2012 IEEE Signal Processing Society Summer School

Compressive Sensing for Cognitive Radio

Prof. Zhi (Gerry) Tian

Michigan Technological University

Houghton, MI, USA

ztian@mtu.edu

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Outline

- Basis of Compressive Sensing (CS)
- Motivation of CS for Cognitive Radio (CR)
- Compressive Spectrum Sensing for CR
  - Compressive sampling of sparse signals
  - Multi-CR cooperative compressive sensing
  - Compressive cyclic feature detection
  - Compressive sensing framework for random processes
- Sparsity-constrained Dynamic Resource Allocation and Waveform Design
- References

Ack: Prof. Georgios B. Giannakis, Univ. of Minnesota, USA
Prof. Geert Leus, Delft Univ. of Technology, the Netherlands

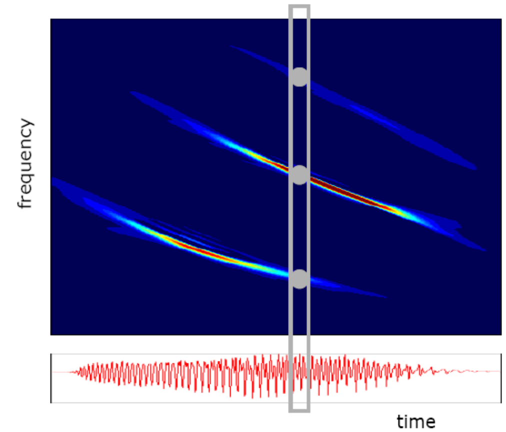
Outline

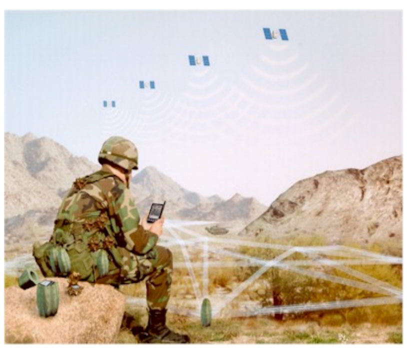
- Basis of Compressive Sensing (CS)
  - Motivating applications
  - Theory
  - Algorithms
- Motivation of CS for Cognitive Radio (CR)
- Compressive Spectrum Sensing for CR
- Sparsity-constrained Dynamic Resource Allocation and Waveform Design
- References

Sparse Signals & Underdetermined Systems

Broadly, how useful is it to study signal sparsity?

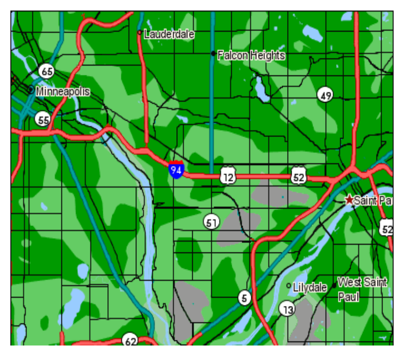
- Many signals are sparse in some basis
  - medical imaging
  - biosensing for DNA microarray
  - remote sensing, astronomy
  - target tracking



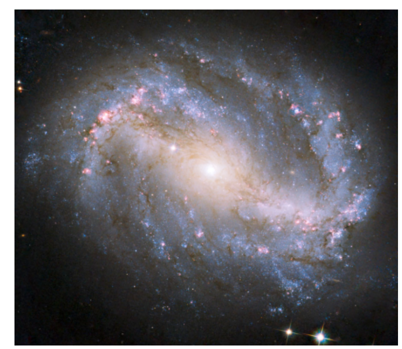


event detection
target tracking

DSP and
communication



imagery and
tomography



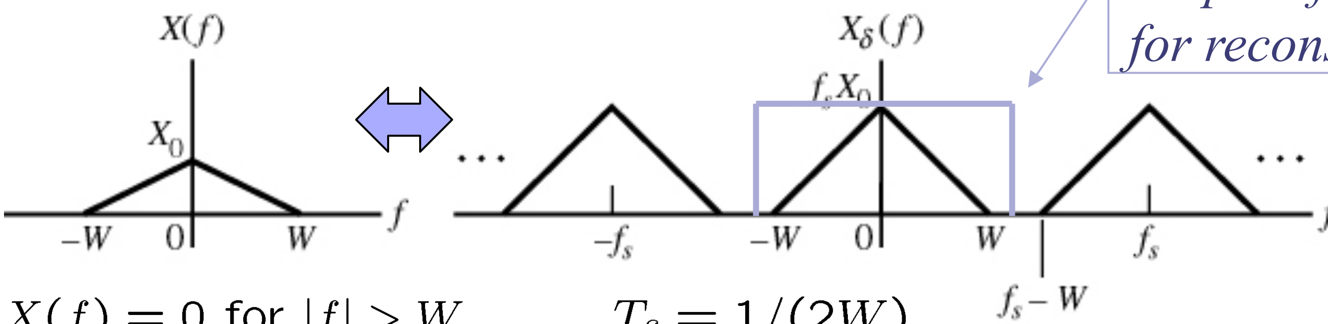
remote sensing
and astronomy

DSP 101: Nyquist-rate Sampling

Q: How fast shall we sample (ADC)? How to recover (DAC)?

A: Nyquist theorem; linear interpolation formula

lowpass filtering for reconstruction



$X(f) = 0$ for $|f| > W$ $\quad T_s = 1/(2W)$

$$\boxed{f_s \geq 2W}$$

- Sampled signal: $x_\delta(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

$$X_\delta(f) = \sum_{n=-\infty}^{\infty} x[nT_s] e^{-j2\pi n f T_s} = \sum_{n=-\infty}^{\infty} x\left[\frac{n}{2W}\right] e^{-j\pi n f/W}$$

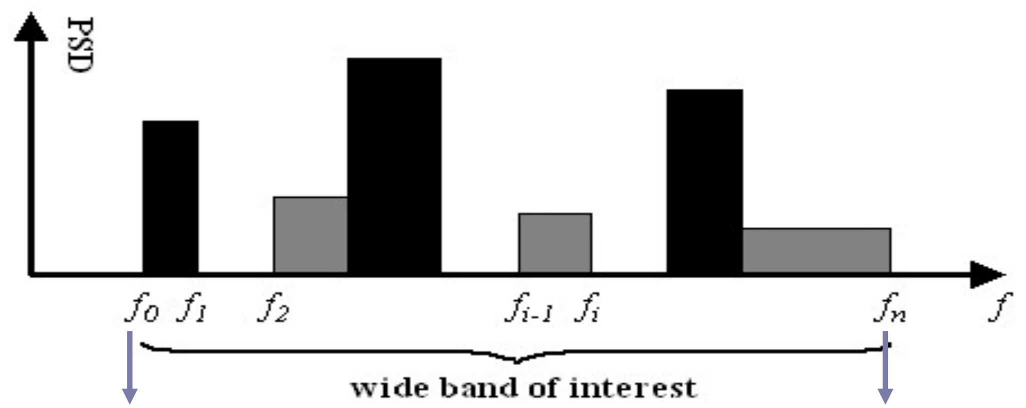
- Recovered signal:

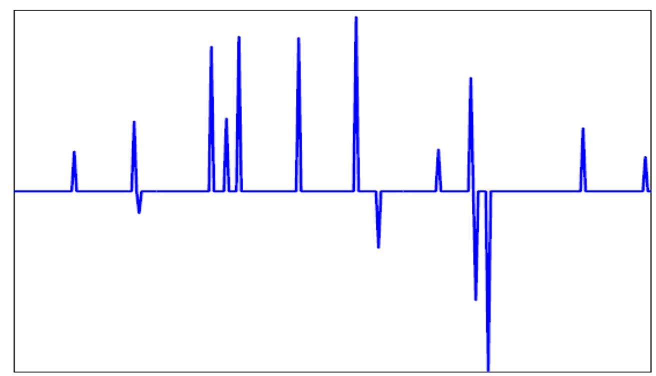
$$X(f) = T_s X_\delta(f), \quad |f| \leq W \quad \Rightarrow \quad \boxed{x(t) = \sum_{n=-\infty}^{\infty} x\left[\frac{n}{2W}\right] sinc(2Wt - n)}$$

Compressive Sampling in DSP

- State of the Art in DSP
  - Trends and demands: wider spectrum; higher data dimension
  - Limitations: high-speed, high-resolution ADC is costly or infeasible

Ex: some signals are band-limited, but with spectrum holes





Lowest frequency Highest frequency

Overall nonzero frequency band: $W_{NZ} << W$

***Q**: What is the minimum # samples needed?*
How to sample? How to recover?

Statistics 101: Linear Regression

- Linear subset regression
  - Goal: find important regressors/predictors/bases
  - Assumption: some regressors are irrelevant

Data *Model*

$$\boxed{\mathbf{b} = \mathbf{A}\,\mathbf{x} + \mathbf{e}}$$

$$A = \begin{pmatrix} | & | & & | \\ \boldsymbol{a}_1 & \boldsymbol{a}_2 & \dots & \boldsymbol{a}_N \\ | & | & & | \end{pmatrix}$$

Regression Matrix

Noise

Regressor

Regression Coefficients

  - Approach: look for sparse $\mathbf{x}$ by solving L_1-regularized least squares

- Applications: gene selection in microarray data analysis, medical diagnosis, stock selection, ……

Sparse Model Selection: Factors That Cause Diabetes

- Goal: which are the major risk factors for diabetes
- Output: quantitative measure of disease progression
- Input: data collected from $K = 442$ patients
  - can be over-determined (interpretability desired), or under-determined (e.g., gene expression analysis)

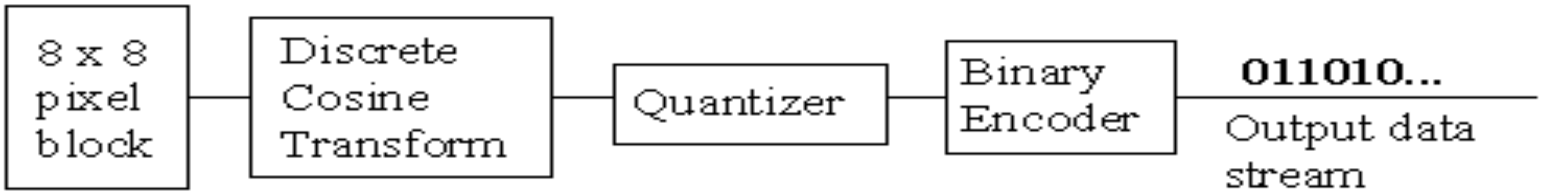
Test Data

	AGE	SEX	BMI	BP	··· Serum	Measurements ···				Response
Patient	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	y
1	59	2	32.1	101	157	93.2	38	4	4.9	87	151
2	48	1	21.6	87	183	103.2	70	3	3.9	69	75
3	72	2	30.5	93	156	93.6	41	4	4.7	85	141
4	24	1	25.3	84	198	131.4	40	5	4.9	89	206
5	50	1	23.0	101	192	125.4	52	4	4.3	80	135
6	23	1	22.6	89	139	64.8	61	2	4.2	68	97
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
441	36	1	30.0	95	201	125.2	42	5	5.1	85	220
442	36	1	19.6	71	250	133.2	97	3	4.6	92	57

Predictors

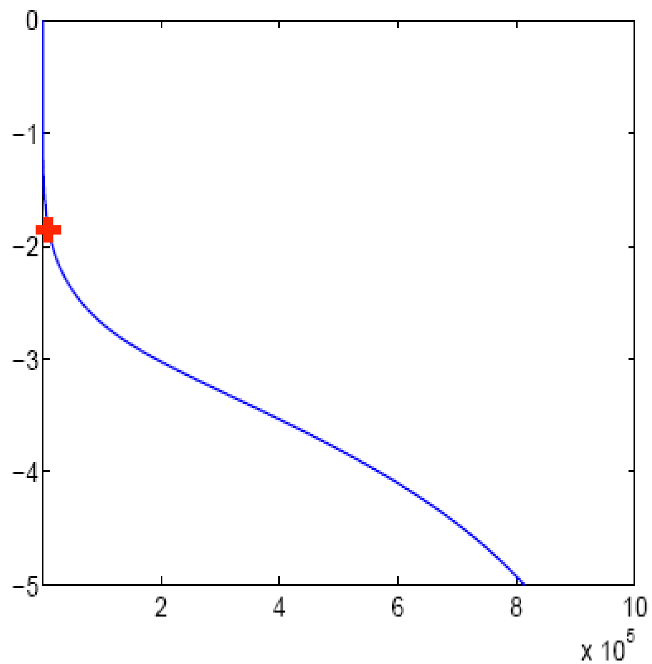
Image Compression 101

JPEG Compression



1 megapixel image

25k term approx



Scaled K-Term Approximation Error

2.5% allowed (within 2 digits in MSE)

Traditional Compression

- Typical Signal Acquisition Scenario

$\underline{x} \in \Re^n$ → Sense all the n samples → Compress raw data → ••• → Decompress for later processing

  - Costs in Storage? Processing? Transmission? Acquisition?

