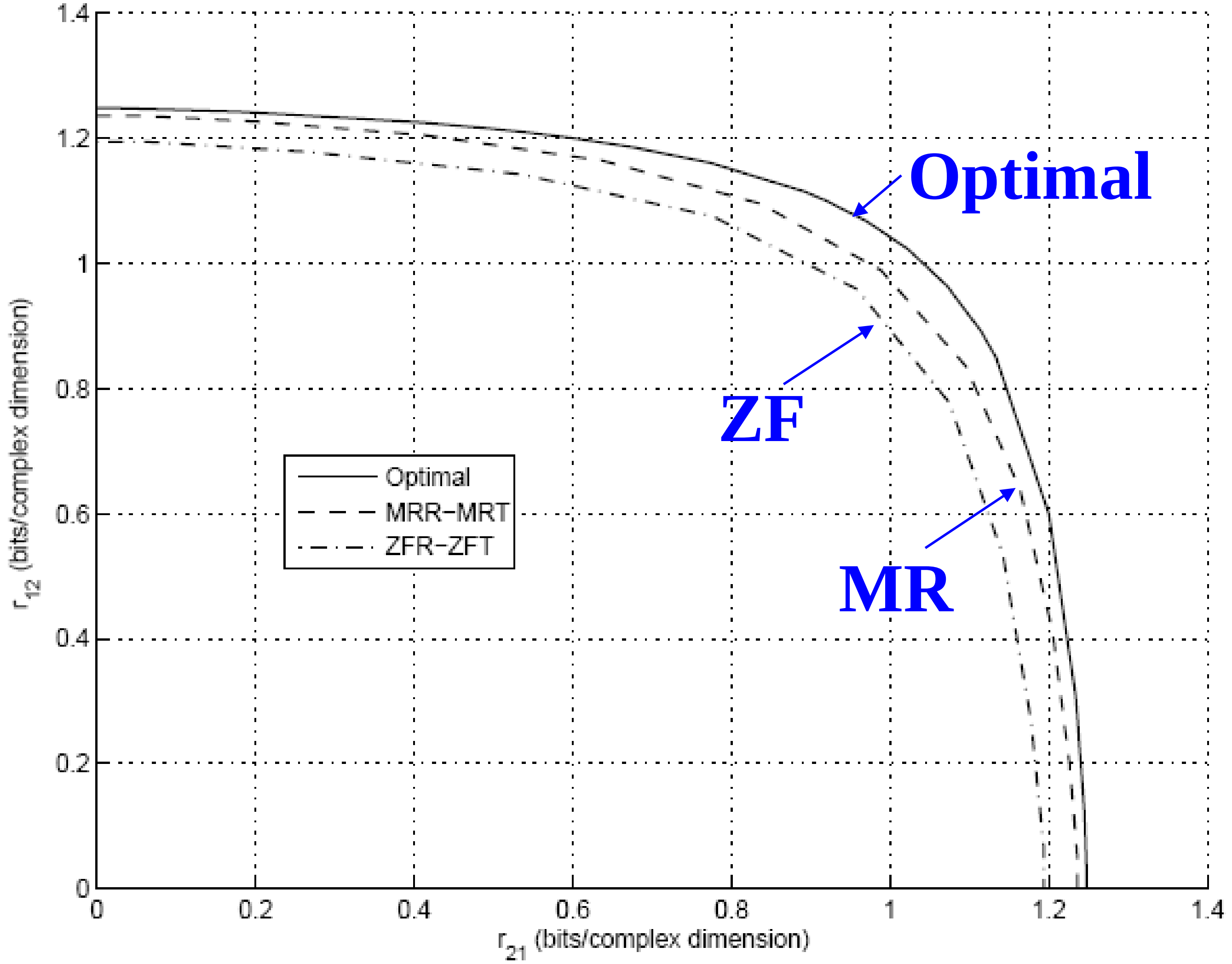
$$\boxed{\boldsymbol{H}_{\mathrm{UL}} = [\boldsymbol{h}_1, \boldsymbol{h}_2]} \qquad \boxed{\boldsymbol{H}_{\mathrm{DL}} = [\boldsymbol{h}_2, \boldsymbol{h}_1]^T}$$

- **Zero-Forcing** (ZF) Relay Beamforming

$$\boxed{\boldsymbol{A}_{\mathrm{ZF}} = \boldsymbol{H}_{\mathrm{DL}}^{\dagger} \begin{bmatrix} a_{\mathrm{ZF}} & 0 \\ 0 & b_{\mathrm{ZF}} \end{bmatrix} \boldsymbol{H}_{\mathrm{UL}}^{\dagger}}$$

Performance Comparison (1)

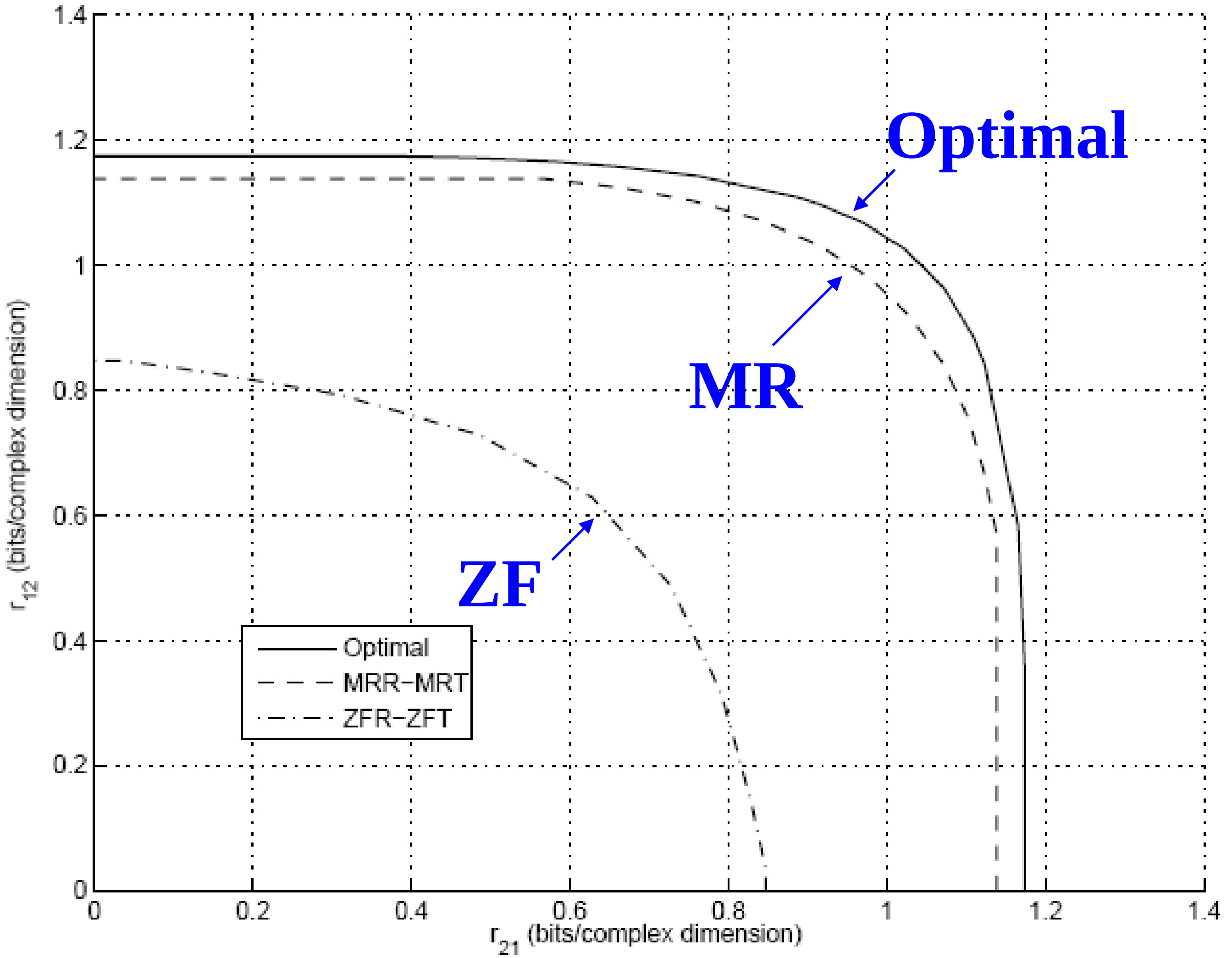
$$M = 4, P_1 = P_2 = P_R = 10$$



$$\rho(\boldsymbol{h}_1, \boldsymbol{h}_2) = 0.1$$

Performance Comparison (2)

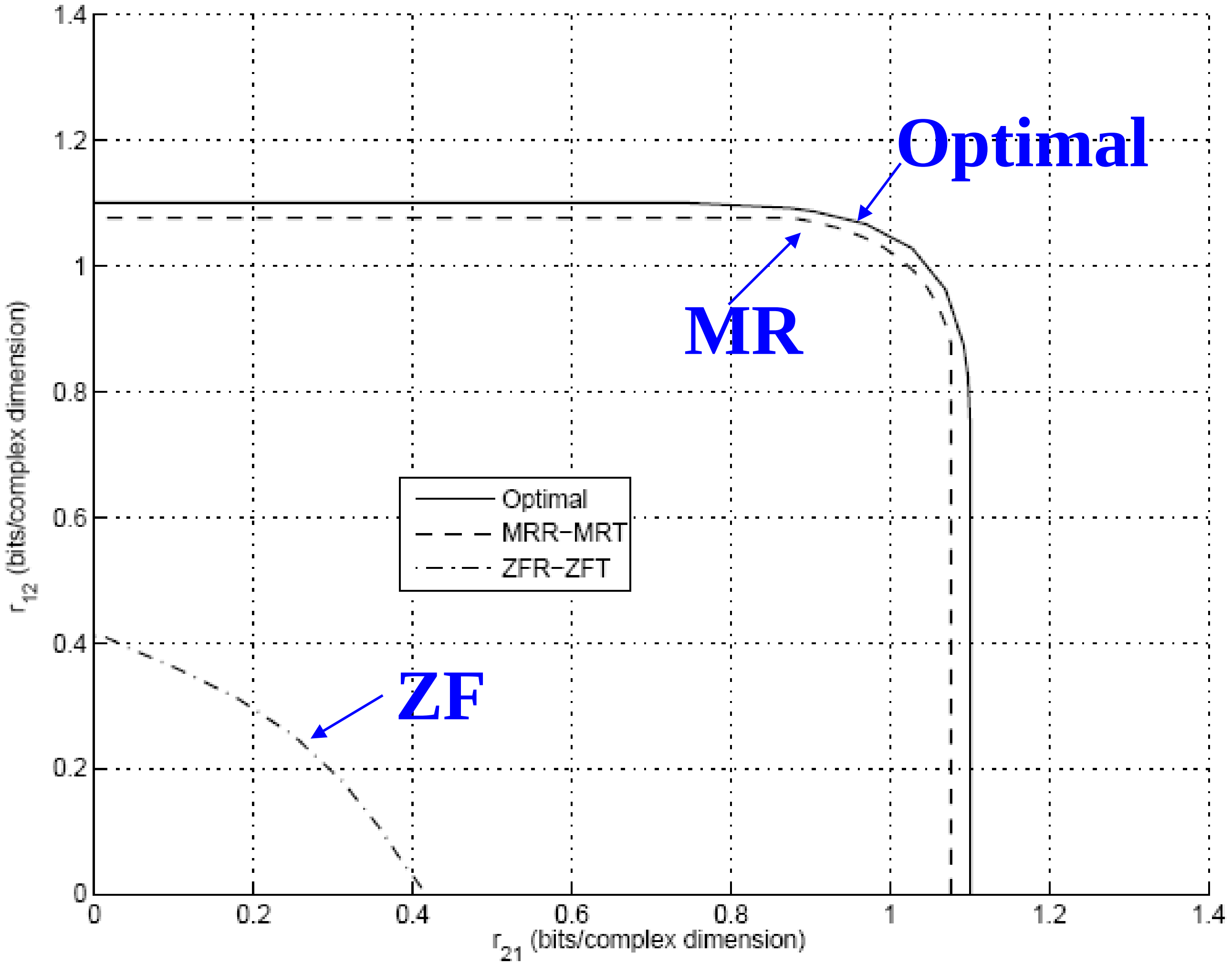
$$M = 4, P_1 = P_2 = P_R = 10$$



$$\rho(\boldsymbol{h}_1, \boldsymbol{h}_2) = 0.5$$

Performance Comparison (3)

$$M = 4, P_1 = P_2 = P_R = 10$$

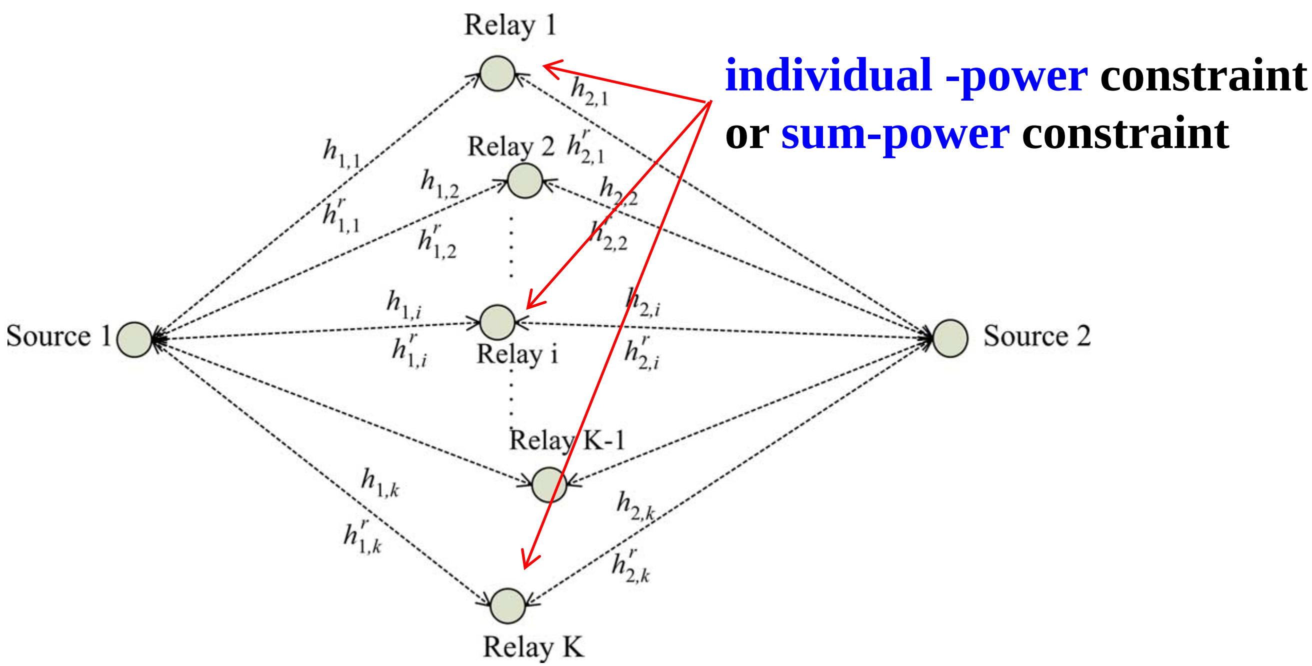


$$\rho(\boldsymbol{h}_1, \boldsymbol{h}_2) = 0.8$$

Concluding Remarks on Two-Way Relay Beamforming

- Optimal two-way relay beamforming for analogue network coding
- Rate region characterization: non-convex problem
- Global optimal solution achieved via rate-profile + SDP relaxation
- Low-complexity schemes: MR performers better than ZF
  - ✓ non-wise to suppress interference at relay due to source self-interference cancellation
- Similar results hold for *non-reciprocal* source-relay channels
- Many possible extensions
  - ✓ multiple relays
  - ✓ multi-antenna source nodes
  - ✓ multiple source nodes
  - ✓ multi-hop