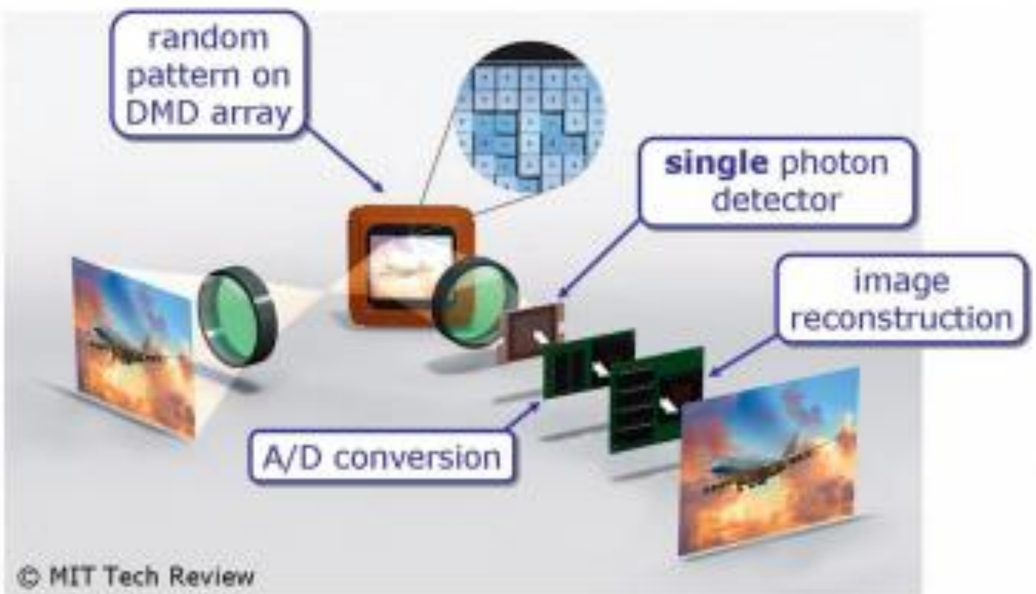
- This 6.1 Mega-Pixels digital camera senses 6.1e+6 samples to construct an image.
- The image is then compressed using JPEG to an average size smaller than 1MB – a compression ratio of ~20.

Q: Can we sample less data in acquisition, with accurate recovery?

Compression in Acquisition

- Compressive Sampling: combine sensing with compressing

$\underline{x} \in \Re^n$ → Sense $K<<n$ values $\{b_i(\mathbf{x})\}_{i=1}^K$ → These K values $\mathbf{x}$ in a compressed form • • • → Reconstruct $\underline{x}$ from $\{b_i(\mathbf{x})\}$ for later processing



Single-pixel camera @ Rice University

- Single photon detector/sensor

What is Compressed Sensing (CS)?

Sparsity-enforcing: $\min \quad \underbrace{||\mathbf{x}||_0}_{\text{Sparse Signal}}$

Measurement equation: $s.t. \quad \underbrace{\mathbf{A}}_{\text{Measurement Matrix}}\mathbf{x} = \underbrace{\mathbf{b}}_{\text{Measurement}}$

Q1: *how to design the sampling matrix* **A** *for perfect recovery?*

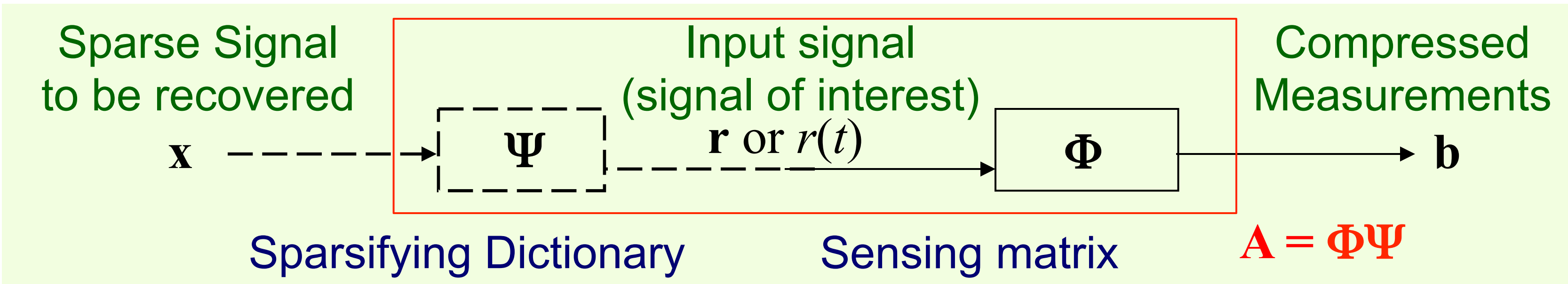
Q2: *how to optimize the non-convex program?*

- CS gives the design rule of the (non-adaptive) sampling matrix
- CS shows when the following programs are equivalent

Non-Convex $\begin{array}{ll} \min & ||\mathbf{x}||_0 \\ s.t. & \mathbf{A}\mathbf{x} = \mathbf{b} \end{array} \longleftrightarrow \begin{array}{ll} \min & ||\mathbf{x}||_1 \\ s.t. & \mathbf{A}\mathbf{x} = \mathbf{b} \end{array}$ Convex

[Chen-Donoho-Saunders'98], [Candès et al'04-06]

Sampling via Linear Projections

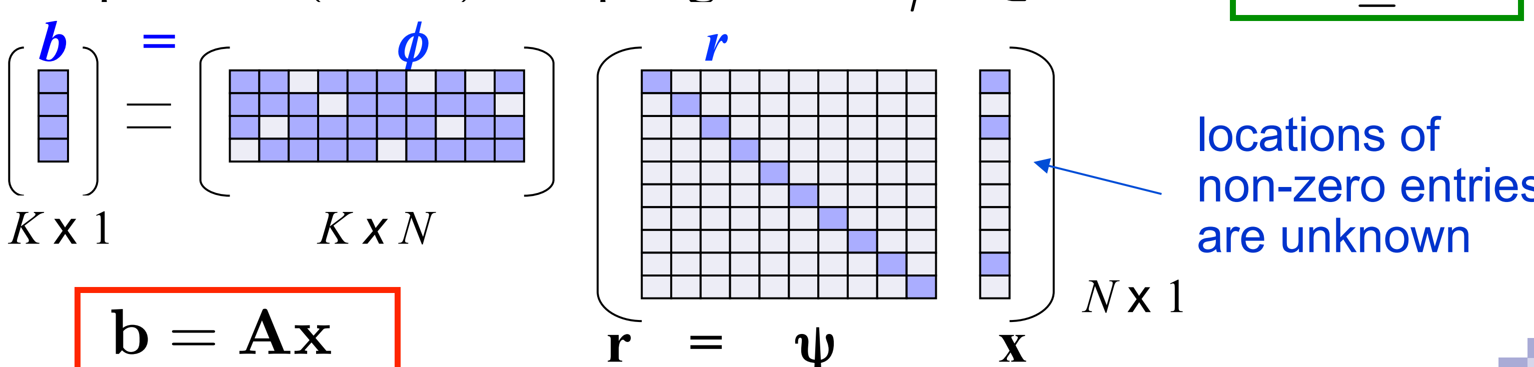


- Sparsity on Transform Domain (Sparsifying Dictionary)

$$\mathbf{r} = \psi \mathbf{x} \in \mathcal{R}^N$$

  - sparsity measure: $||\mathbf{x}||_l \leq R$, for some $l \in [0, 2)$, $0 < R \ll \infty$
  - exactly S-sparse signal $\mathbf{r}$ ($l = 0$): $\boxed{S := ||\mathbf{x}||_0 \ll N}$

- Measurements collected from linear projections
  - Compressive (linear) Sampling: $\mathbf{b} = \phi \mathbf{r} \in \mathcal{R}^K$ $\boxed{K \leq N}$



$$\boxed{\mathbf{b} = \mathbf{A}\mathbf{x}}$$

CS Theory

Q: *what sampling functions to choose and how many measurements to take so as to enable error-free recovery?*

- Restricted Isometric Property (RIP)
  - Conditions on **A** for *noise-free*, *compressible* and *noisy* signals
    - **Sparsity** of signal: *local, coherent*
    - **Incoherence** of measurements: *global, incoherent*
  - RIP implies *tractable, robust* and *stable* recovery
- Random matrices satisfy the RIP with high probability
  - each sample picks up a little (new) info about each signal component

Noise-free case

(True x)

(P0) $$\min \ \|\mathbf{x}\|_0 \quad s.t. \ \mathbf{A}\mathbf{x} = \mathbf{b}$$

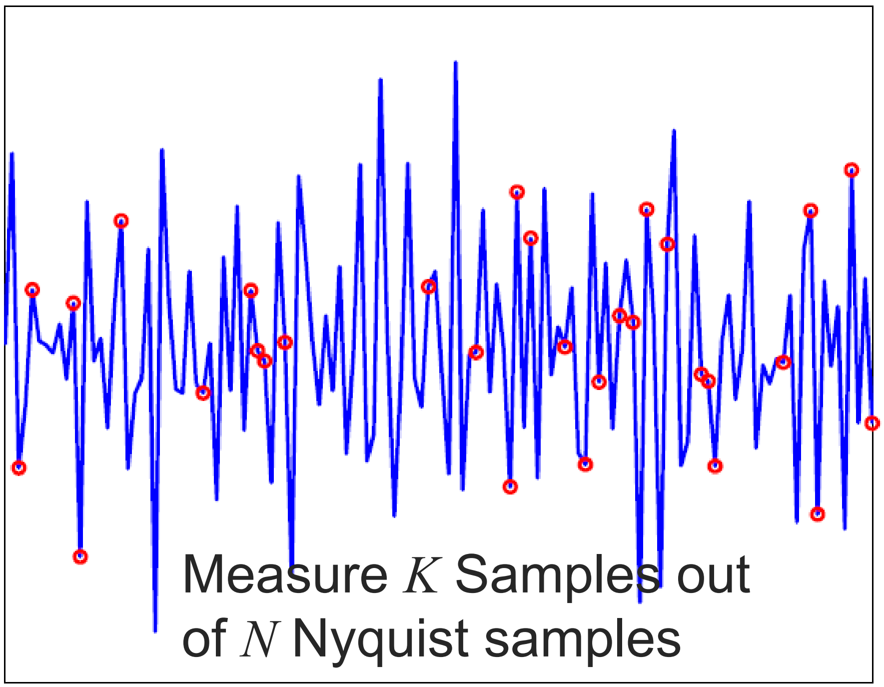
(P1) $$\min \ \|\mathbf{x}\|_1 \quad s.t. \ \mathbf{A}\mathbf{x} = \mathbf{b}$$

Noisy case

(P2) $$\min \ \|\mathbf{x}\|_1 \quad s.t. \ \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2 \leq \epsilon$$