Extension: Collaborative BF for Distributed Two-Way Relay Networks [ZengZhangCui11]



- **Case of Reciprocal Source-Relay Channel**
  - ✓ only relay power allocation needs to be optimized
- **Case of Non-Reciprocal Source-Relay Channel**
  - ✓ both relay power and phase need to be optimized

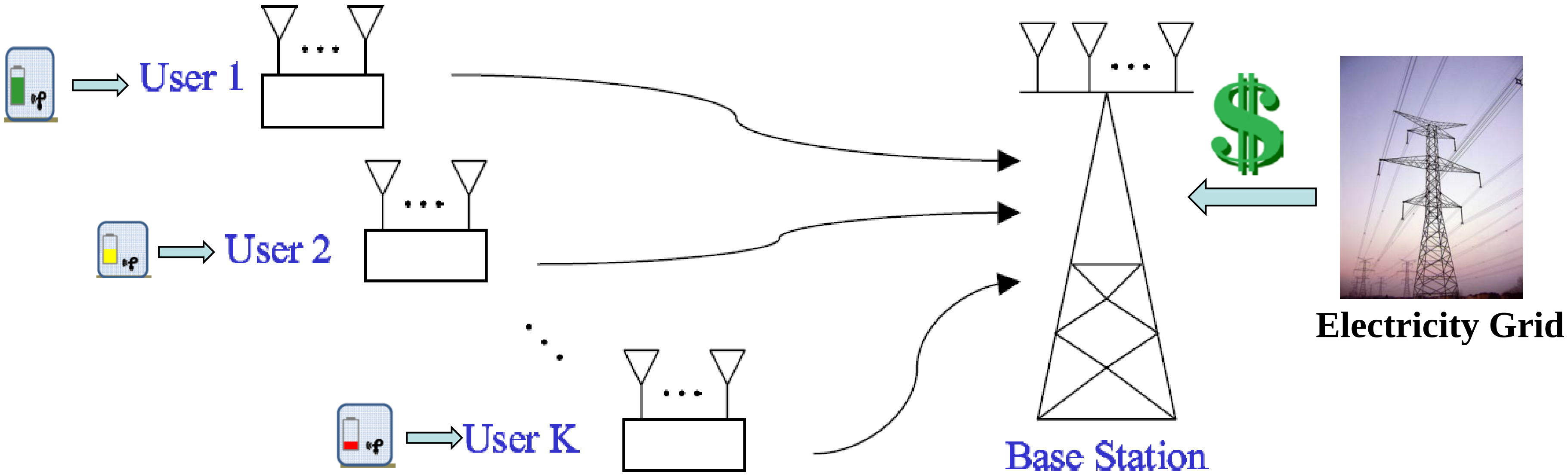
[ZengZhangCui11]: M. Zeng, R. Zhang, and S. Cui, "On design of collaborative beamforming for two-way relay networks," *IEEE Transactions on Signal Processing*, May 2011.

Agenda

- ❑ Overview of the talk
- ❑ Exploiting multi-antennas in
  - ➢ Cognitive Radio Networks
  - ➢ Cooperative Multi-Cell
  - ➢ Two-Way Relay Networks
  - ➢ **Green Cellular Networks**
  - ➢ Wireless Information and Power Transfer
- ❑ Concluding remarks

Topic #4: Power Minimization in MU-MIMO

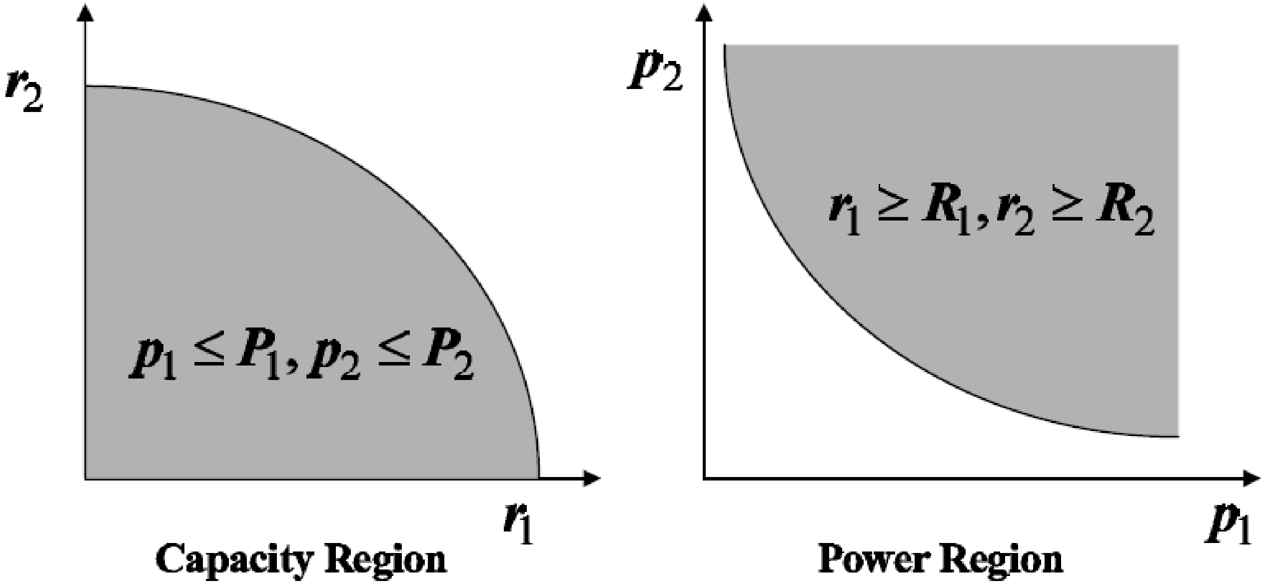
“Green” Cellular Networks



- **Energy consumption reduction at base station**
  - electricity cost, environmental concerns
- **Energy consumption reduction at mobile terminals**
  - limited battery capacity, operation time maximization
- **A design paradigm shift in wireless communication**
  - from “throughput/rate maximization” to “energy/power minimization”

Fundamental Limits

- Capacity Region vs. Power Region



Power Minimization in MU-MMO [MohseniZhangCioffi06]

- **Power Region Characterization for Cellular Uplink (MIMO-MAC)**
  - weighed sum-power minimization (W-SPmin)
    - ✓ AWGN channel
    - ✓ fading channel

- **BS Power Minimization for Cellular Downlink (MIMO-BC)**
  - apply uplink results with MIMO MAC-BC duality (details omitted)

[MohseniZhangCioffi06]: M. Mohseni, R. Zhang, and J. M. Cioffi, "Optimized transmission of fading multiple-access and broadcast channels with multiple antennas," ***IEEE Journal on Selected Areas in Communications*****, Aug. 2006.**

Channel Model of AWGN MIMO-MAC

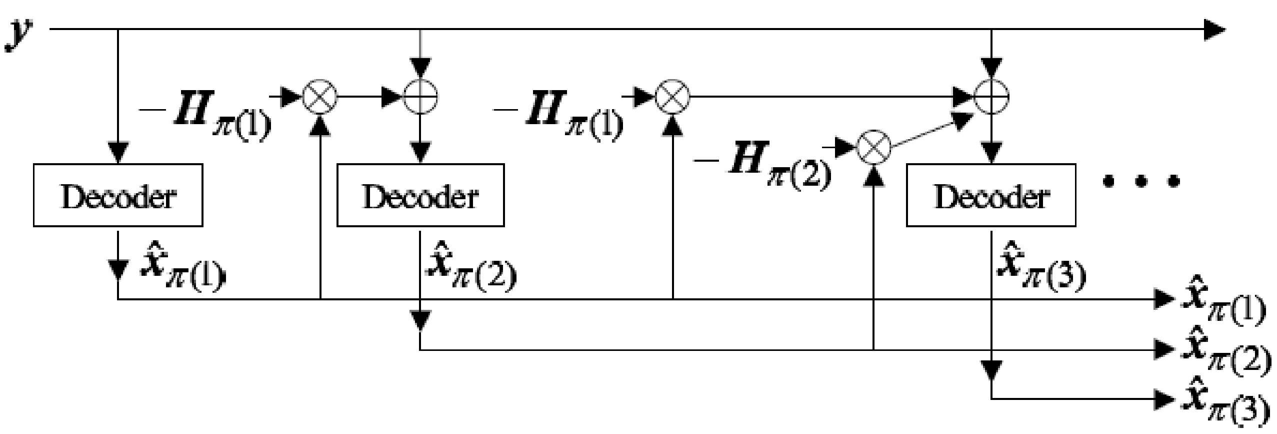
$$\boldsymbol{y} = [\boldsymbol{H}_1 \cdots \boldsymbol{H}_K] \begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_K \end{bmatrix} + \boldsymbol{z}$$

- $\boldsymbol{y}$ is $r \times 1$ received signal vector at base station
- $\boldsymbol{H}_k$ is $r \times t_k$ channel matrix for user k
- $\boldsymbol{x}_k$ is $t_k \times 1$ transmitted signal vector for user k
- $\boldsymbol{z}$ is $r \times 1$ additive Gaussian noise vector at receiver. $\boldsymbol{z} \sim \mathcal{CN}(0, \boldsymbol{S}_z)$

Assumption

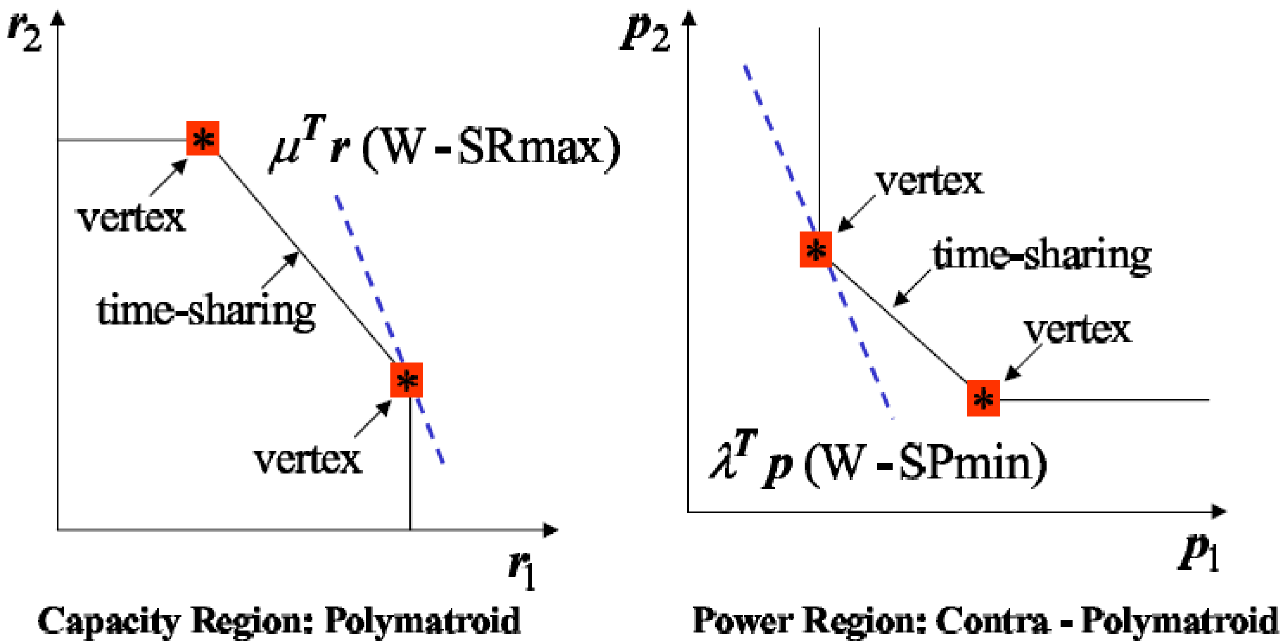
- Optimum Gaussian encoder at each transmitter
  - $\boldsymbol{x}_k \sim \mathcal{CN}(0, \boldsymbol{S}_k), \forall k$
  - $\boldsymbol{S}_k \triangleq \mathbb{E}[\boldsymbol{x}_k \boldsymbol{x}_k^\dagger]$: transmit covariance matrix (or spatial spectrum) of user k

- Optimum successive decoder at receiver
  - $\boldsymbol{\pi}$: decoding order vector, permutation over $\{1, 2, \cdots, K\}$
  - e.g., user $\boldsymbol{\pi}(1)$ is decoded first , user $\boldsymbol{\pi}(2)$ is decoded second , ...