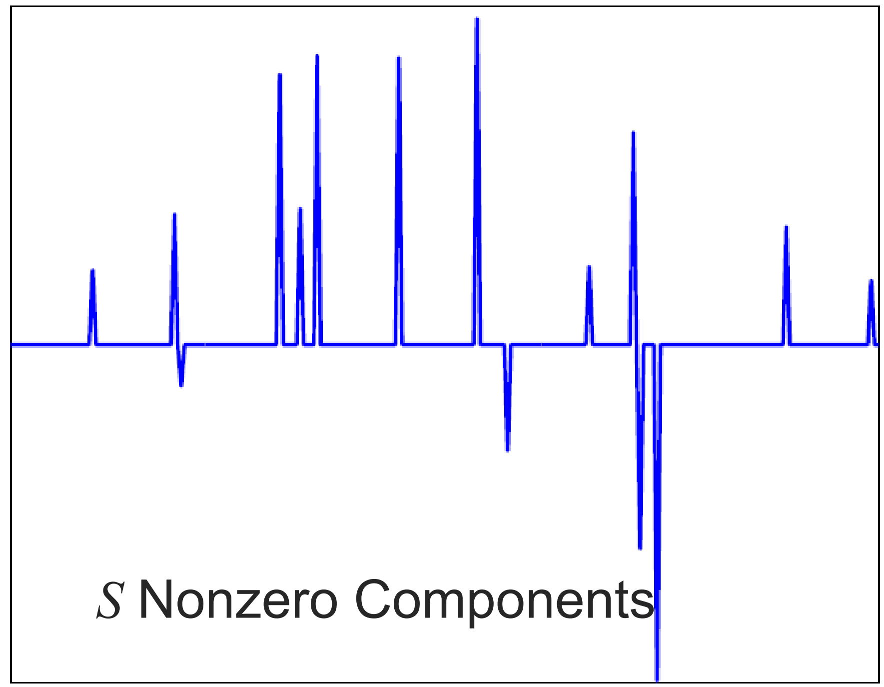
Example: Fourier Measurements

Sampling basis: Time Domain



$$f(t) = \sum_{i=1}^{S} x_i e^{j\omega_i t} \qquad t_1, \ldots, t_K$$

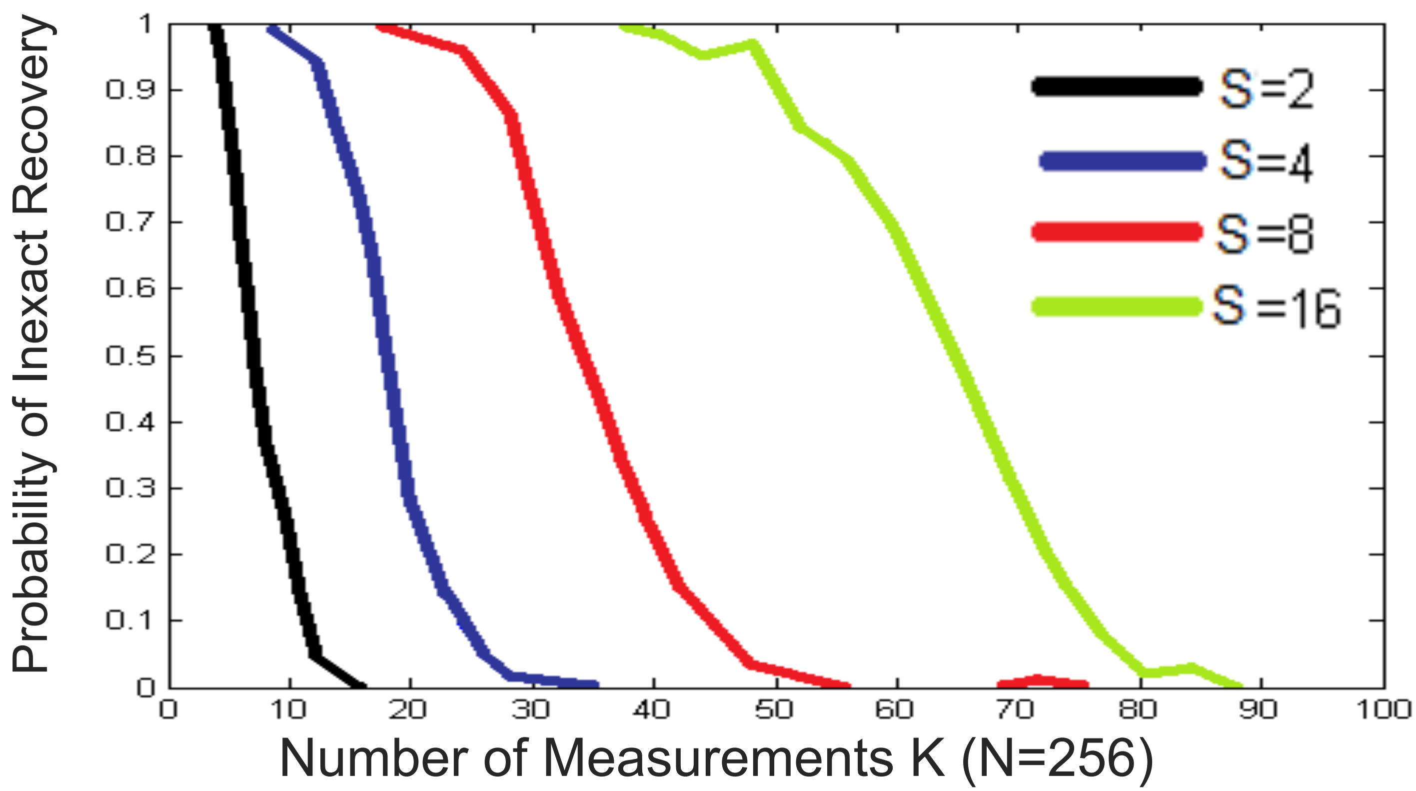
Sparsity basis: Frequency Domain



$$F(\omega) = \sum_{i=1}^{S} x_i \delta(\omega - \omega_i)$$

Example: Fourier Measurements (cont'd)

- Performance (Prob. of inexact recovery) vs. # measurements K

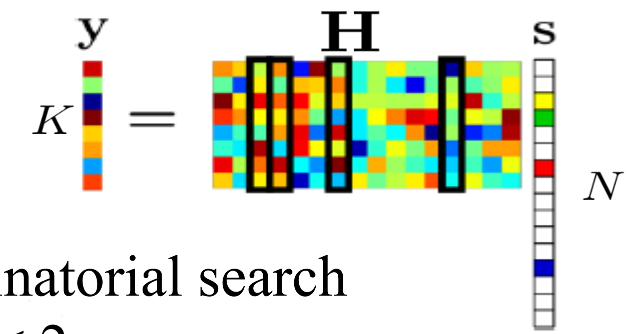


- (K; N, S): depends on *measurement matrix* and *recovery method*
- RIP conditions are sufficient rather than necessary

CS Algorithms (1)

- Optimization-based algorithms
  - L_p-norm minimization: (P1), (P2)

$$\min \ ||\mathbf{x}||_p^p, \qquad \mathbf{A}\mathbf{x} = \mathbf{b}$$



    - $p = 0$: exact sparsity; but requires combinatorial search
    - sparsity weakens as p increases, until $p < 2$
    - p in [0, 2): p: [0,1) → nonconvex; $p \geq 1$ → convex
  - computational load: $O(N^3)$; # measurements: $O(S \log(1+N/S))$

Sparse regression [Tibshirani'96], [Tipping'01]

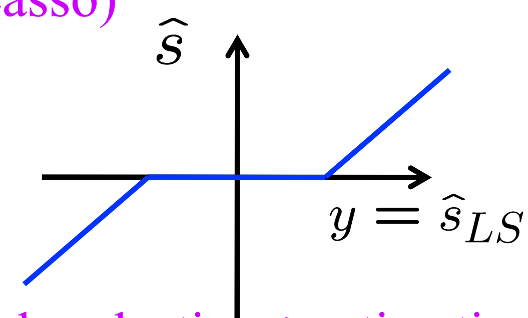
$$\mathbf{y} = \mathbf{H}\ \mathbf{s} + \mathbf{w}$$

- Least-absolute shrinkage selection operator (Lasso)

$$\hat{\mathbf{s}} = \arg\min \frac{1}{2}\|\mathbf{y} - \mathbf{H}\mathbf{s}\|_2^2 + \lambda\|\mathbf{s}\|_1$$

Ex. (scalar case) closed-form solution

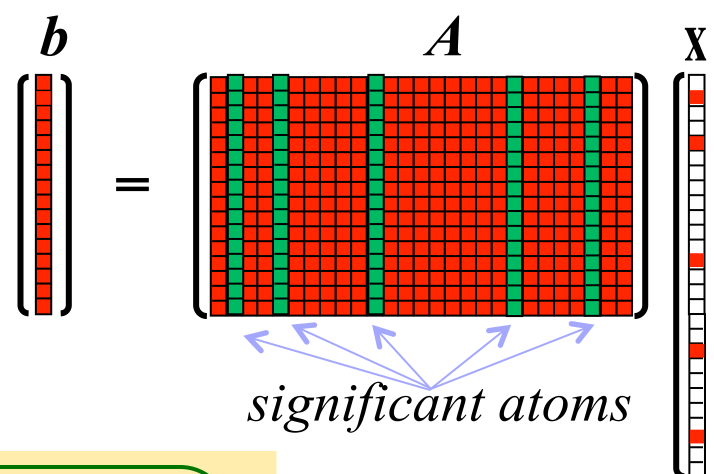
$$\begin{aligned}\hat{s} &= \arg\min \tfrac{1}{2}(y-s)^2 + \lambda|s| \\ &= \mathrm{sign}(y)\ \ (|y| - 2\lambda)_+\end{aligned}$$



variable selection + estimation

CS Algorithms (2)

- **Greedy algorithms**
  - Matching Pursuit and its variants (MP, OMP, TOMP, ….)
  - Idea: iteratively identify columns of **A** (atoms) that are associated with non-zero entries of **x**
  - Suboptimal performance, low complexity, more samples needed
    - Complexity: $O(NS^2)$; # measurements: $O(S\log N)$
- **Sparse Bayesian learning**
- **Fast algorithms**
  - Iterative reweighted algorithms
  - Iterative shrinkage/thresholding
  - Iterative support detection



$$\mathbf{b} = \mathbf{A}\mathbf{x} + \mathbf{w}, \quad \mathbf{A} = \mathbf{\Phi}\mathbf{\Psi} \;\Rightarrow\; \boxed{\hat{\mathbf{x}} = CS(\mathbf{b}; \mathbf{A}), \quad \hat{\mathbf{r}} = \mathbf{\Psi}\hat{\mathbf{x}}}$$

Quiz: True or False

1. [T] [F] Given a vector signal **u**, its sparsity order is fixed
2. [T] [F] *Compressed Sensing is a new technology that can offer stronger compression than traditional compression techniques*
3. [T] [F] *CS theory on the RIP conditions reveals the minimal number of measurements for recovering a sparse signal from any measurement systems*
4. [T] [F] *To compress during the sensing process, the sensing matrix needs to be random*
5. [T] [F] *A random measurement matrix is likely to enable signal recovery from compressed samples*
6. [T] [F] *When using L1-minimization to recover an unknown vector* $\boldsymbol{x}$ *from* $\boldsymbol{b} = \boldsymbol{A}\,\boldsymbol{x}$*, the formulation works under two conditions: 1)* $\boldsymbol{x}$ *is sparse, 2)* $\boldsymbol{A}$ *is a fat matrix (under-determined) and satisfies RIP*

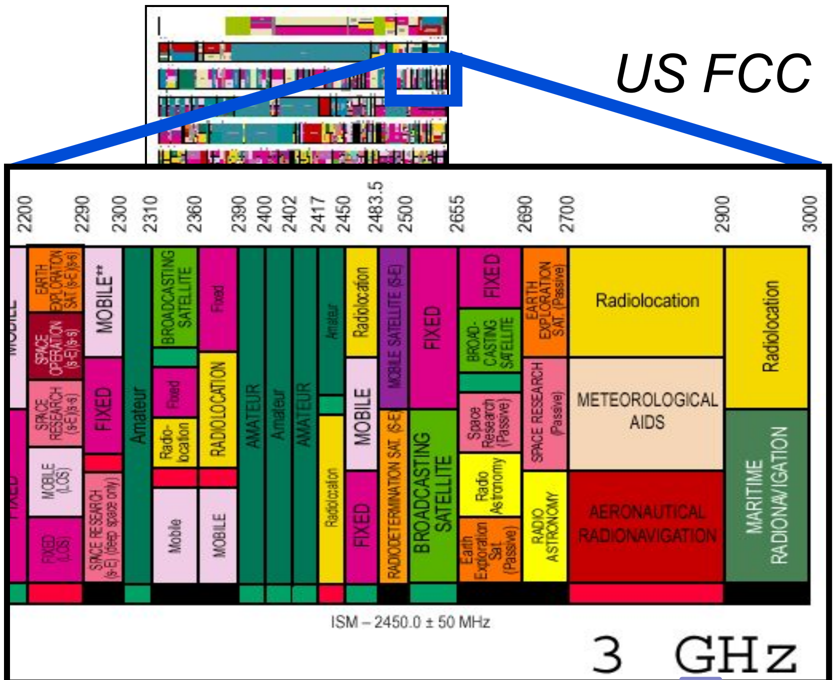
Outline

- Basis of Compressive Sensing (CS)
- Motivation of CS for Cognitive Radio (CR)
  - Introduction to CR
  - Technical challenges in wideband spectrum sensing
  - Roadmap
- Compressive Spectrum Sensing for CR
- Sparsity-constrained Dynamic Resource Allocation and Waveform Design
- References

Introduction on CR: Spectrum Scarcity

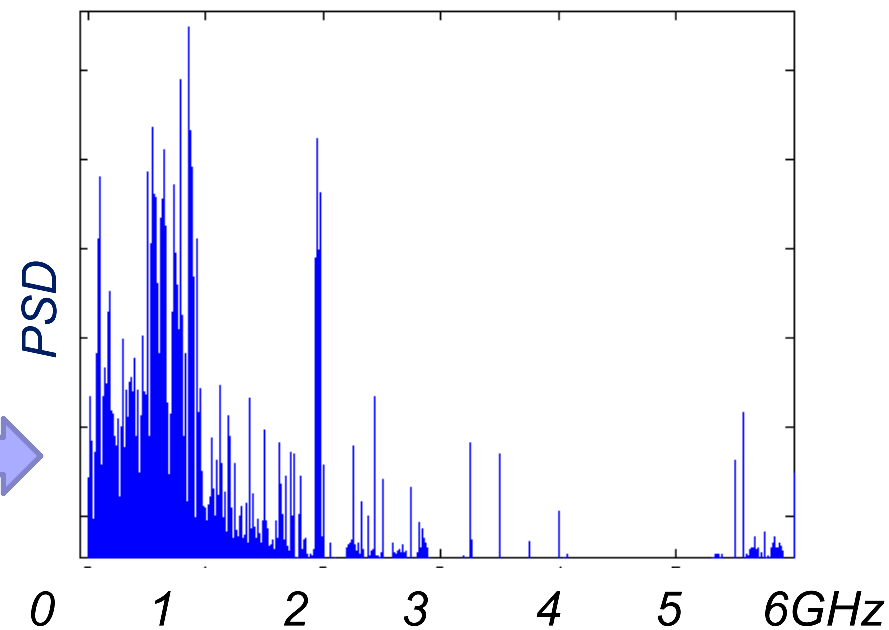
"Scarcity vs. Underutilization Dilemma"

US FCC



☹ fixed spectrum access policies have useful radio spectrum pre-assigned

inefficient utilization

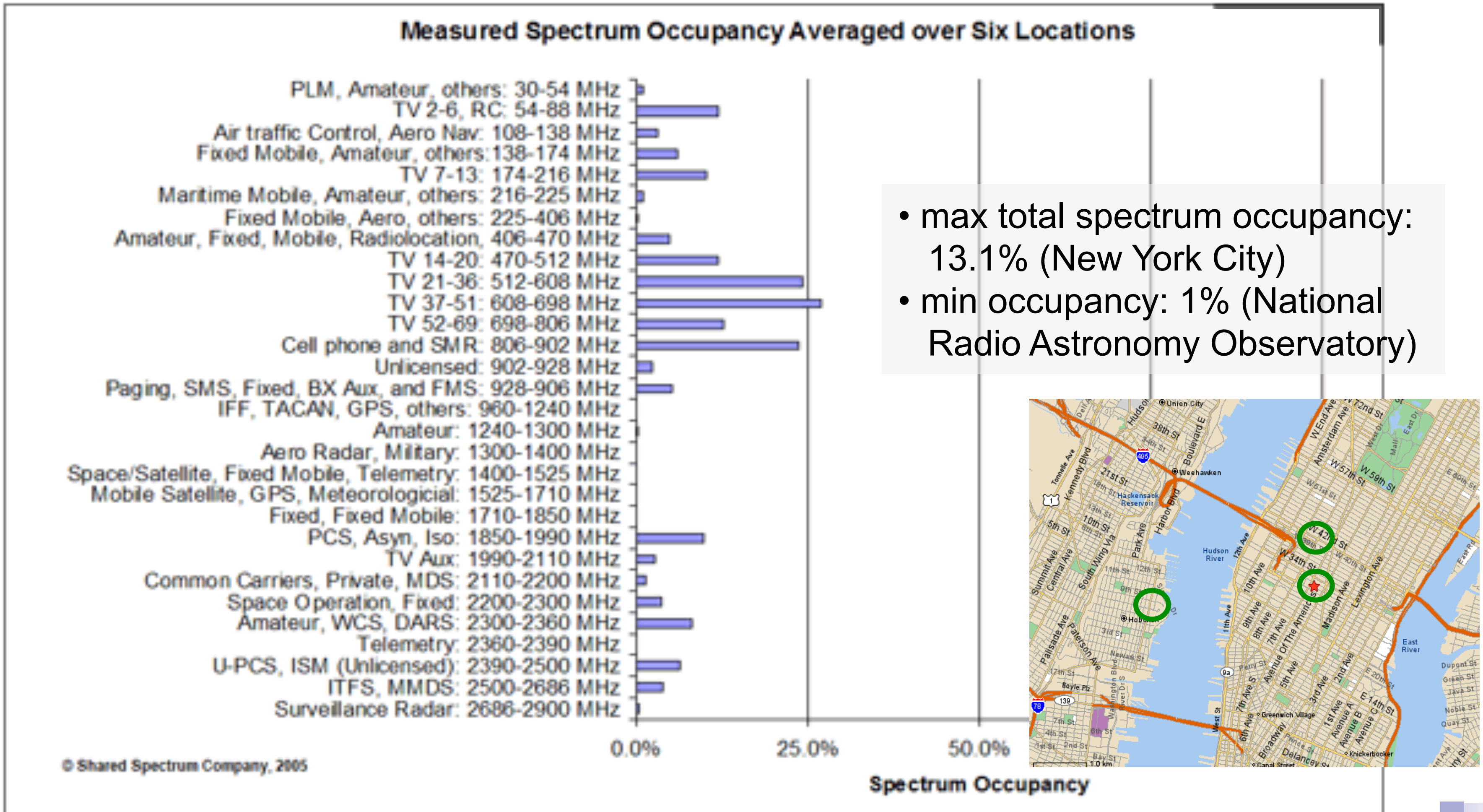


Source: Spectrum Sharing Inc.

☺ at any *time and location,* most *spectrum* is unused

Spectrum Opportunities & *Sparsity*

- Spectrum usage measurements averaged over six locations [SSC]
  - average occupancy over all of the locations: 5.2%; Jan.' 04-Aug.' 05



- max total spectrum occupancy: 13.1% (New York City)
- min occupancy: 1% (National Radio Astronomy Observatory)

Motivating Applications

- Future pervasive networks: dynamic spectrum access (DSA)

Licensed networks

<u>Cellular, PCS band</u>

Improved spectrum efficiency

Improved capacity

Secondary markets

<u>Public safety band</u>

Voluntary agreements between licensees and third party

Limited QoS

Third party access in licensed networks

<u>TV bands (400-800 MHz)</u>

Non-voluntary third party access

Licensee sets a protection threshold

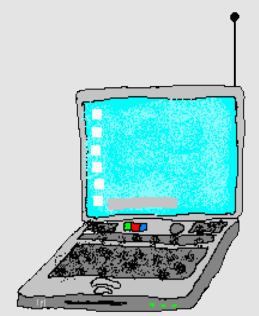
Unlicensed networks

<u>ISM, UNII, Ad-hoc</u>

Automatic frequency coordination

Interoperability

Co-existence

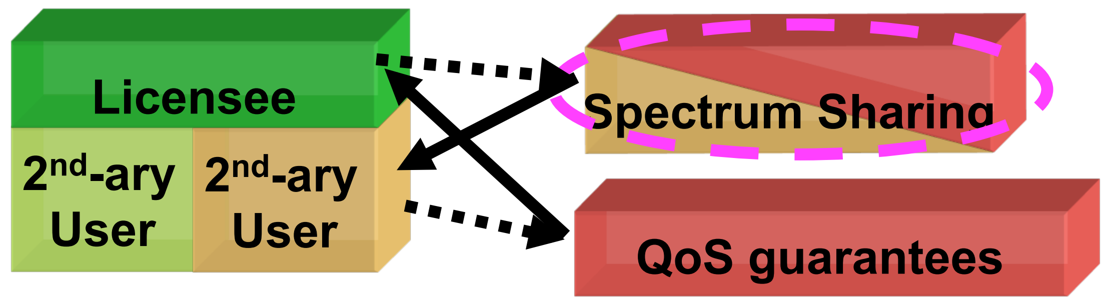


√ more users/services √ higher rates √ better quality √ less interference

DSA under User Hierarchy

Primary User (PU)

Secondary User (SU)



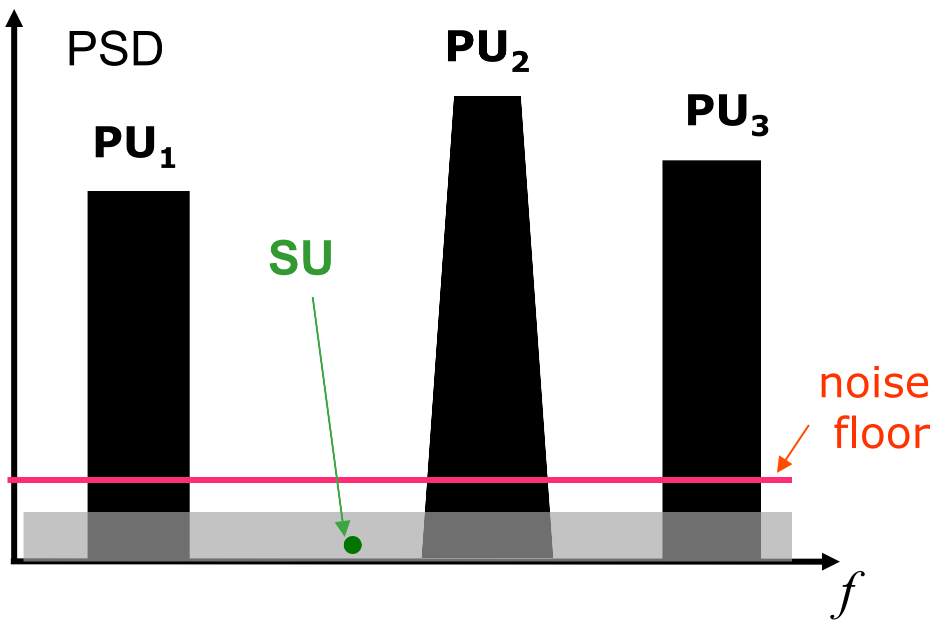
FCC diagram

min rate, prob. of collisions, interference, outage rate

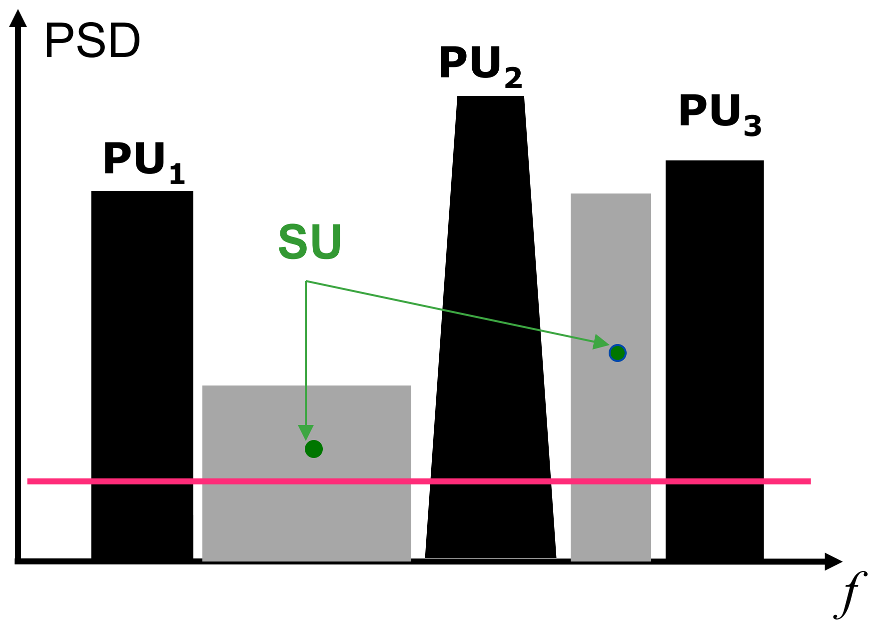
- DSA access models for SUs
  - Spectrum Underlay
    - restriction on transmit-power levels
    - operation over ultra wide bandwidths
  - Spectrum Overlay
    - constraints on when and where to transmit
    - avoid interference to PUs via sensing and adaptive allocation