Special Case: SISO-MAC

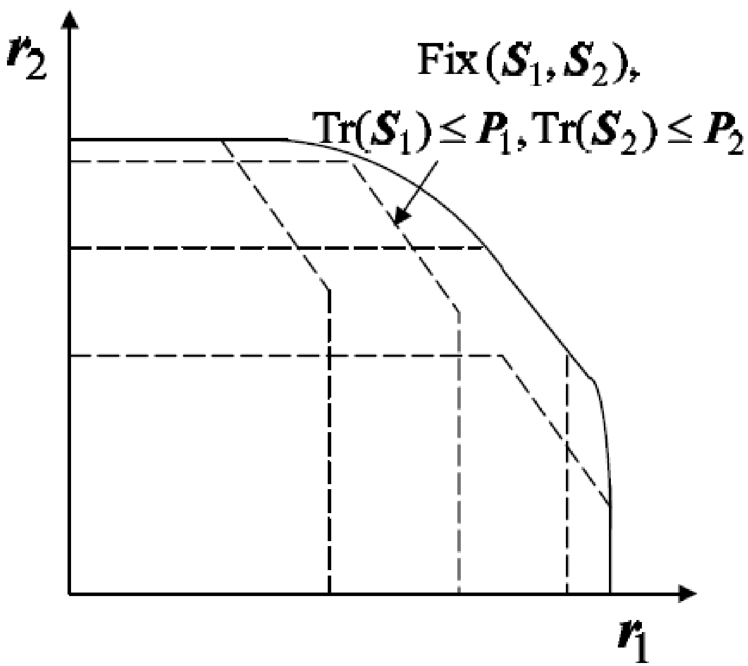
$$y = h_1 x_1 + h_2 x_2 + z$$



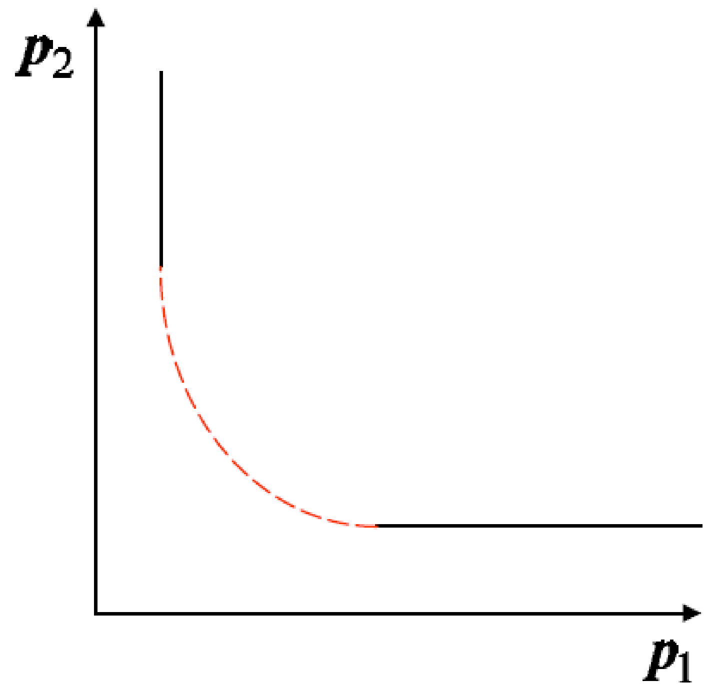
- W-SRmax: weighted sum-rate maximization
- W-SPmin: weighted sum-power minimization

Power Region of MIMO-MAC

$$y = H_1 x_1 + H_2 x_2 + z$$



Capacity Region: Union of Polymatroids



Power Region: ?

Capacity Polymatroid (convex set)

- I: mutual information
- Rate Inequalities for MAC:

$$\sum_{k \in \mathcal{J}} r_k \leq I\left(\{\boldsymbol{x}_k\}_{k \in \mathcal{J}}; \boldsymbol{y} | \{\boldsymbol{x}_{k'}\}_{k' \notin \mathcal{J}}\right), \forall \mathcal{J} \subseteq \{1, \ldots, K\}$$

  – Ahlswede ('71), Liao ('72), Cover-Wyner ('73)

- Capacity polymatroid given $\{\boldsymbol{S}_1, \cdots, \boldsymbol{S}_K\}$:

$$\mathcal{C}(\{\boldsymbol{S}_k\}) \triangleq \left\{ \boldsymbol{r} \in \mathbb{R}_+^K : \sum_{k \in \mathcal{J}} r_k \leq \log \left| \sum_{k \in \mathcal{J}} \boldsymbol{H}_k \boldsymbol{S}_k \boldsymbol{H}_k^\dagger + \boldsymbol{S}_z \right|, \forall \mathcal{J} \subseteq \{1, \ldots, K\} \right\}$$

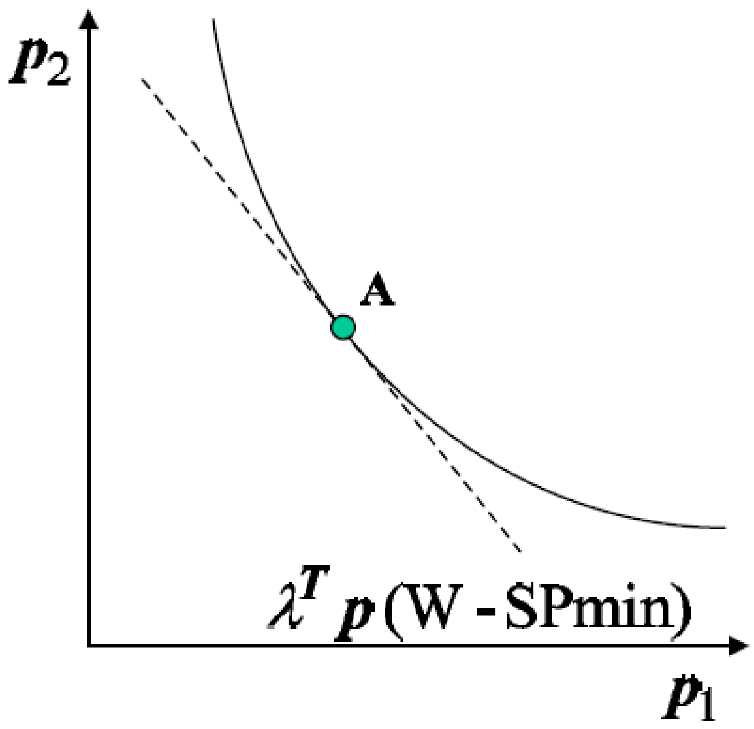
Power Region Definition

Definition 1. *Given user's rate constraint* $\boldsymbol{R} = (R_1, R_2, \cdots, R_K)$, *a transmit power-tuple* $\boldsymbol{p} = (p_1, p_2, \cdots, p_K)$ *is in the power region* $\mathcal{P}(\boldsymbol{R})$ *iff there exits a set of* $\{\boldsymbol{S}_k\}$, $k = 1, \cdots, K$ *such that*

- $p_k = \mathtt{Tr}(\boldsymbol{S}_k), \forall k$
- $\boldsymbol{R} \in \mathcal{C}(\{\boldsymbol{S}_k\})$

Power Region Characterization via W-SPmin

- $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \cdots, \lambda_K) \in \mathbb{R}_+^K$: power prices



W-SPmin Problem Formulation

- Variables:
  - transmit rates: $\boldsymbol{r} = (r_1, r_2, \cdots, r_K)$
  - transmit covariance matrices: $\boldsymbol{S}_1, \boldsymbol{S}_2, \cdots, \boldsymbol{S}_K$

- Problem formulation:

$$\begin{array}{ll} \texttt{Minimize} & \displaystyle\sum_{k=1}^{K} \lambda_k \texttt{Tr}\left(\boldsymbol{S}_k\right) \\ \texttt{Subject to} & r_k \geq R_k \quad \forall k \\ & \boldsymbol{r} \in \mathcal{C}(\{\boldsymbol{S}_k\}) \\ & \boldsymbol{S}_k \succeq 0 \quad \forall k \end{array}$$

implicit rate constraints

- Convex problem, but not directly solvable due to implicit rate constraints

Heuristic Approach

- Step 1:
  Fix decoding order $\boldsymbol{\pi}$, find $\{\boldsymbol{S}_k\}$ to minimize $\sum_k \lambda_k \mathrm{Tr}\left(\boldsymbol{S}_k\right)$
  - For user $\boldsymbol{\pi}(k)$, $r_{\boldsymbol{\pi}(k)}$ is expressed as

$$\log\left|\sum_{i=k}^{K} \boldsymbol{H}_{\boldsymbol{\pi}(i)} \boldsymbol{S}_{\boldsymbol{\pi}(i)} \boldsymbol{H}_{\boldsymbol{\pi}(i)}^{\dagger} + \boldsymbol{S}_z\right| - \log\left|\sum_{i=k+1}^{K} \boldsymbol{H}_{\boldsymbol{\pi}(i)} \boldsymbol{S}_{\boldsymbol{\pi}(i)} \boldsymbol{H}_{\boldsymbol{\pi}(i)}^{\dagger} + \boldsymbol{S}_z\right|$$

  - Caution : Constraint $r_{\boldsymbol{\pi}(k)} \geq R_{\boldsymbol{\pi}(k)}$ is non-convex
- Step 2:
  Over all possible $(K!)$ decoding orders, find $\boldsymbol{\pi}$ to minimize $\sum_k \lambda_k \mathrm{Tr}\left(\boldsymbol{S}_k\right)$
  - Caution : Excludes time-sharing of decoding orders

Proposed Optimal Solution

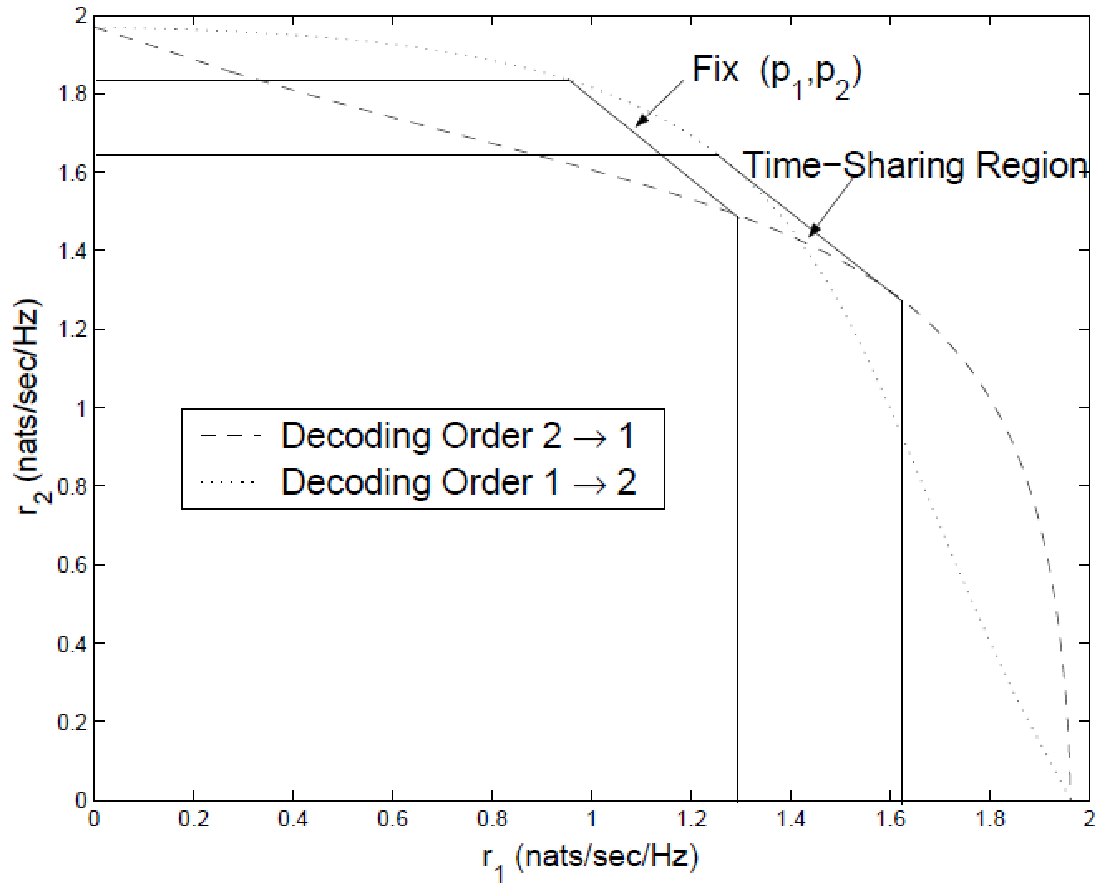
- Goal : joint optimization of transmit powers, transmit covariance matrices, decoding orders, and (if necessary) time-sharing factors

- Approach : duality between power region and capacity region under *weighted sum-power (W-SP) constraint*

- Implementation : Lagrange duality

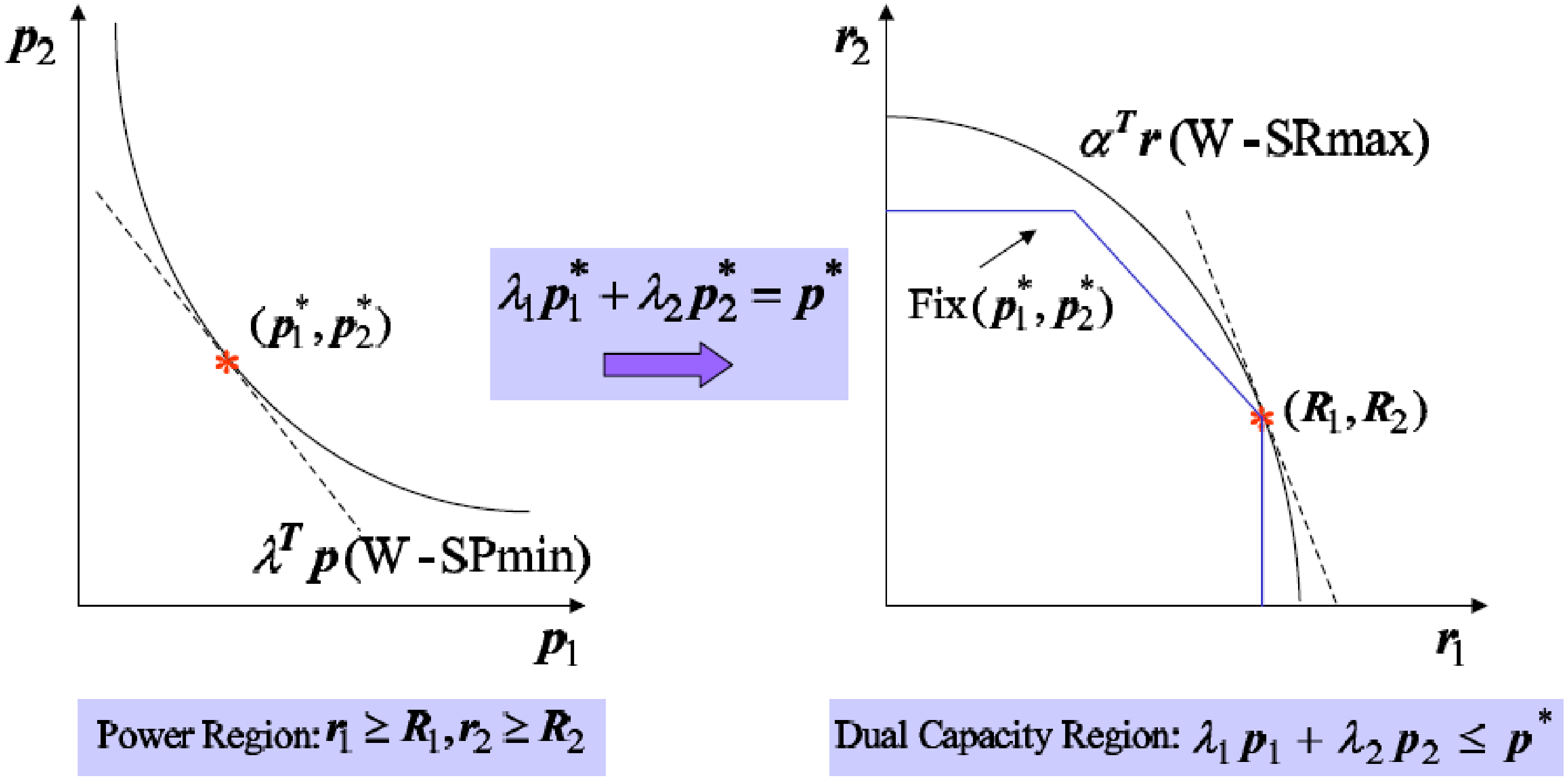
Capacity Region under W-SP Constraint

- Example: 2-user single transmit and multiple receive antenna (SIMO) MAC



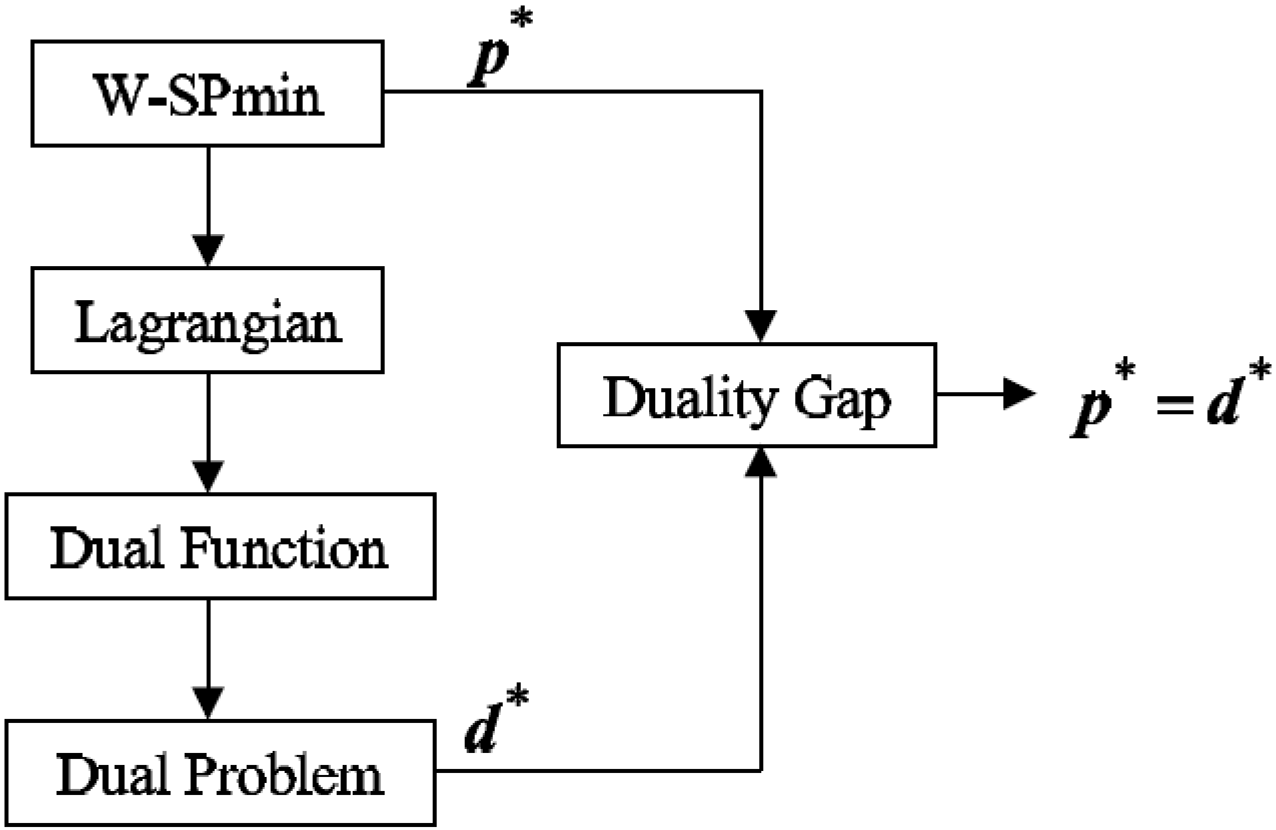
$$\lambda_1 p_1 + \lambda_2 p_2 \leq p^*$$

Power/Capacity Region Duality



- W-SPmin in power region $\Rightarrow$ W-SRmax in dual capacity region
- How to find α ? Lagrange duality

Lagrange Duality



Lagrangian

- Primal (original) problem :

$$\begin{aligned} \texttt{Minimize} \quad & p = \sum_{k=1}^{K} \lambda_k \mathrm{Tr}\left(\boldsymbol{S}_k\right) \\ \texttt{Subject to} \quad & r_k \geq R_k \;\; \forall k \\ & \boldsymbol{r} \in \mathcal{C}(\{\boldsymbol{S}_k\}) \\ & \boldsymbol{S}_k \succeq 0 \quad \forall k \end{aligned}$$

- Dual variables: μ_k w.r.t. $r_k \geq R_k$, $k = 1, \ldots, K$

- Lagrangian :

$$\mathcal{L}(\{\boldsymbol{S}_k\}, \{r_k\}, \boldsymbol{\mu}) = \sum_{k=1}^{K} \lambda_k \mathrm{Tr}(\boldsymbol{S}_k) - \sum_{k=1}^{K} \mu_k (r_k - R_k)$$

Dual Function

$$
\begin{aligned}
g(\boldsymbol{\mu}) &= \min_{\{\boldsymbol{S}_k\},\{r_k\}} \mathcal{L}(\{\boldsymbol{S}_k\},\{r_k\},\boldsymbol{\mu}) \\
&= \min_{\{\boldsymbol{S}_k\},\{r_k\}} \sum_{k=1}^{K} \lambda_k \mathtt{Tr}(\boldsymbol{S}_k) - \sum_{k=1}^{K} \mu_k r_k + \sum_{k=1}^{K} \mu_k R_k
\end{aligned}
$$

- Equivalent problem:

$$
\begin{array}{ll}
\mathtt{Maximize} & \sum_{k=1}^{K} \mu_k r_k - \sum_{k=1}^{K} \lambda_k \mathtt{Tr}(\boldsymbol{S}_k) \\
\mathtt{Subject\ to} & \boldsymbol{r} \in \mathcal{C}(\{\boldsymbol{S}_k\})
\end{array}
$$

- Weighted sum-rate maximization (W-SRmax) over $\mathcal{C}(\{\boldsymbol{S}_k\})$

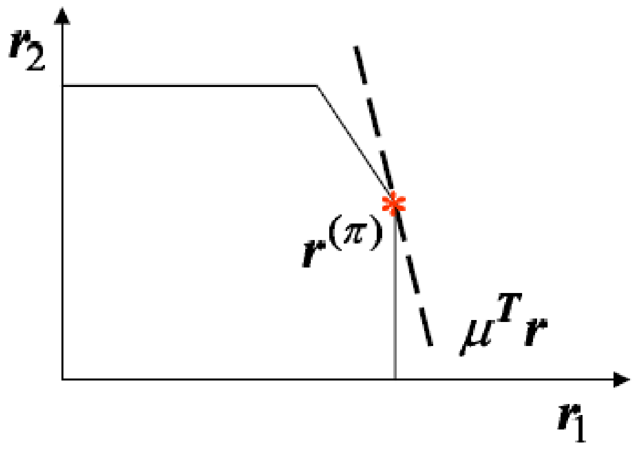
Polymatroid Structure of $\mathcal{C}(\{\boldsymbol{S}_k\})$

Lemma 1. [Tse-Hanly('98)] *The solution for the W-SRmax over $\mathcal{C}(\{\boldsymbol{S}_k\})$:*

$$\begin{array}{ll} \texttt{Maximize} & \displaystyle\sum_{k=1}^{K} \mu_k r_k \\ \texttt{Subject to} & \boldsymbol{r} \in \mathcal{C}(\{\boldsymbol{S}_k\}) \end{array}$$

is attained by a vertex $\boldsymbol{r}^{(\boldsymbol{\pi})}$ *of* $\mathcal{C}(\{\boldsymbol{S}_k\})$, *for which*

- $\boldsymbol{\pi}$ *is given by* $\mu_{\boldsymbol{\pi}(1)} \leq \mu_{\boldsymbol{\pi}(2)} \leq \cdots \leq \mu_{\boldsymbol{\pi}(K)}$
- $r_{\boldsymbol{\pi}(k)}^{(\boldsymbol{\pi})} = \log\left|\sum_{i=k}^{K} \boldsymbol{H}_{\boldsymbol{\pi}(i)}\boldsymbol{S}_{\boldsymbol{\pi}(i)}\boldsymbol{H}_{\boldsymbol{\pi}(i)}^{\dagger} + \boldsymbol{S}_z\right| - \log\left|\sum_{i=k+1}^{K} \boldsymbol{H}_{\boldsymbol{\pi}(i)}\boldsymbol{S}_{\boldsymbol{\pi}(i)}\boldsymbol{H}_{\boldsymbol{\pi}(i)}^{\dagger} + \boldsymbol{S}_z\right|$



Obtain $g(\boldsymbol{\mu})$

$$\begin{aligned} \texttt{Maximize} \quad & \sum_{k=1}^{K} \mu_k r_k - \sum_{k=1}^{K} \lambda_k \mathtt{Tr}(\boldsymbol{S}_k) \qquad (1) \\ \texttt{Subject to} \quad & \boldsymbol{r} \in \mathcal{C}(\{\boldsymbol{S}_k\}) \end{aligned}$$

- By Lemma 1 , (1) simplifies as the maximization of

$$\sum_{k=1}^{K} (\mu_{\boldsymbol{\pi}(k+1)} - \mu_{\boldsymbol{\pi}(k)}) \log \left| \sum_{i=k}^{K} \left(\boldsymbol{H}_{\boldsymbol{\pi}(i)} \boldsymbol{S}_{\boldsymbol{\pi}(i)} \boldsymbol{H}_{\boldsymbol{\pi}(i)}^{\dagger} \right) + \boldsymbol{S}_z \right| - \sum_{k=1}^{K} \lambda_k \mathtt{Tr}(\boldsymbol{S}_k)$$

- Twice continuously differentiable and concave function of $\{\boldsymbol{S}_k\}$

- Solvable by using gradient-based method, e.g., Newton's method

Dual Problem

- $\{\boldsymbol{S}'_k\}$ and $\{r'_k\}$ are Lagrangian minimizers:

$$g(\boldsymbol{\mu}) = \sum_{k=1}^{K} \lambda_k \mathrm{Tr}(\boldsymbol{S}'_k) - \sum_{k=1}^{K} \mu_k (r'_k - R_k)$$

- Dual problem:

$$d^* = \max_{\boldsymbol{\mu} \succeq 0} g(\boldsymbol{\mu}) \triangleq g(\boldsymbol{\mu}^*)$$

- Search μ_k towards μ_k^*:
  - $R_k - r'_k$ is a sub-gradient for μ_k , $k = 1, \ldots, K$
  - Update μ_k by using sub-gradient based method, e.g., Ellipsoid method

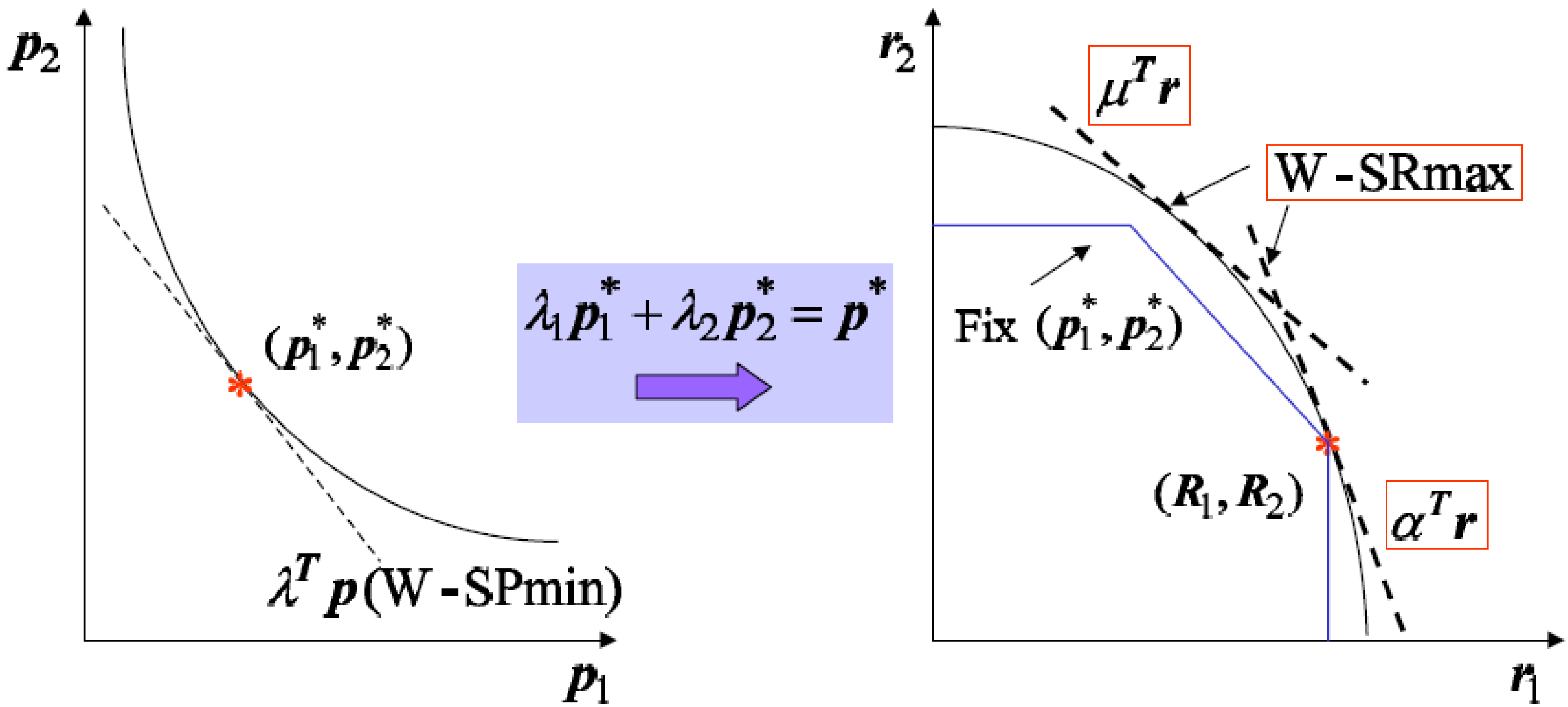
Algorithm

NUS National University of Singapore

$$p^* = d^* = \max_{\boldsymbol{\mu}} \min_{\{\boldsymbol{S}_k\},\{r_k\}} \sum_{k=1}^{K} \lambda_k \mathtt{Tr}(\boldsymbol{S}_k) - \sum_{k=1}^{K} \mu_k \left(r_k - R_k\right)$$