Underlay vs. Overlay

- Spectrum Underlay (UWB)
  - regulatory and dynamic spectrum masks
- Spectrum Overlay (CR)
  - *Opportunistic*: spectrum is used when PU is idle
  - *Cooperative*: real-time negotiation with PU



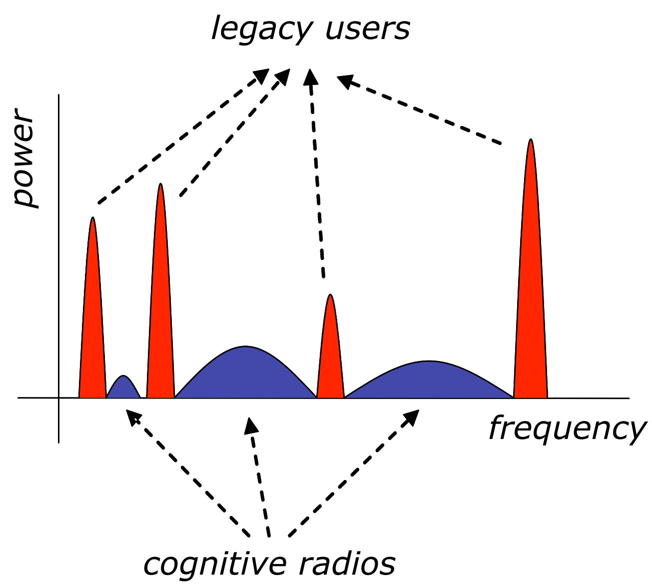
US FCC: 3.1-10.6GHz

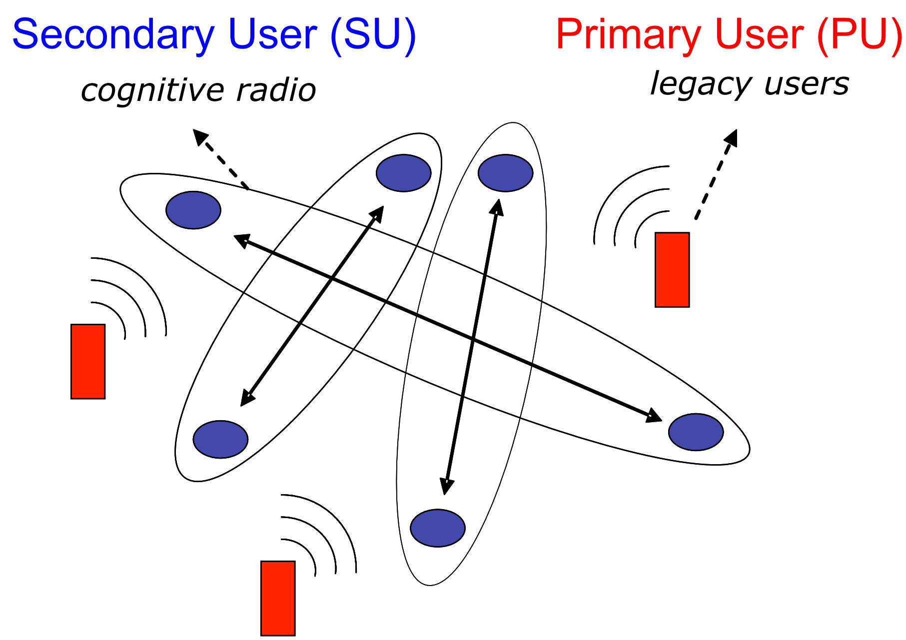


US FCC: 54-700 MHz

Cognitive Radio (CR)

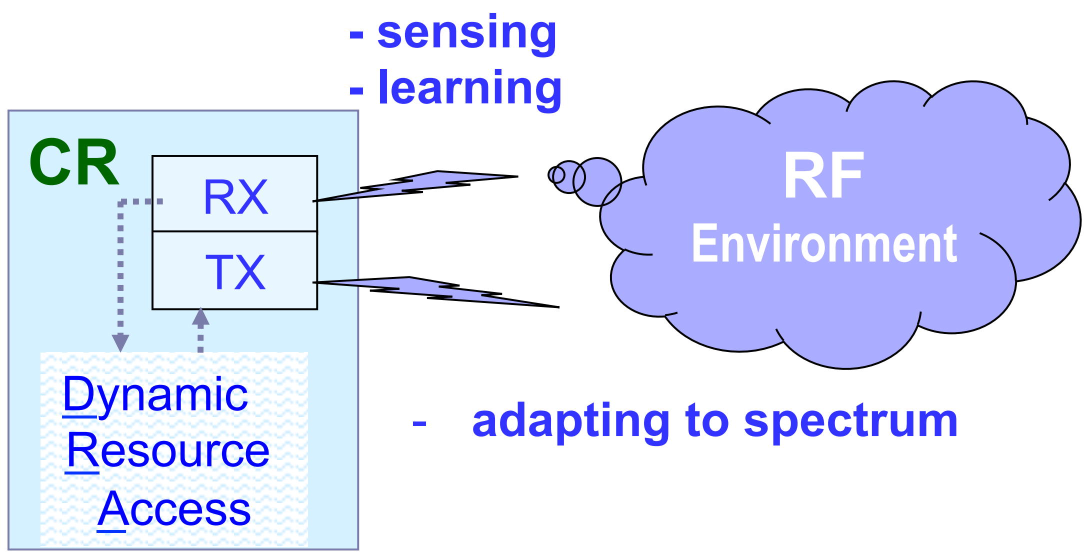
- CRs opportunistically use the spectrum





- Cognitive radio network problems
  - Finding holes in the spectrum: wideband spectrum sensing
  - Allocating the open spectrum: dynamic resource allocation
  - Adjusting the transmit waveforms: waveform adaptation

DSA Implications on CR Capabilities

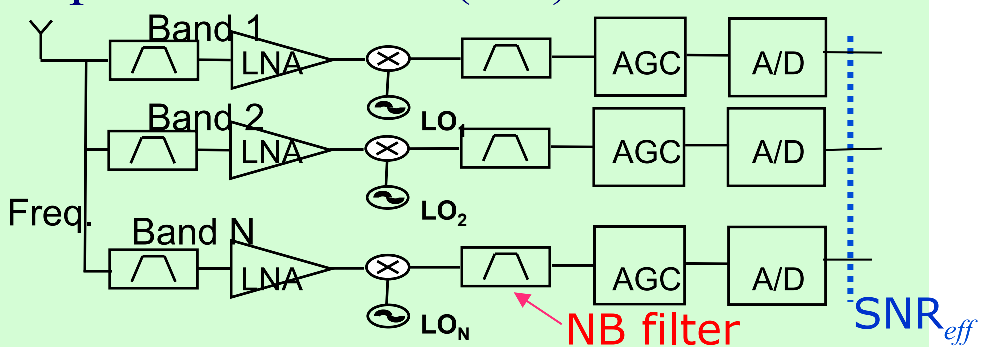


- ***Cognizant receiver***
  - Observe: **sensing** with high **sensitivity** and over wide freq. range
  - Learn: radio etiquettes, traffic pattern, spectrum opportunities statistics
- ***Agile transmitter***
  - Act: **wideband frequency agility**, fast **adaptation**, dynamic range
- ***Intelligent DRA***
  - Decide, plan, & negotiate: **spectrum access** and radio re-configuration
  - MAC and networking capabilities that support DRA intelligence

Challenge 1: Wideband Signal Acquisition

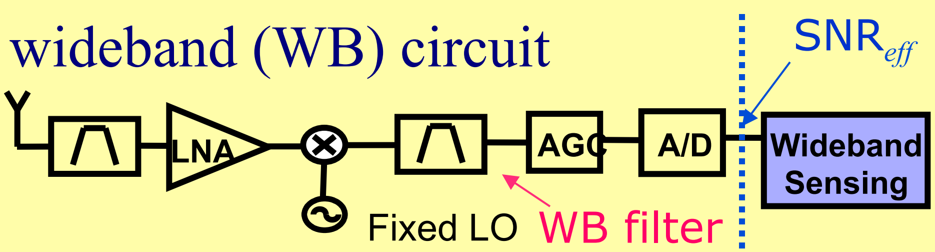
❑ Choices for RF Circuits: *multiple NB or single WB* ?

multiple narrowband (NB) circuits



- multiple RF chains, BPFs
- number of bands fixed
- LO filter range is preset
- simple (energy/feature) detection within each BPF

wideband (WB) circuit



- single RF chain
- flexible to dynamic PSD
- burden on A/D: $f_s \sim$ GHz
- complex wideband sensing

- Effective SNR (SNR_{eff}) for DSP determined by front-end circuits

Q: ***How can we alleviate DSP burden on wideband circuit design?***

Challenge 2: User Hierarchy

"IEEE 802.22 requires CRs to sense PU signals as low as -114dBm"

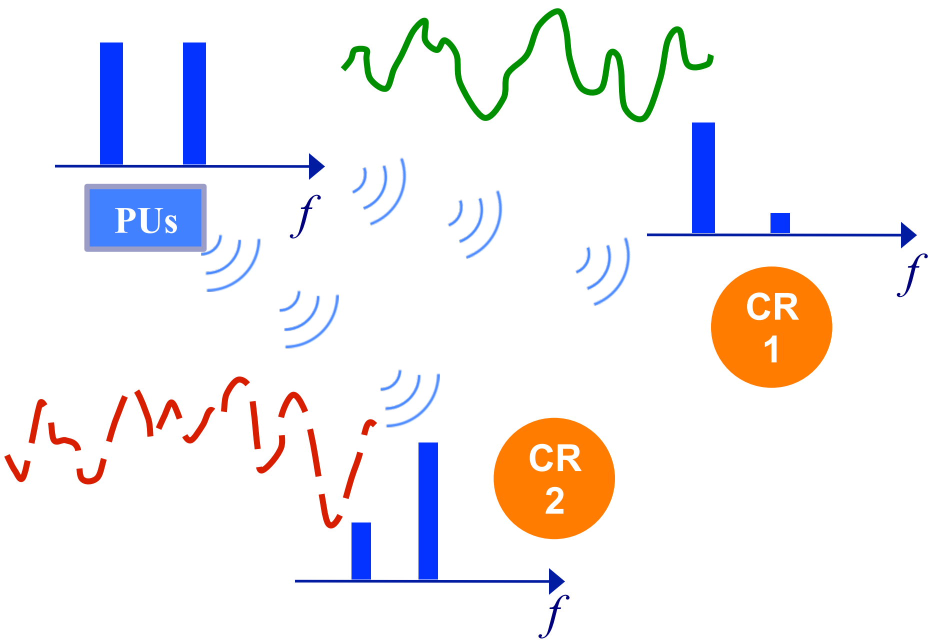
Operating Conditions	Technical Challenges
Protection of primary systems	Sensing at low SNR
	Modulation classification
	Short sensing time
Random sources of interference and noise	Robustness to noise uncertainty
	Interference identification