- "min": Fix $\boldsymbol{\mu}$, obtain $g(\boldsymbol{\mu})$
- "max": Update $\boldsymbol{\mu}$ towards $\boldsymbol{\mu}^*$
- Iterates the above two until the algorithm converges

Illustration via Lagrange Duality

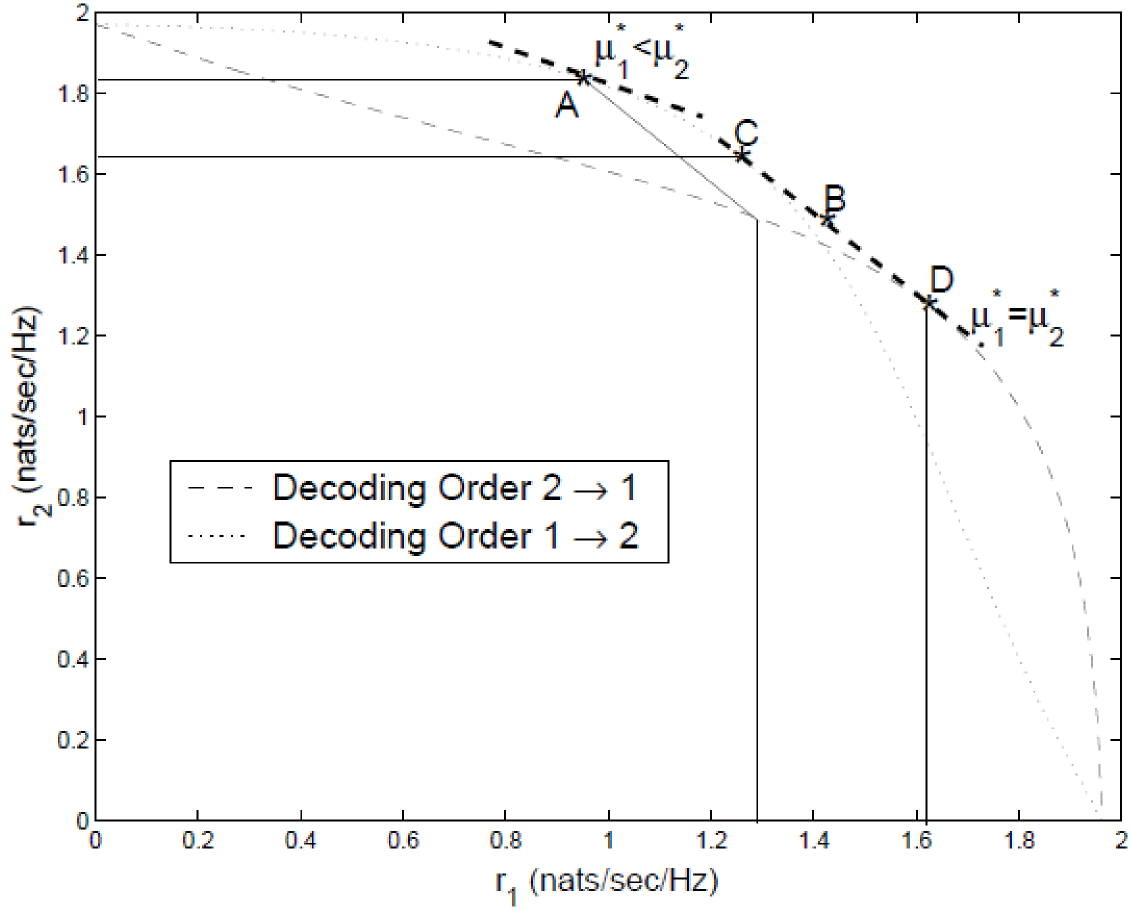


Power Region: $r_1 \geq R_1, r_2 \geq R_2$

Dual Capacity Region: $\lambda_1 p_1 + \lambda_2 p_2 \leq p^*$

Lagrange duality: find $\mu^* = \alpha$

Optimal Decoding Order (K=2)



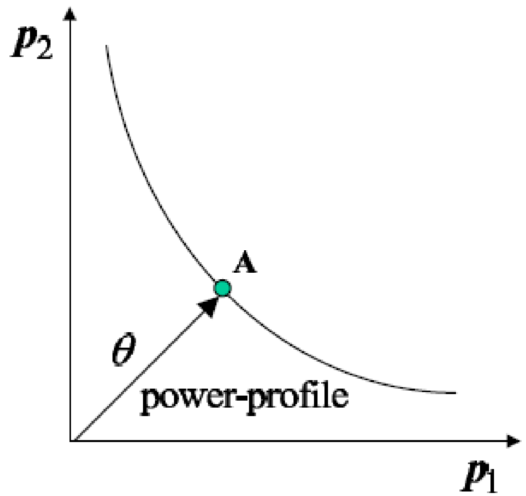
- Case I: $\mu_1^* < \mu_2^*$: $\boldsymbol{\pi}^*$ is $1 \rightarrow 2$, e.g., R as Point A
- Case II: $\mu_1^* = \mu_2^*$: $\boldsymbol{\pi}^*$ is time-sharing of $1 \rightarrow 2$ and $2 \rightarrow 1$, e.g., R as Point B

Optimal Decoding Order (arbitrary K)

- Case I:
  - If all $\{\mu_k^*\}$ are distinct ...
  - $\boldsymbol{\pi}$ is given by $\mu^*_{\boldsymbol{\pi}(1)} < \mu^*_{\boldsymbol{\pi}(2)} < \cdots < \mu^*_{\boldsymbol{\pi}(K)}$
- Case II:
  - If $\{\mu_k^*\}$ are equal for all $k \in \mathcal{J}, \mathcal{J} \subseteq \{1, 2, \cdots, K\}$ ...
  - $\boldsymbol{\pi}_{\mathcal{J}}$ is given by time-sharing of at most $|\mathcal{J}|$ different decoding orders

Power Region Characterization via Power Profile Approach

- Given:
  - rate constraint: $R_1, R_2, \ldots, R_K$
  - power profile vector : $\boldsymbol{\theta} = (\theta_1, \theta_2, \cdots, \theta_K) \in \mathbb{R}_+^K, \sum_{k=1}^{K} \theta_k = 1$
- Goal: find minimum $\{p_1, p_2, \cdots, p_k\}$ such that $\frac{p_k}{p_{k'}} = \frac{\theta_k}{\theta_{k'}}$ $, \forall k, k' \in \{1, 2, \cdots, K\}$
- Applications: proportionally-fair power consumption

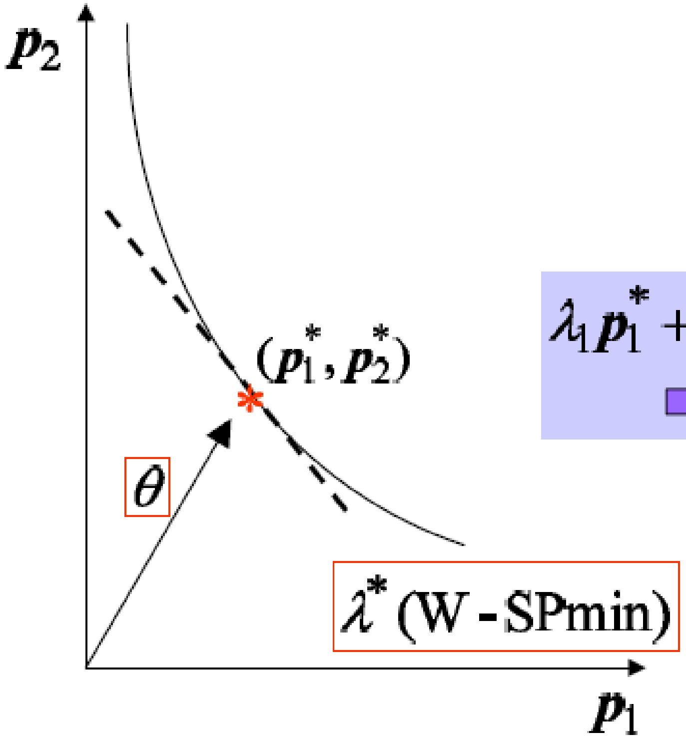


Sum-Power Minimization under Power Profile Constraints

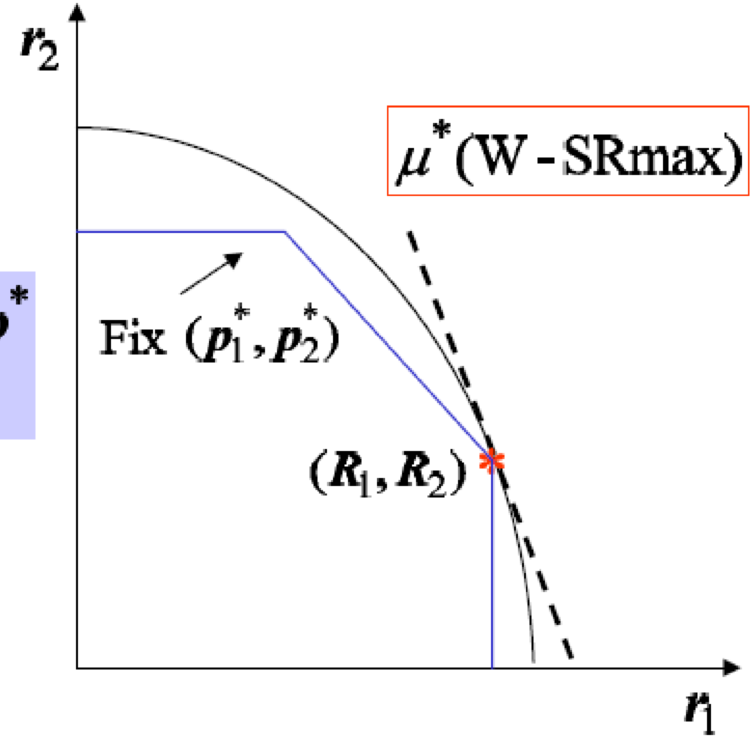
$$
\begin{array}{rl}
\texttt{Minimize} & P \\
\texttt{Subject to} & r_k \geq R_k \quad \forall k \\
& \boldsymbol{r} \in \mathcal{C}(\{\boldsymbol{S}_k\}) \\
& \boldsymbol{S}_k \succeq 0 \;\; \forall k \\
& \texttt{Tr}\left(\boldsymbol{S}_k\right) \leq \theta_k P \quad \forall k \\
& P \geq 0
\end{array}
$$

- Solutions implemented via Lagrange duality
- Dual variables:
  - μ_k w.r.t. $r_k \geq R_k$, $k = 1, \ldots, K$
  - λ_k w.r.t. $\texttt{Tr}\left(\boldsymbol{S}_k\right) \leq \theta_k P$, $k = 1, \ldots, K$

Illustration via Lagrange Duality



$$\lambda_1 p_1^* + \lambda_2 p_2^* = p^*$$



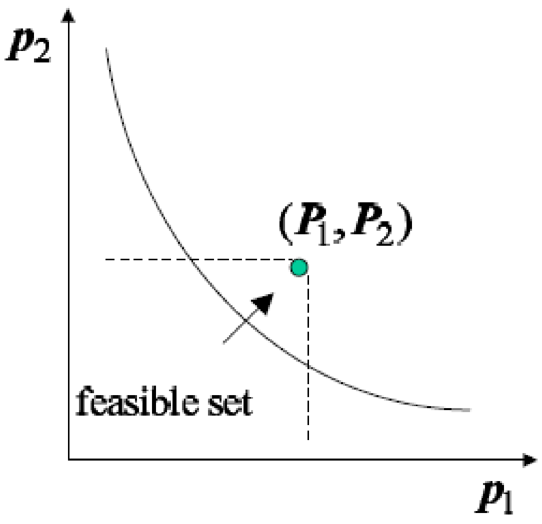
Power Region: $r_1 \geq R_1, r_2 \geq R_2$

Dual Capacity Region: $\lambda_1 p_1 + \lambda_2 p_2 \leq p^*$

Lagrange duality: find both λ^* and μ^*

Admission Problem

- Given:
  - rate constraint: $R_1, R_2, \ldots, R_K$
  - maximum power constraint : $\boldsymbol{P} = (P_1, P_2, \cdots, P_K) \in \mathbb{R}_+^K$
- Goal: check whether $\boldsymbol{P} \in \mathcal{P}(\boldsymbol{R})$
  - If yes, find a feasible set of powers
  - If no, find a proof for infeasibility