Q: ***How can we alleviate noise uncertainty effects at low SNR?***

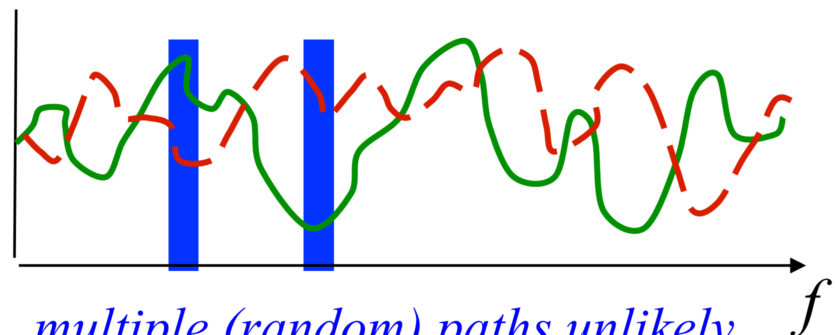
Challenge 3: Wireless Fading

- If no energy detected on a band, can CR assume PU is absent?
  - Detection performance limited by received signal strength
  - Wireless: deep fading, shadowing, local interference

    ⇨ missed detection, hidden terminal problem



Spatial diversity against fading



multiple (random) paths unlikely to fade simultaneously

Q: ***How can we collect cooperation gain at affordable overhead?***

Road Map for Wideband Sensing

Local Compression + Network Cooperation

- Compressed Sensing with sub-Nyquist-rate sampling
  - Exploiting the Sparsity in the received signal (in freq. domain)
  - Making use of Compressive Sampling to reduce sampling rates
- Multiple-CR Cooperative Sensing
  - Centralized vs. distributed; with vs. without channel knowledge
- Compressed Cyclic Feature based Sensing
  - Exploiting the Sparsity in both freq. & cyclic-freq. domains
  - Making use of Cyclic Statistics for robustness to noise uncertainty and low SNR conditions
- Compressive sensing for non-sparse, random signals

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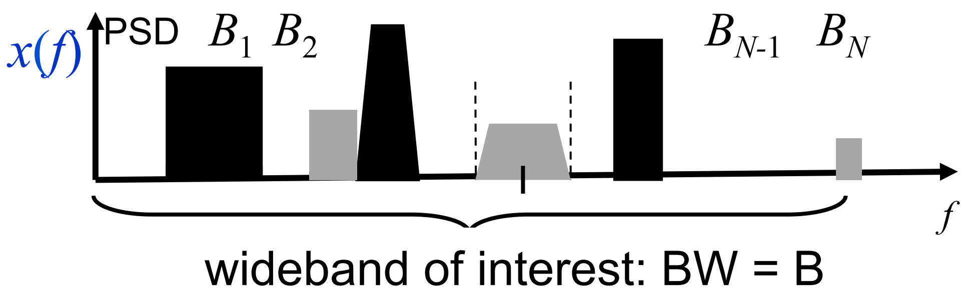
- Motivation of CS for Cognitive Radio (CR)
- Compressive Spectrum Sensing (CSS) for CR
  - Compressive sampling of sparse signals
  - Multi-CR cooperative compressive sensing
    - Consensus-based distributed optimization
    - Cooperative support detection (MRM, row-Lasso)
  - Compressive cyclic feature detection
  - Compressive sensing framework for random processes
    - Direct extraction of useful 2^{nd}-order statistics
    - Sampler design
- Sparsity-constrained Dynamic Resource Allocation and Waveform Design
- References

❶ Compressive Sampling of Analog Signals

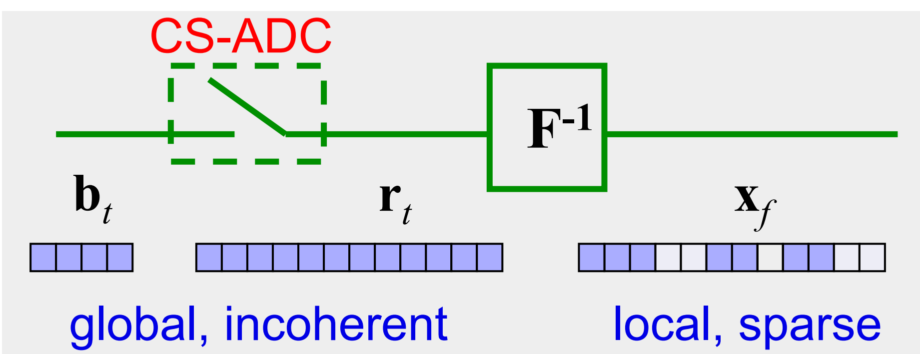
- Context: Wideband Spectrum Sensing in Cognitive Networks



Spectrum occupancy ratio

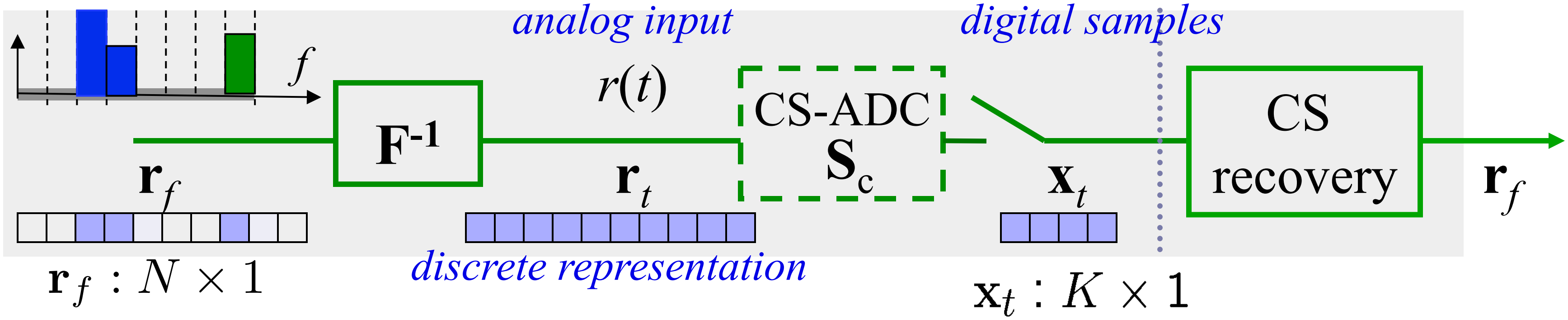
$$r_{nz} = B_{eff}/B \ll 1$$

- Goal: recover frequency spectrum $\mathbf{x}_f$ from samples $\mathbf{b}_t$
  - lower-than-Nyquist-rate sampling
  - recovery without distortion or losing frequency resolution



- *How to sample?*
- *How to compress?*
- *What is minimum f_s for*
  - reconstruct CR signals?
  - identify spectrum bands?
  - extract useful statistics?

Sub-Nyquist-rate Sampling



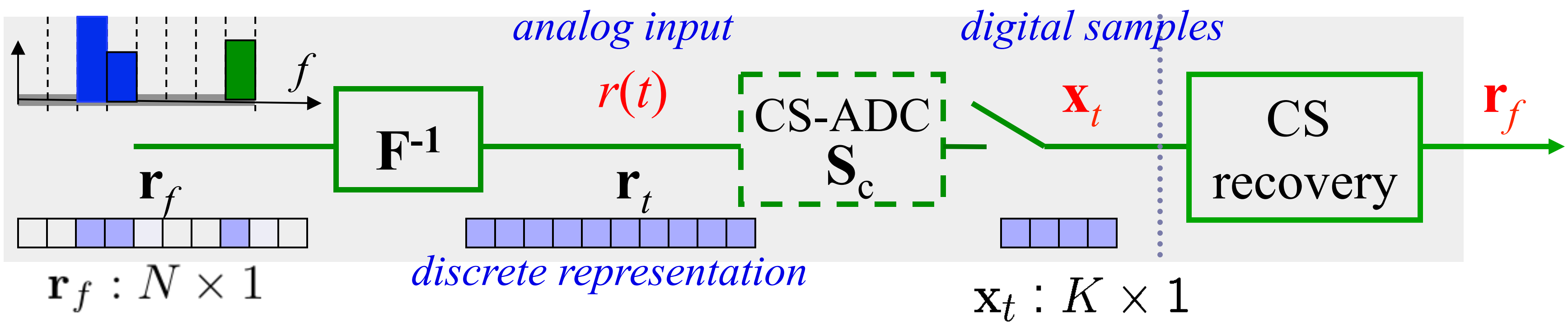
- **Received signal** $r(t) : t \in [0, NT_s]$
  - Fine-resolution (Nyquist-rate) representation: $\mathbf{r}_t \leftrightarrow \mathbf{r}_f = \mathbf{F}\mathbf{r}_t$
  - Sparsity in frequency: $N_{nz} = \|\mathbf{r}_f\|_0 \ll N$
- **Linear sampling** $\mathbf{x}_t = \mathbf{S}_c \mathbf{r}_t$ $\quad x_t(k) = \int S_{c,i}(t) r(t) dt$
  - Compression in time (*M/N*): $\mathbf{S}_c : K \times N$ $\quad N_{nz} \leq K \leq N$

$$\boxed{\mathbf{x}_t = \mathbf{S}_c\ \mathbf{r}_t = \mathbf{S}_c \mathbf{F}^{-1} \mathbf{r}_f}$$

$\mathbf{A} = \mathbf{S}_c \mathbf{F}^{-1}$ is rank-deficient

  - Various designs of random samplers [Kirolos etal'06, Hoyos etal'08, Mishali-Eldar'10]

CS for Frequency Spectrum Recovery



- **Sparse spectrum recovery**
  - Sub-Nyquist rate random sampling: $r(t) \longrightarrow \mathbf{x}_t$
  - Sparse signal recovery

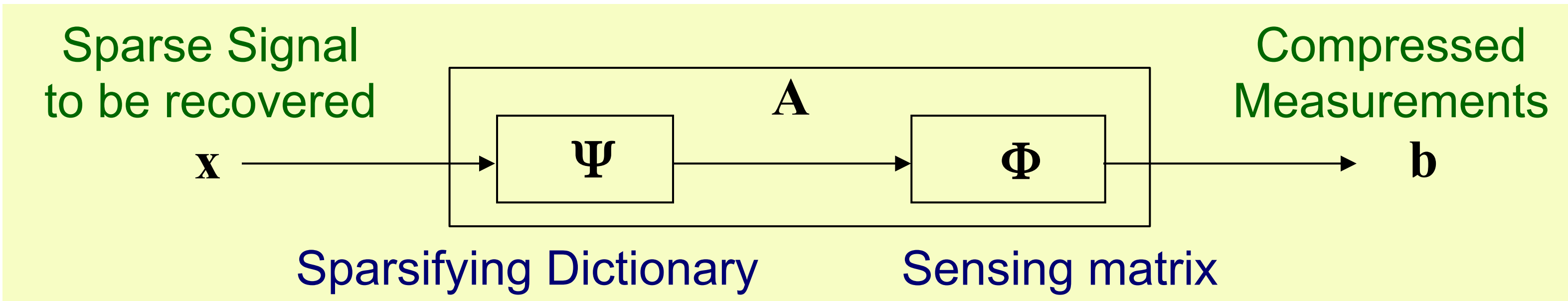
Compressive samples

$$\mathbf{x}_t = \mathbf{S}_c\ \mathbf{r}_t = \mathbf{S}_c \mathbf{F}^{-1} \mathbf{r}_f$$

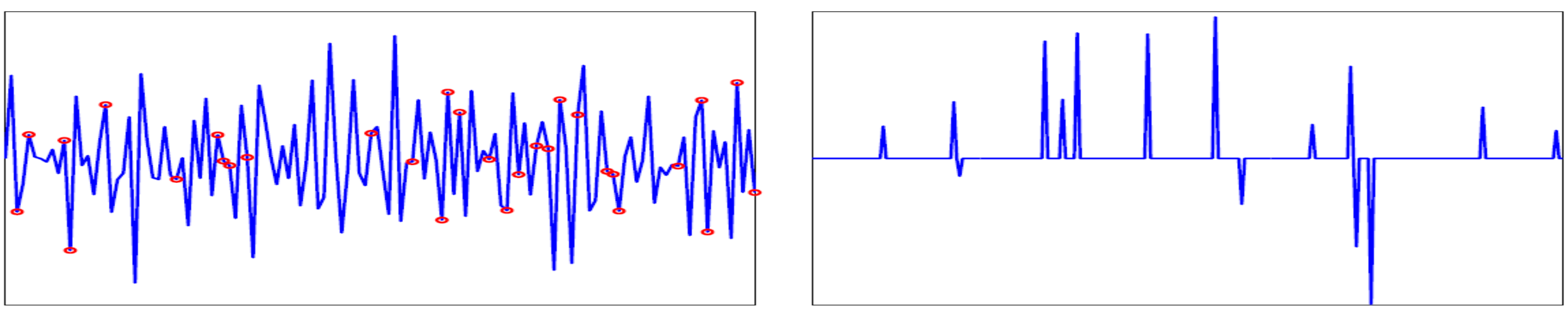
wideband frequency spectrum

$$\hat{\mathbf{r}}_f = \arg\min_{\mathbf{r}_f} \ \frac{1}{2} \left\| \mathbf{x}_t - \mathbf{S}_c \mathbf{F}^{-1} \mathbf{r}_f \right\|_2^2 + \lambda \left\| \mathbf{r}_f \right\|_1$$

CS – Sensing Matrix (1)



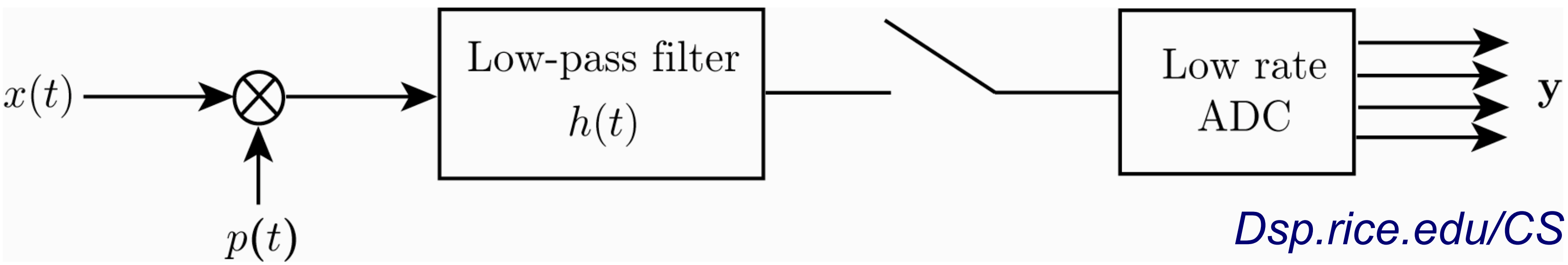
- Measurement matrix **A**: Random Fourier Measurements
  - Sparsifying Matrix: $\mathbf{\Psi} = \mathbf{F}^{-1}$ (DFT)
  - Sensing matrix: $\mathbf{\Phi} = \{0,1\}^{K \times N}$ that takes K samples out of N Nyquist samples in time

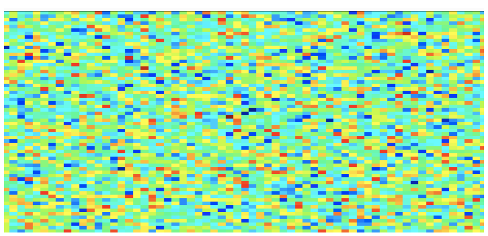


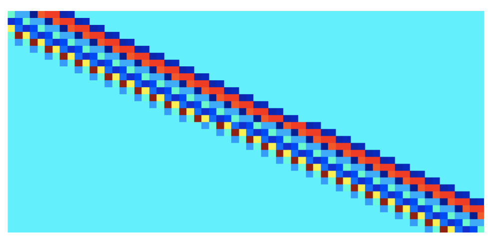
Non-uniform sampling: avg. rate reduced, but peak rate = Nyquist

CS – Sensing Matrix (2)

- Sensing matrix Φ: Analog to Information Converter (AIC)
  - Pseudo-random modulation with maximal-length PN sequence, followed by low-pass filter and down-sampler [Kirolos etal 2006]
  - Uniform reduced-rate sampling; w/ wideband filtering





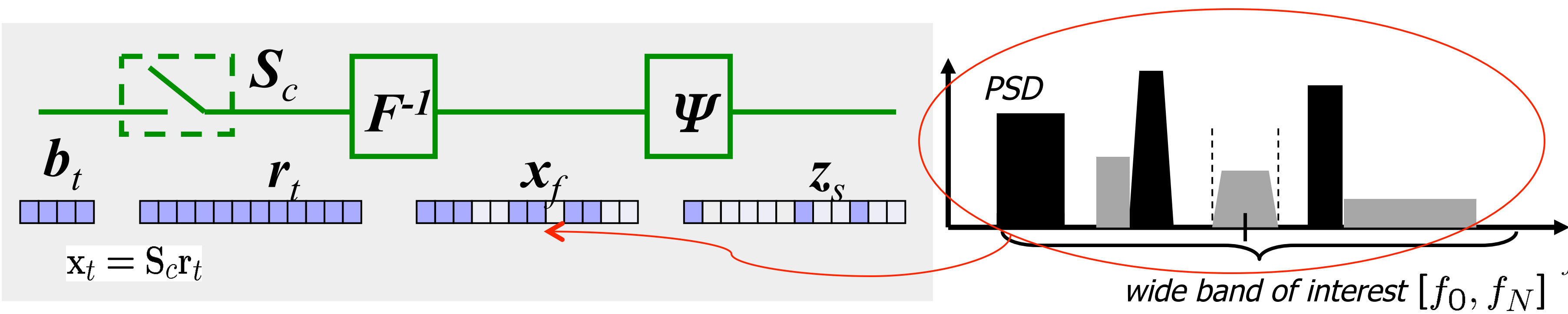


banded structure for real-time streaming

Sensing: tradeoffs in RIP/incoherence and hardware constraints

CS – Sparsifying Matrix (1)

***Q**: Given the signal of interest, is the sparsifying matrix fixed?*



- Let's say that the spectrum is localized over sub-bands
  - Spectral hole detection in cognitive radio applications: how to coarsely identify which sub-bands are occupied?

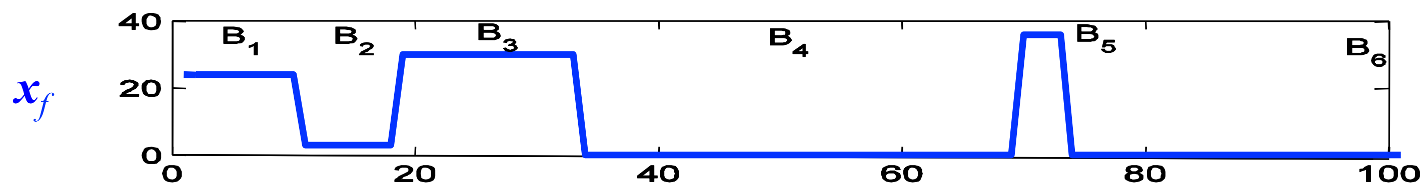
***Q**: how can we rapidly estimate* $N, \{f_n\}_{n=1}^{N-1}, \{\alpha_n^2\}_{n=1}^{N}$

- *Modeling assumption: spectrum is block sparsity or approx. piecewise smooth → sparse in the wavelet domain*

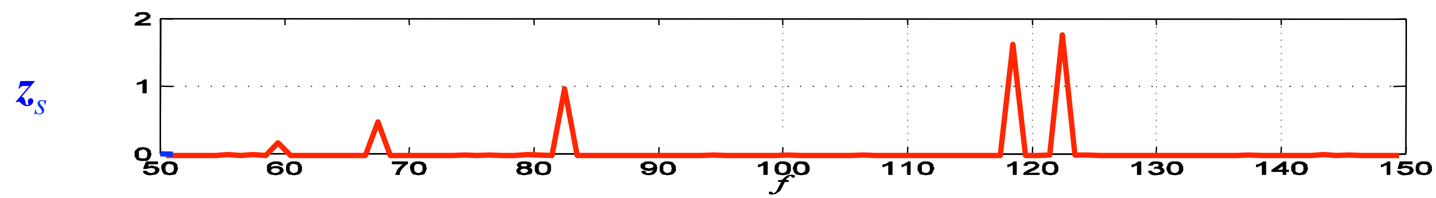
***A**: (spectral) edge detection + CS via wavelets*

CS – Sparsifying Matrix (2)

- Multi-Step CS (MSCS) – estimating spectrum $\mathbf{r}_f$ then $\mathbf{z}_s$
  - compression ratio K/M determined by effective bandwidth B_{eff}/B



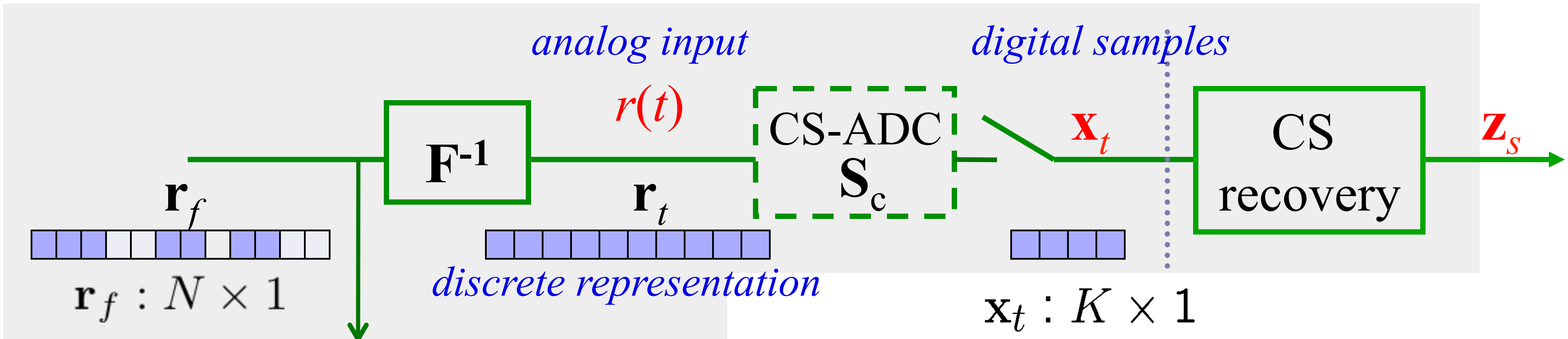
- One-Step CS (OSCS) – directly estimating edges $\mathbf{z}_s$

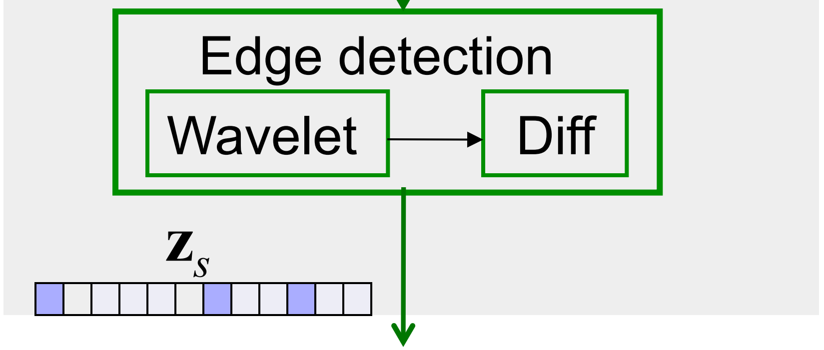


  - Permissible compression ratio K/M is determined by #bands
  - Improved performance and convergence given the same #samples
  - Simple to implement