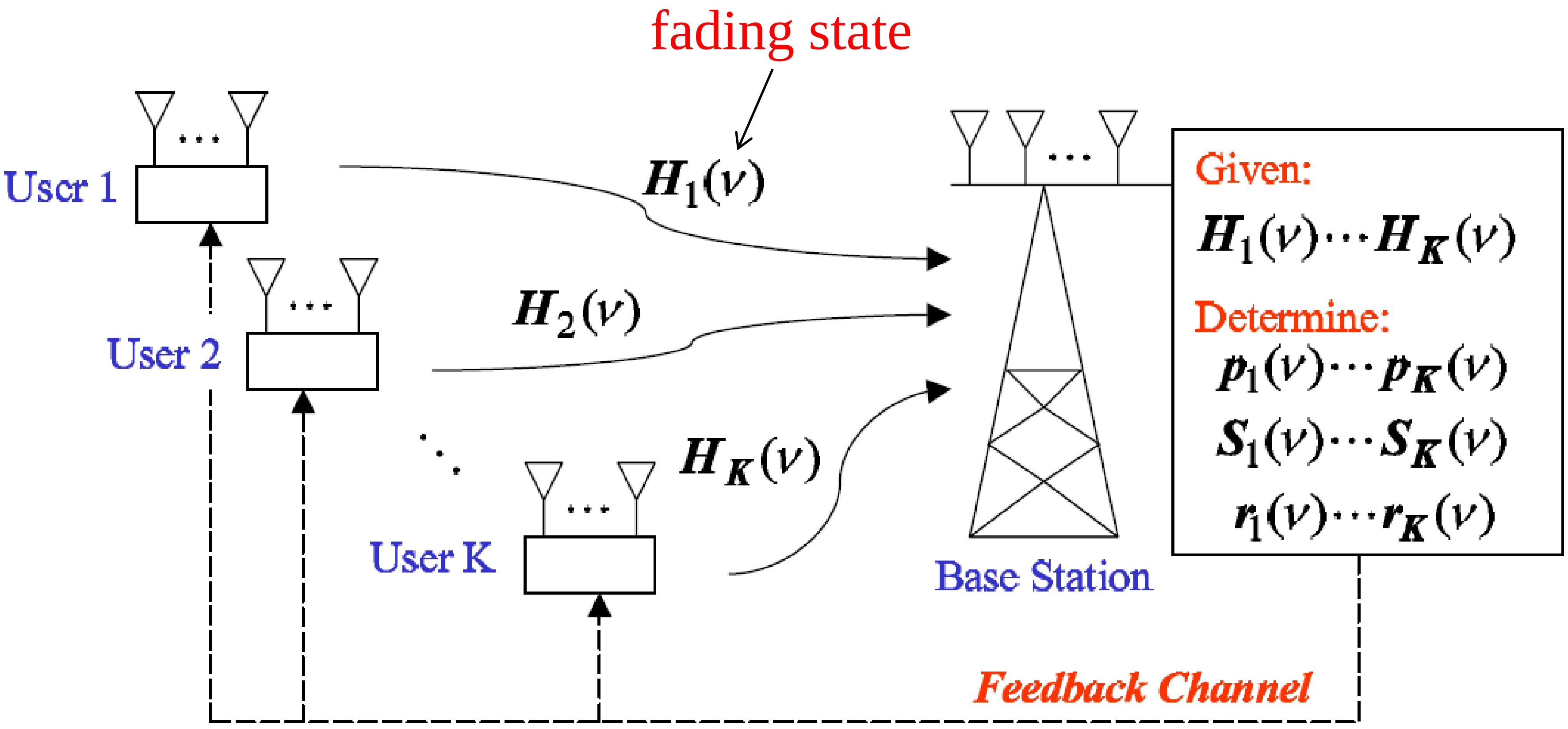


Feasibility Test for Admission Problem

$$
\begin{array}{rl}
\texttt{Minimize} & 0 \\
\texttt{Subject to} & r_k \geq R_k \quad \forall k \\
& \boldsymbol{r} \in \mathcal{C}(\{\boldsymbol{S}_k\}) \\
& \texttt{Tr}(\boldsymbol{S}_k) \leq P_k \quad \forall k \\
& \boldsymbol{S}_k \succeq 0 \quad \forall\ k
\end{array}
$$

Solvable by Lagrange duality, similarly as Sum-Power Minimization under Power Profile Constraints

Extension: Fading MIMO-MAC



Channel Model of Fading MIMO-MAC

$$\boldsymbol{y} = [\boldsymbol{H}_1(\nu) \ \cdots \boldsymbol{H}_K(\nu)\ ] \begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_K \end{bmatrix} + \boldsymbol{z}$$

$\boldsymbol{\nu}$: fading state

- state space is continuous and infinite
- state process is stationary and ergodic

W-SPmin for Fading MIMO-MAC

- Both $\boldsymbol{S}_k(\nu)$ and $\boldsymbol{r}(\nu)$ depend on ν

- Problem formulation:

$$\begin{aligned} \texttt{Minimize} \quad & \sum_{k=1}^{K} \lambda_k \mathbb{E}_\nu \left[\texttt{Tr} \left(\boldsymbol{S}_k(\nu) \right) \right] \\ \texttt{Subject to} \quad & \mathbb{E}_\nu [r_k(\nu)] \geq R_k \quad \forall k \\ & \boldsymbol{r}(\nu) \in \mathcal{C}_\nu \left(\{ \boldsymbol{S}_k(\nu) \} \right) \quad \forall \nu \\ & \boldsymbol{S}_k(\nu) \succeq 0 \quad \forall \nu, k \end{aligned}$$

- Solutions implemented via Lagrange dual decomposition

- Dual variables: μ_k w.r.t. $\mathbb{E}_\nu [r_k(\nu)] \geq R_k$, $k = 1, \ldots, K$

Lagrange Dual Decomposition

- Lagrangian:

$$\mathcal{L}(\{\boldsymbol{S}_k(\nu)\}, \{r_k(\nu)\}, \boldsymbol{\mu}) = \sum_{k=1}^{K} \lambda_k \mathbb{E}_\nu \left[\mathrm{Tr}\left(\boldsymbol{S}_k(\nu)\right)\right] - \sum_{k=1}^{K} \mu_k \left(\mathbb{E}_\nu[r_k(\nu)] - R_k\right)$$

- Dual function:

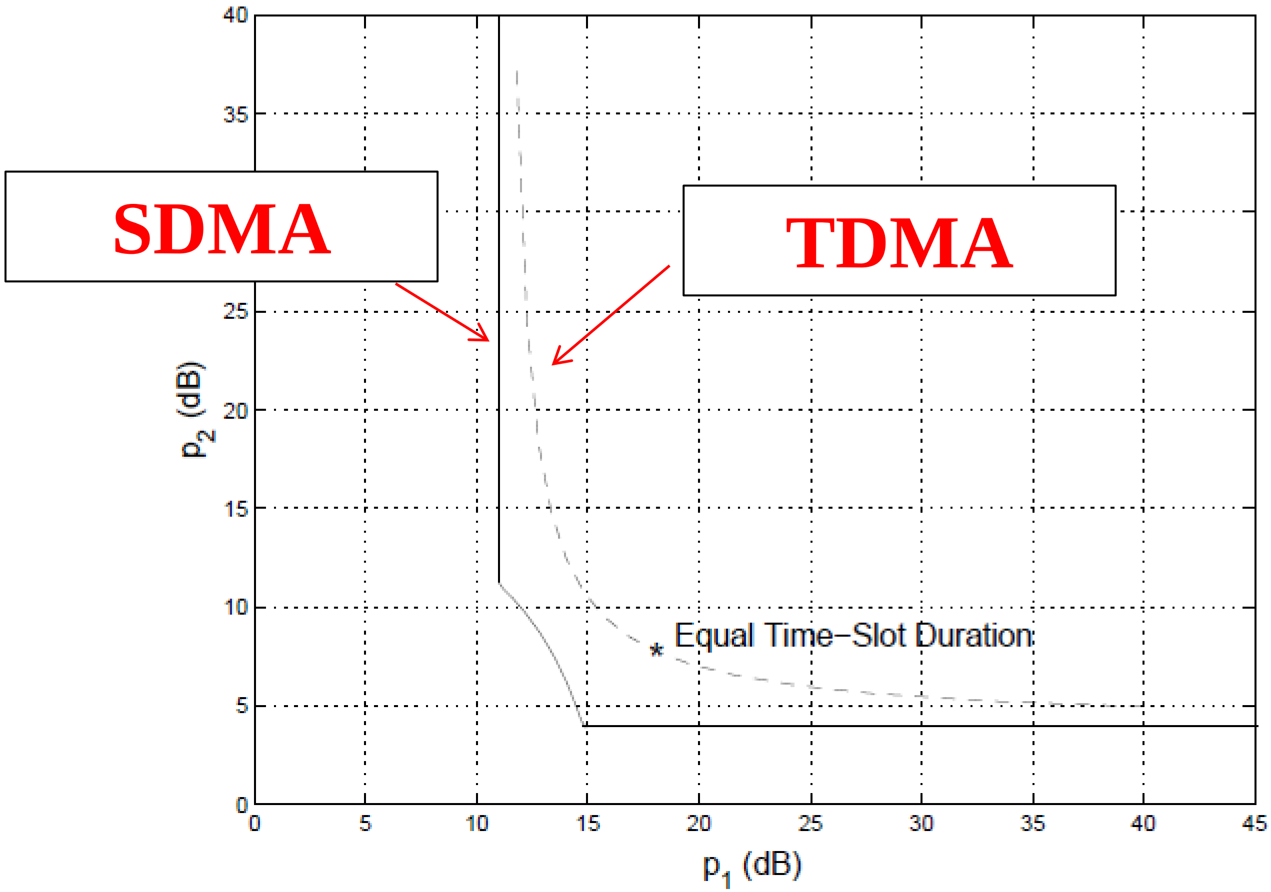
$$\begin{aligned} g(\boldsymbol{\mu}) &= \min_{\boldsymbol{S}_k(\nu), r_k(\nu), \forall k, \nu} \mathcal{L}(\{\boldsymbol{S}_k(\nu)\}, \{r_k(\nu)\}, \boldsymbol{\mu}) \\ &= \mathbb{E}_\nu \left[\underbrace{\min_{\boldsymbol{S}_k(\nu), r_k(\nu), \forall k} \left\{ \sum_{k=1}^{K} \lambda_k \mathrm{Tr}\left(\boldsymbol{S}_k(\nu)\right) - \sum_{k=1}^{K} \mu_k r_k(\nu) \right\}}_{\triangleq g_\nu(\boldsymbol{\mu})} \right] + \sum_{k=1}^{K} \mu_k R_k \end{aligned}$$

- Dual problem:

$$d^* = \max_{\boldsymbol{\mu} \succeq 0} g(\boldsymbol{\mu})$$

Power Region Comparison: SDMA vs. TDMA

$$\boldsymbol{R} = [2 \quad 1] \ \text{nats/sec/Hz}$$



- Two-user fading MIMO-MAC
- Number of transmit antennas: 2, $k = 1, 2$
- Number of receive antennas: 2
- $\boldsymbol{H}_k(\nu) = \boldsymbol{H}_w \boldsymbol{R}_{tk}^{1/2}, \ k = 1, 2$

$$\boldsymbol{R}_{t1} = \begin{bmatrix} 1 & 0.4 \\ 0.4 & 1 \end{bmatrix}$$

$$\boldsymbol{R}_{t2} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

Concluding Remarks on Power Minimization in MU-MIMO

- **Power region characterization for MIMO-MAC via**
  - W-SPmin
  - power profile
- **Power/capacity region duality via Lagrange duality**
- **Lagrange dual decomposition**
  - a general tool for optimal resource allocation over parallel (e.g., fading, multi-carrier) channels

Agenda

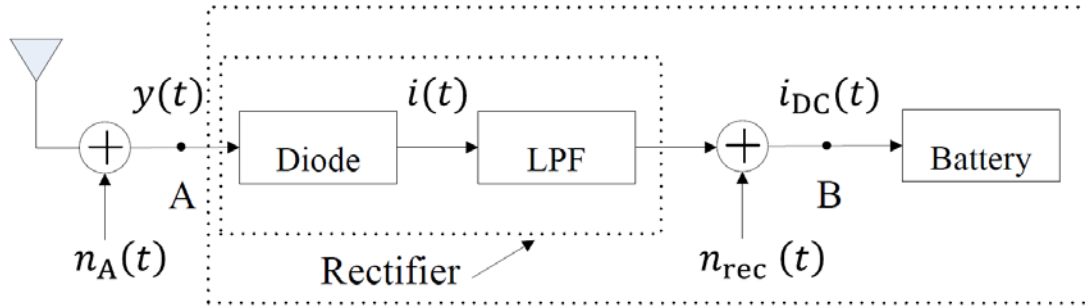
- Overview of the talk
- Exploiting multi-antennas in
  - Cognitive Radio Networks
  - Cooperative Multi-Cell
  - Two-Way Relay Networks
  - Green Cellular Networks
  - **Wireless Information and Power Transfer**
- Concluding remarks

Topic #5: MIMO Broadcasting for Wireless Information and Power Transfer

RF-Based Wireless Power Transfer

Energy Transmitter

Energy Receiver



RF Energy Receiver Architecture

- Why **RF-based** Wireless Power Transfer (WPT)?
  - longer transmission distance than near-field WPT (e.g., RFID)
  - many advantages over traditional batteries and energy harvesting
    - lower cost: no need to replace/dispose batteries
    - safer: in e.g. toxic environment
    - more robust: overcome lack of light, temp. diff., or vibration (for energy harvesting)
    - more convenient: controllable, continuous, schedulable on demand
  - abundant applications in emerging wireless sensor networks
    - building automation, healthcare, smart grid, structural monitoring…..
  - current limitation
    - low received power (<1uW at distance > 5m and transmit power <1W)

High-Efficiency WPT: An Energy Beamforming Approach

- Transmit covariance matrix:

$$\boldsymbol{S} = \mathbb{E}[\boldsymbol{x}(n)\boldsymbol{x}^H(n)]$$

- Optimization problem (convex):

$$\boxed{\begin{aligned} \max_{\boldsymbol{S}} \quad & Q := \mathtt{tr}\left(\boldsymbol{G}\boldsymbol{S}\boldsymbol{G}^H\right) \\ \mathtt{s.t.} \quad & \mathtt{tr}(\boldsymbol{S}) \le P, \boldsymbol{S} \succeq 0. \end{aligned}}$$

- Beamforming is optimal:

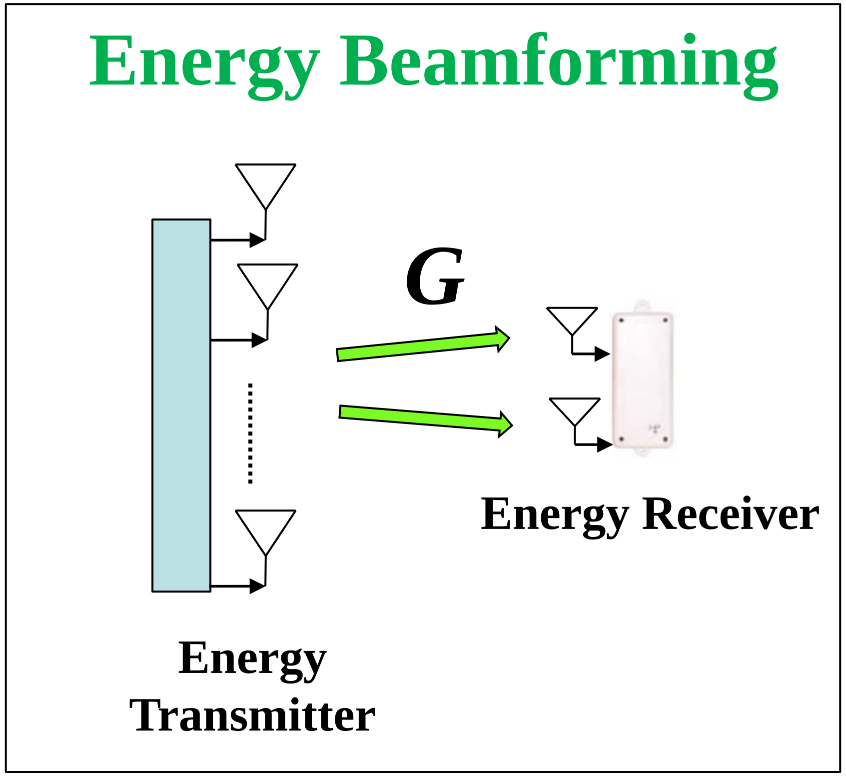
$$\boldsymbol{S}_{\mathrm{EH}} = P\boldsymbol{v}_1\boldsymbol{v}_1^H$$

$\boldsymbol{v}_1$: eigenvector of $\boldsymbol{G}^H\boldsymbol{G}$ corresponding to the largest eigenvalue g_1

- Maximum received power:

beamforming gain

$$Q_{\max} = g_1 P$$

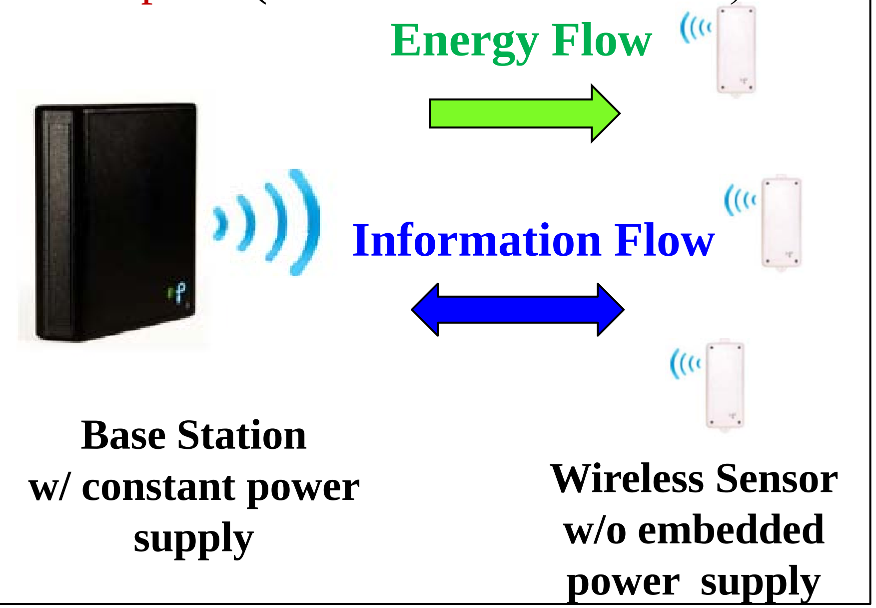


Wireless Information and Power Transfer: A Unified Study

- **Hybrid Information/Energy Flow:**
  - “asymmetric” downlink/uplink transmissions
- **Technical Challenges:**
  - **joint energy and communication scheduling**
    - energy-aware communication
    - communication-aware energy transfer
  - **information and power transfer (downlink)**
    - orthogonal transmissions
    - simultaneous transmissions (more efficient)
      - ✓ circuit limitation: existing energy receivers cannot decode information directly
      - ✓ possible solutions:
        - ❖ **MIMO broadcasting** **[ZhangHo11]**
        - ❖ “opportunistic” energy harvesting **[LiuZhangChua12]**
        - ❖ “integrated” energy/information receivers **[ZouZhangHo12]**

RF-Powered Wireless Sensor Network

- Downlink (Base Station → Sensors)
- Uplink (Sensors → Base Station)



[ZhangHo11]: R. Zhang and C. K. Ho, “MIMO broadcasting for simultaneous wireless information and power transfer,” IEEE Globecom, 2011. (Available Online at http://arxiv.org/abs/1105.4999)

[LiuZhangChua12]: L. Liu, R. Zhang, and K. C. Chua, “Wireless information transfer with opportunistic energy harvesting,” IEEE ISIT, 2012. (Available Online at http://arxiv.org/abs/1204.2035)

[ZouZhangHo12]: X. Zhou, R. Zhang, and C. K. Ho, “Wireless information and power transfer: architecture design and rate-energy tradeoff,” submitted to IEEE Globecom, 2012. (Available Online at http://arxiv.org/abs/1205.0618)

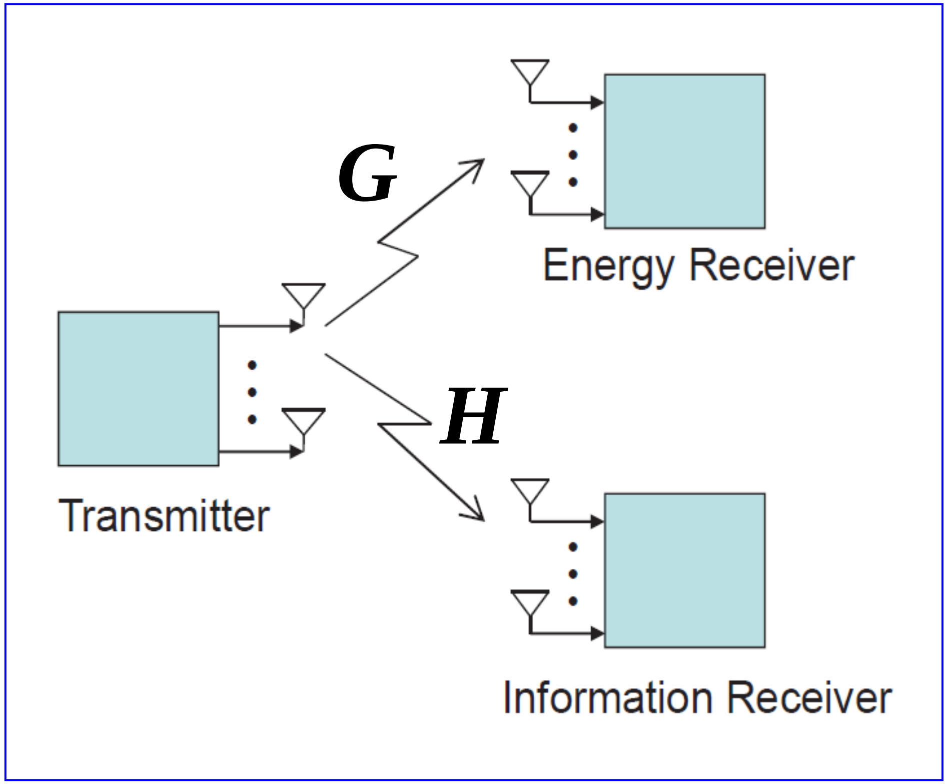
MIMO Broadcasting for Wireless Information and Power Transfer [ZhangHo11]

- Two scenarios:
  - **separated receivers**: G ≠ H
  - **co-located receivers**: G = H
- Objective: characterize "rate-energy" region
  - extension of capacity-energy function of SISO AWGN channels [Varshney08], [GroverSahai10]
- Optimization problem (convex):

$$\begin{aligned} \max_{\boldsymbol{S}} \quad & \log \left| \boldsymbol{I} + \boldsymbol{H}\boldsymbol{S}\boldsymbol{H}^H \right| \\ \text{s.t.} \quad & \mathtt{tr}\left(\boldsymbol{G}\boldsymbol{S}\boldsymbol{G}^H\right) \geq \bar{Q} \\ & \mathtt{tr}(\boldsymbol{S}) \leq P \\ & \boldsymbol{S} \succeq 0. \end{aligned}$$

generalized linear transmit power constraint

$$\boldsymbol{G} \in \mathbb{C}^{N_{\text{EH}} \times M}, \ \boldsymbol{H} \in \mathbb{C}^{N_{\text{ID}} \times M}$$



A three-node MIMO broadcast system with perfect CSIT/CSIR

Separated Receiver Case (G ≠ H)

- Semi-closed-form optimal solution:

$$\boxed{\boldsymbol{S}^* = \boldsymbol{A}^{-1/2}\tilde{\boldsymbol{V}}\tilde{\boldsymbol{\Lambda}}\tilde{\boldsymbol{V}}^H\boldsymbol{A}^{-1/2}}$$

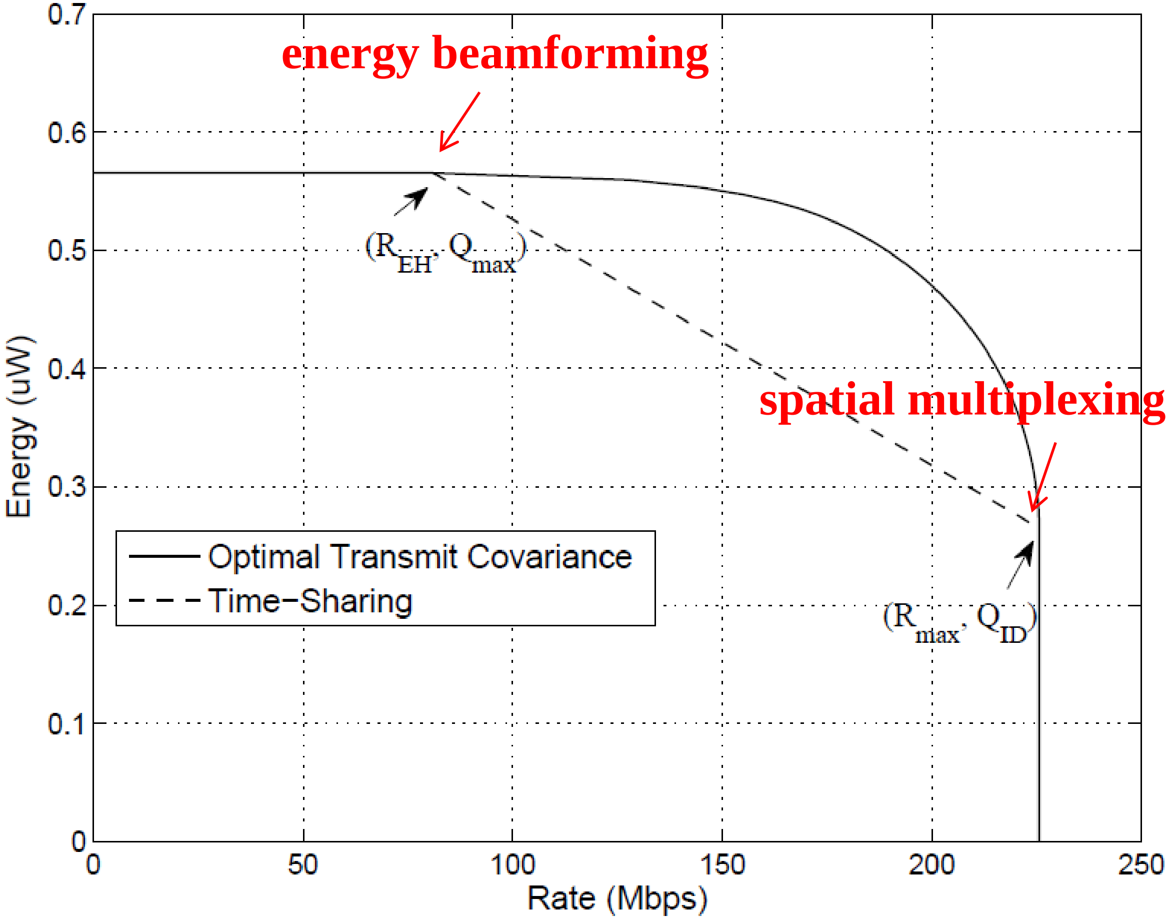
where

- μ^*: optimal dual variable for transmit power constraint
- λ^*: optimal dual variable for receive power constraint
- $\boldsymbol{A} = \mu^*\boldsymbol{I} - \lambda^*\boldsymbol{G}^H\boldsymbol{G}$
- $\mu^* > \lambda^* g_1$ (largest eigenvalue of $\boldsymbol{G}^H\boldsymbol{G}$)
- $\tilde{\boldsymbol{V}}$: obtained from the (reduced) SVD $\boldsymbol{H}\boldsymbol{A}^{-1/2} = \tilde{\boldsymbol{U}}\tilde{\boldsymbol{\Gamma}}^{1/2}\tilde{\boldsymbol{V}}^H$
- $\tilde{\boldsymbol{\Gamma}} = \mathtt{diag}(\tilde{h}_1, \ldots, \tilde{h}_T) \succeq 0$, $T = \min(M, N_{\mathrm{ID}})$
- $\tilde{\boldsymbol{\Lambda}} = \mathtt{diag}(\tilde{p}_1, \ldots, \tilde{p}_T)$, with $\tilde{p}_i = (1 - 1/\tilde{h}_i)^+, i = 1, \ldots, T$

- Optimal solution obtained by Lagrange duality method

Rate-Energy Region (Separated Receiver)

$$\mathcal{C}_{\mathrm{R-E}}(P) \triangleq \left\{ (R, Q) : R \leq \log |\boldsymbol{I} + \boldsymbol{H}\boldsymbol{S}\boldsymbol{H}^H|, Q \leq \mathtt{tr}(\boldsymbol{G}\boldsymbol{S}\boldsymbol{G}^H), \mathtt{tr}(\boldsymbol{S}) \leq P, \boldsymbol{S} \succeq 0 \right\}$$



- $M = N_{\mathrm{EH}} = N_{\mathrm{ID}} = 4$
- $P = 0.1\mathrm{W}$ (20dBm)
- $f_c = 900\mathrm{MHz}$, $B_w = 10\mathrm{MHz}$
- $d = 10\mathrm{m}$ (60dB signal power attenuation)
- $\boldsymbol{G}$, $\boldsymbol{H}$: i.i.d Rayleigh fading
- $N_0 = -130\mathrm{dBm/Hz}$
- per-antenna average received power: 100nW
- per-antenna average received SNR: 20dB
- energy conversion efficiency: 50%

Co-Located Receiver Case (G=H)

- Optimal solution simplified as

$$\boxed{\boldsymbol{S}^* = \boldsymbol{V}_H \boldsymbol{\Sigma} \boldsymbol{V}_H^H}$$

- $\boldsymbol{V}_H$: obtained from the (reduced) SVD $\boldsymbol{H} = \boldsymbol{U}_H \boldsymbol{\Gamma}_H^{1/2} \boldsymbol{V}_H^H$
- $\boldsymbol{\Gamma}_H = \texttt{diag}(h_1, \ldots, h_T) \succeq 0$, $T = \min(M, N_{\text{ID}})$
- $\boldsymbol{\Sigma} = \texttt{diag}(\hat{p}_1, \ldots, \hat{p}_T)$, with $\hat{p}_i = \left(\frac{1}{\mu^* - \lambda^* h_i} - \frac{1}{h_i}\right)^+, \quad i = 1, \ldots, T$
- $\mu^* > \lambda^* h_1$

- Optimal solution obtained by Lagrange duality method

- Question: Is the corresponding R-E region achievable by practical receivers?

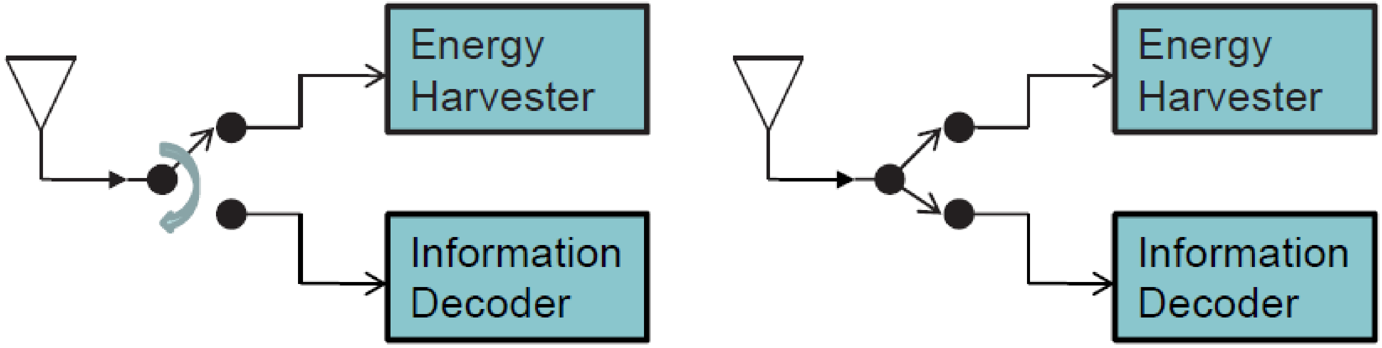
Practical Receivers

❑ Circuit Limitation

- Existing RF-based EH circuits cannot decode information directly
- Thus, previously established rate-energy region only provides performance upper bound

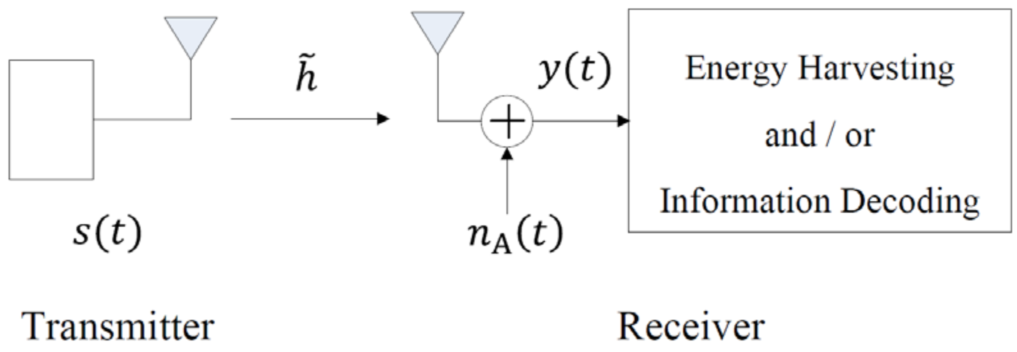
❑ Practical Receiver Design

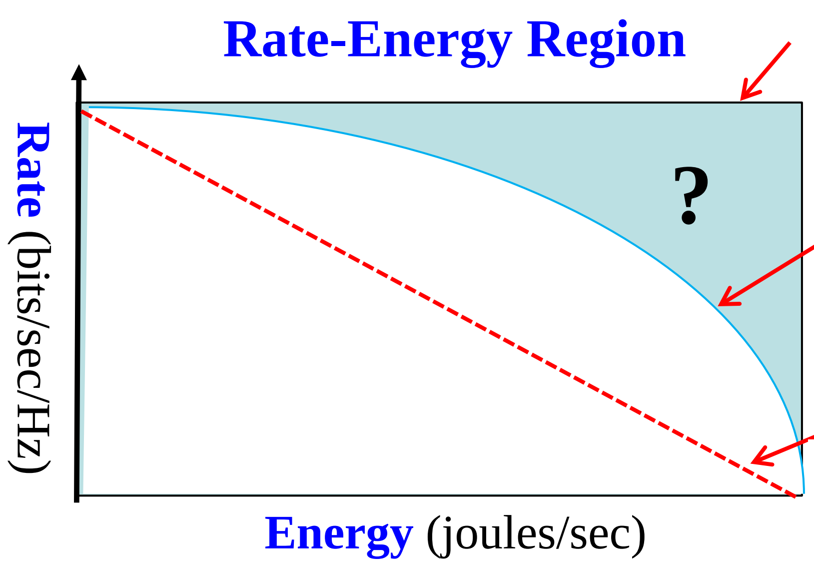
- Time switching
- Power splitting
- Antenna switching (a special case of power splitting)



(a) Time Switching (b) Power Splitting

Special Case: SISO AWGN Channel





R-E Region Upper Bound

$$\mathcal{C}_{\text{R-E}}^{\text{UB}}(P) \triangleq \left\{(R,Q) : R \le \log(1+hP), Q \le hP\right\}$$

Power Splitting

$$\mathcal{C}_{\text{R-E}}^{\text{PS}}(P) \triangleq \bigcup_{\rho:\ 0\le\rho\le 1} \left\{(R,Q) : R \le \log(1+(1-\rho)hP), Q \le \rho hP\right\}$$

Time Switching

$$\mathcal{C}_{\text{R-E}}^{\text{TS}}(P) \triangleq \bigcup_{\alpha:\ 0\le\alpha\le 1} \left\{(R,Q) : R \le (1-\alpha)\log(1+hP), Q \le \alpha hP\right\}$$

MIMO Case

❑ Time Switching

$$\mathcal{C}^{\mathrm{TS}}_{\mathrm{R-E}}(P) \triangleq \bigcup_{\alpha:\ 0\le\alpha\le 1} \Big\{ (R,Q) : R \le (1-\alpha)\log|\boldsymbol{I}+\boldsymbol{H}\boldsymbol{S}_1\boldsymbol{H}^H|, Q \le \alpha\mathtt{tr}(\boldsymbol{H}\boldsymbol{S}_2\boldsymbol{H}^H), \mathtt{tr}(\boldsymbol{S}_1) \le P, \mathtt{tr}(\boldsymbol{S}_2) \le P, \boldsymbol{S}_1 \succeq 0, \boldsymbol{S}_2 \succeq 0 \Big\}$$

❑ Power Splitting

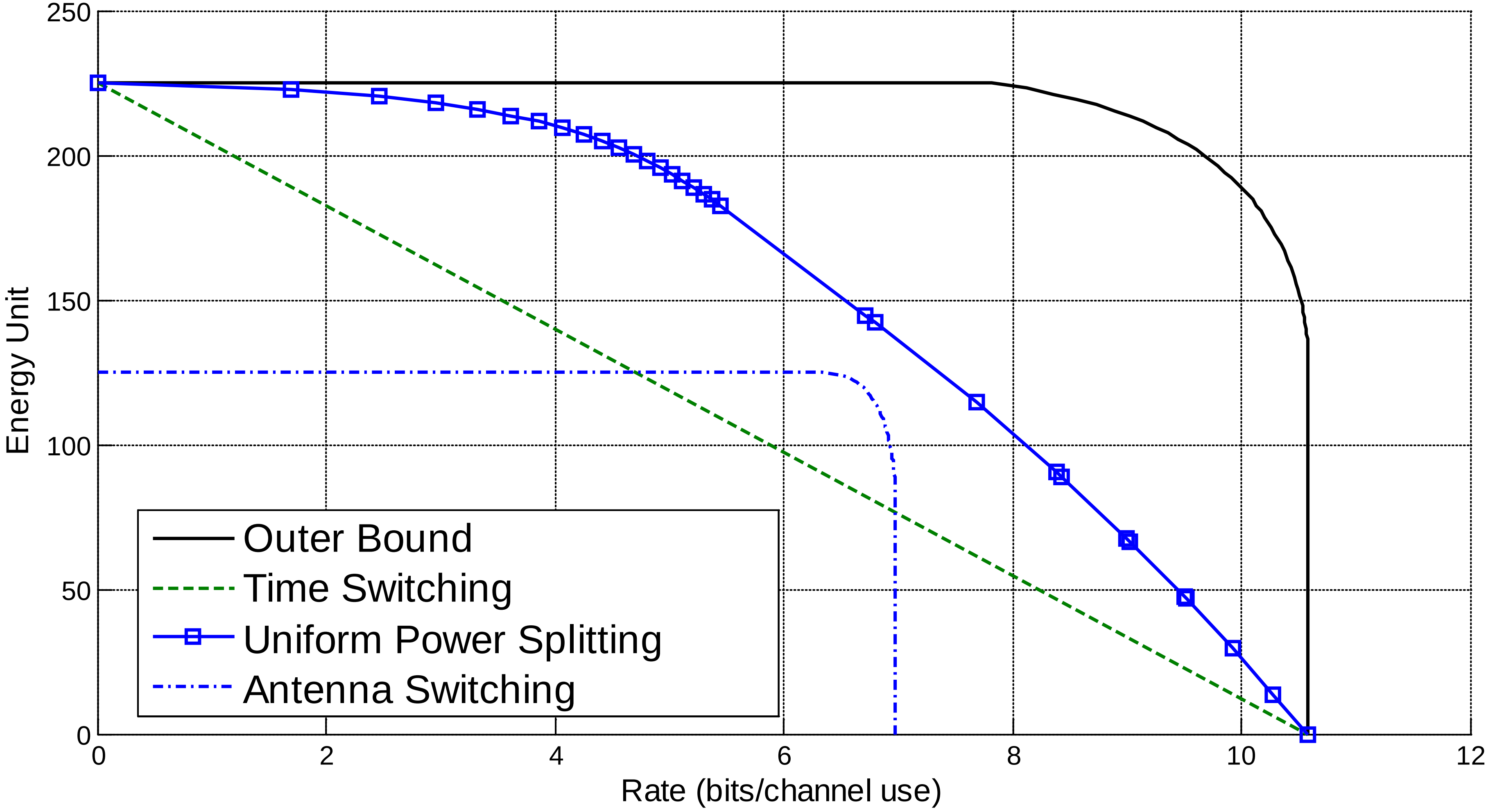
$$\mathcal{C}^{\mathrm{PS}}_{\mathrm{R-E}}(P) \triangleq \bigcup_{\{\rho_i\}:\ 0\le\rho_i\le 1,\forall i} \Big\{ (R,Q) : R \le \log|\boldsymbol{I}+\bar{\boldsymbol{\Lambda}}_\rho^{1/2}\boldsymbol{H}\boldsymbol{S}\boldsymbol{H}^H\bar{\boldsymbol{\Lambda}}_\rho^{1/2}|, Q \le \mathtt{tr}(\boldsymbol{\Lambda}_\rho^{1/2}\boldsymbol{H}\boldsymbol{S}\boldsymbol{H}^H\boldsymbol{\Lambda}_\rho^{1/2}), \mathtt{tr}(\boldsymbol{S}) \le P, \boldsymbol{S} \succeq 0 \Big\}$$

where $\boldsymbol{\Lambda}_\rho = \mathtt{diag}(\rho_1, \ldots, \rho_N)$, $\bar{\boldsymbol{\Lambda}}_\rho = \boldsymbol{I} - \boldsymbol{\Lambda}_\rho$.

- Two Special Cases:
  - Uniform Power Splitting: $\rho_i = \rho, \forall i,\ 0 \le \rho \le 1$
  - On-Off Power Splitting (Antenna Switching): $\rho_i = 0, i \in \Omega;\ \rho_i = 1, i \in \bar{\Omega}$

Rate-Energy Region (Co-Located Receiver)

$$M_t = N_{\mathrm{EH}} = N_{\mathrm{ID}} = 2,\ P = 100 \quad \boldsymbol{G} = \boldsymbol{H} = [1, 0.5; 0.5,\ 1]$$



Concluding Remarks on Wireless Information and Power Transfer

❑ Exploit MIMO broadcasting for wireless information and power transfer

- wireless power transfer: energy beamforming is optimal
- wireless information transfer: spatial multiplexing is optimal
- fundamental tradeoff: rate-energy region
- separated vs. co-located receivers
- "useful" interference (from viewpoint of wireless power transfer)

❑ Practical circuit limitation

- existing energy receiver cannot decode information directly
- practical receiver designs: time switching vs. power splitting
- how to close the gap from R-E region outer bound? (an open problem)

Agenda

NUS National University of Singapore

- ❑ Overview of the talk
- ❑ Exploiting multi-antennas in
  - ➢ Cognitive Radio Networks
  - ➢ Cooperative Multi-Cell
  - ➢ Two-Way Relay Networks
  - ➢ Green Cellular Networks
  - ➢ Wireless Information and Power Transfer
- ❑ **Concluding remarks**

Concluding Remarks

➢ MU-MIMO Optimization

- **New applications**
  - ✓ cognitive radio networks, cooperative multi-cell, two-way relay networks, green cellular networks, wireless information and power transfer….
- **Main challenges**
  - ✓ **generalized linear transmit power constraint**: interference-power constraint, per-antenna power constraint, per-BS power constraint, harvested power constraint…
  - ✓ **non-convex rate maximization:** broadcast channel, interference channel, relay channel…
  - ✓ **distributed implementation**: imperfect sensing/estimation, limited-rate feedback/backhaul, limited computing power….
- **Useful tools**
  - ✓ **optimization theory**: Lagrange duality, nonlinear programming (GP, QCQP, SOCP, SDP), non-convex optimization (branch & bound, monotonic optimization, outer polyblock approximation, sequential convex programming…), alternating/cyclic projection, sub-gradient, ellipsoid method, SDP relaxation, dual decomposition, robust optimization…
  - ✓ **communication and signal processing**: cognitive transmission, cooperative feedback, interference diversity, active interference control, uplink-downlink duality, interference alignment, improper complex Gaussian signaling, symbol extension, rate/power profile approach, power/rate region duality, network coding, compressive sensing…
- **An ongoing very active area of research**
  - ✓ coherently integrating expertise from multiple fields such as optimization, signal processing, communication theory, information theory, and circuit theory

Thank you and please direct your inquiries to

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