Useful to identify a good sparsifying matrix

❷ CS for Edge Spectrum Recovery

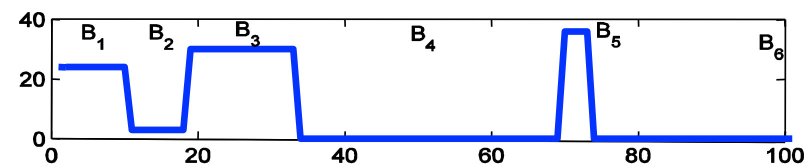




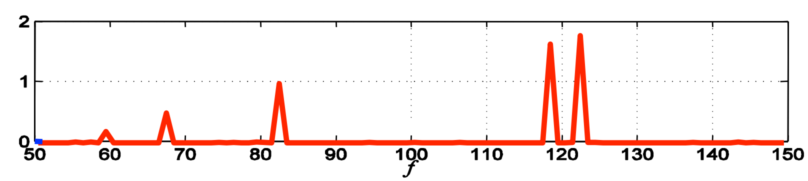
$$\mathbf{x}_t = (\mathbf{S}_c \cdot \mathbf{G})\mathbf{z}_s$$

- bypasses spectral estimation
- effects stronger compression

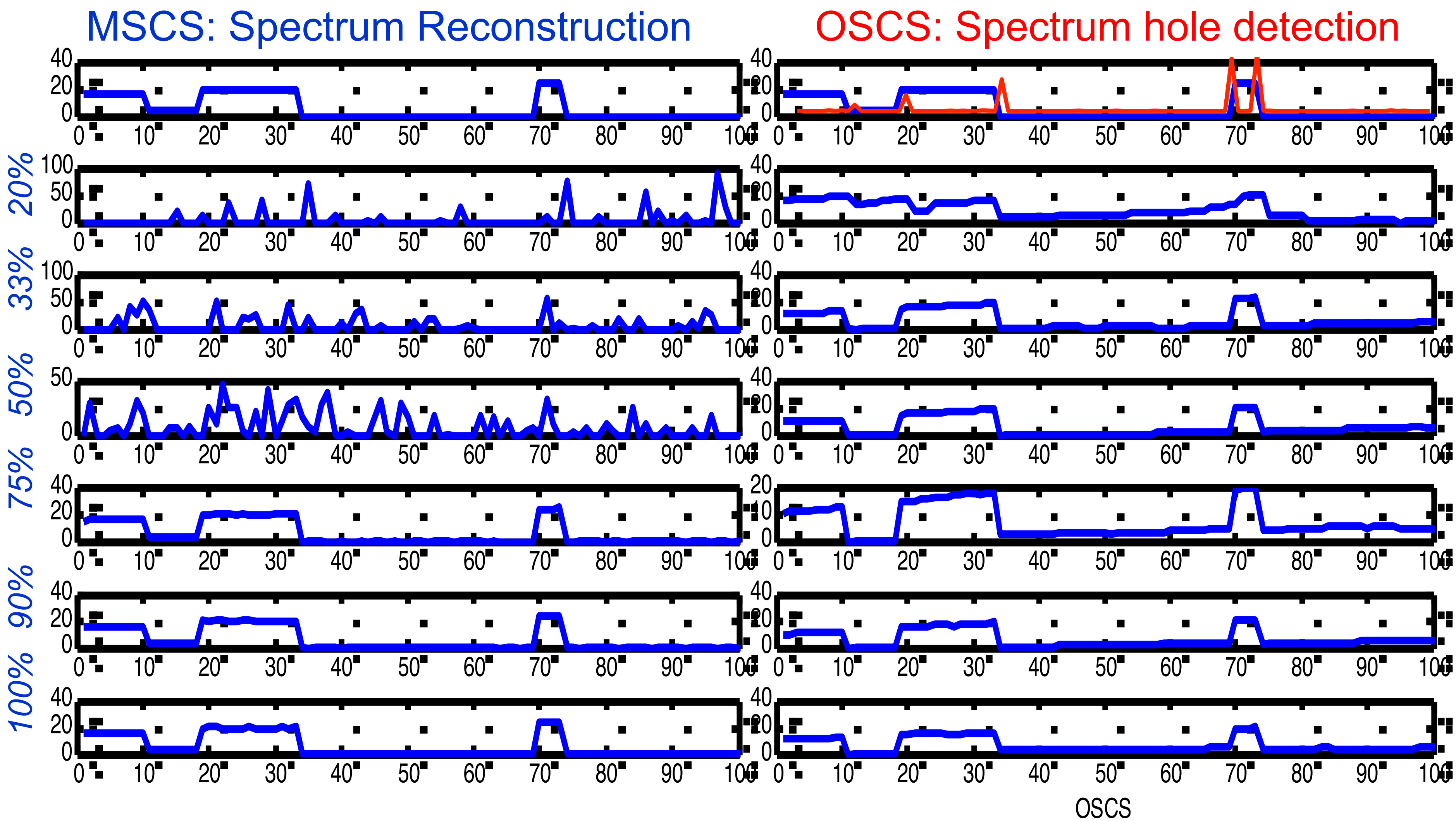
- Frequency spectrum $\mathbf{r}_f$



- Edge Spectrum $\mathbf{z}_s$

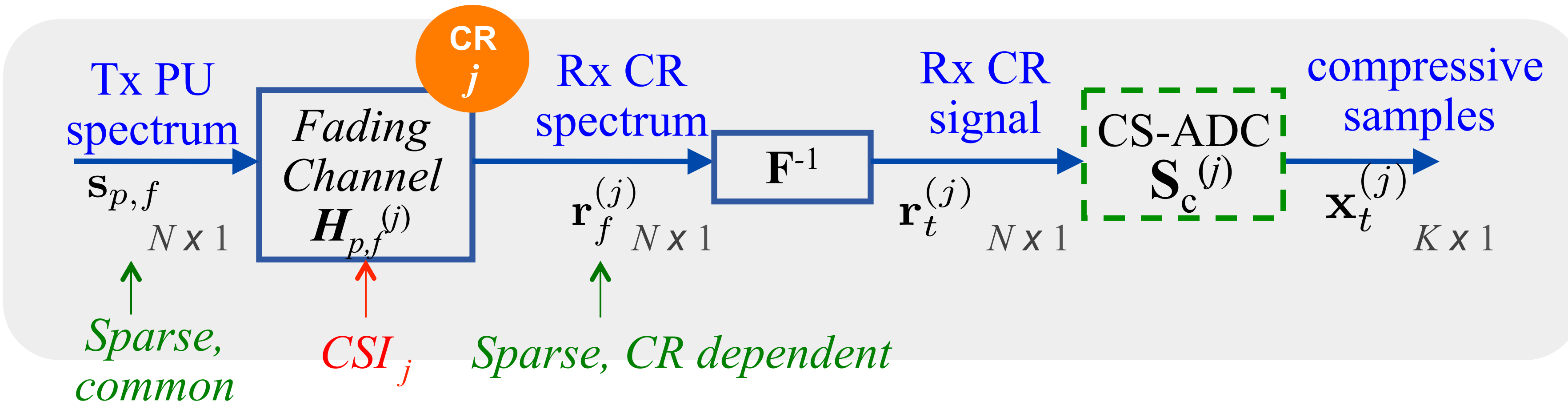


Simulation: Recovered Spectrum



[Tian-Giannakis' 2007]; [Polo-Wang-Pandharipande-Leus' 2009]

3 Cooperative Spectrum Sensing



- Reconstruction of sparse spectrum at individual CRs
  - Spectrum reconstruction for received signal (without CSI)

$$\hat{\mathbf{s}}_{p,f} = CS\left(\mathbf{x}_t^{(j)}; \mathbf{S}_c^{(j)}\mathbf{F}^{-1}\mathbf{H}_{p,f}^{(j)}\right)$$

  - Spectrum reconstruction for transmitted (PU) signals
    - Assumes channel knowledge/estimation (with CSI)

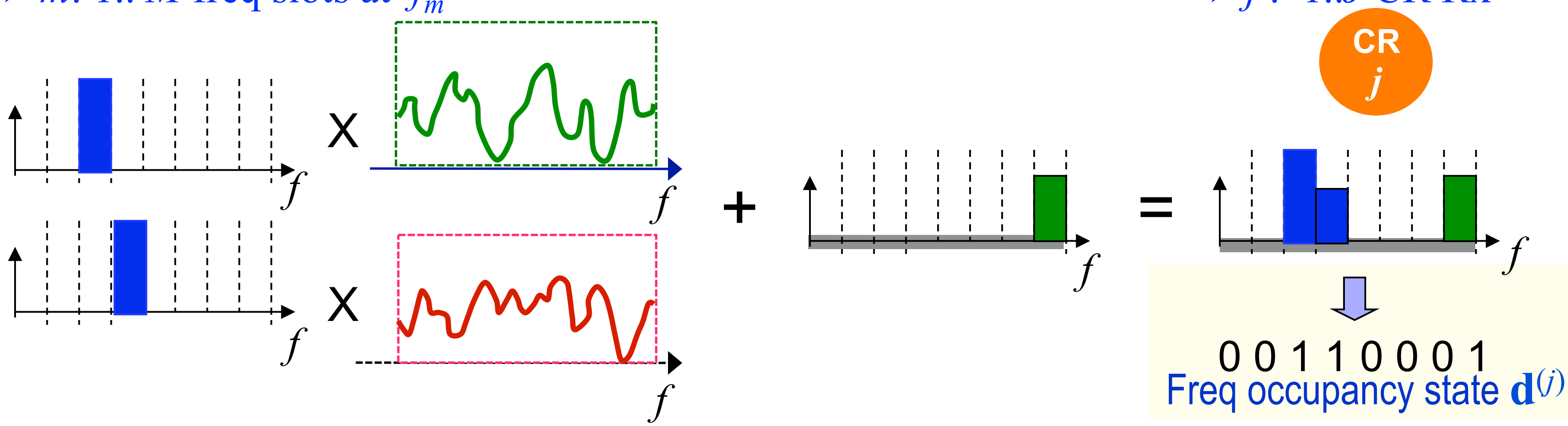
$$\hat{\mathbf{r}}_f^{(j)} = CS\left(\mathbf{x}_t^{(j)}; \mathbf{S}_c^{(j)}\mathbf{F}^{-1}\right)$$

***Q*: How to cooperative with or without CSI?**

Signal Model

"Frequency-domain"

- i : 1..I PU Tx
- m: 1.. M freq slots at f_m
- j : 1..J CR Rx

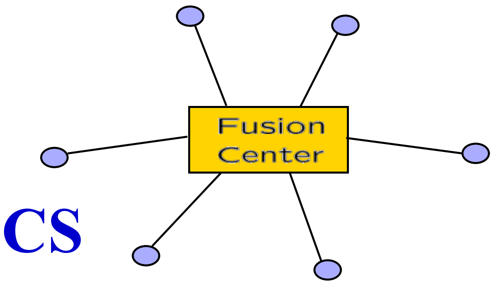


PU Tx signals	Channels (PU→CR)	Innovation + Noise		SU Received Signal
$\mathbf{s}_{p,f}^{(i)}$	$\mathbf{h}_{p,f}^{(i,j)}$	$\mathbf{r}_{c,f}^{(j)}$	$\mathbf{w}_{f}^{(j)}$	$\mathbf{r}_{f}^{(j)}$

CR j:

$$\boxed{\begin{aligned}\mathbf{r}_f^{(j)} &= \textstyle\sum_{i=1}^{I} \operatorname{diag}\left\{\mathbf{h}_{p,f}^{(i,j)}\right\} \mathbf{s}_{p,f}^{(i)} + \mathbf{r}_{c,f}^{(j)} + \mathbf{w}_f^{(j)} \\ &= \mathbf{H}_{p,f}^{(j)} \mathbf{s}_{p,f} + \mathbf{r}_{c,f}^{(j)} + \mathbf{w}_f^{(j)}\end{aligned}}$$

Centralized Cooperative Sensing



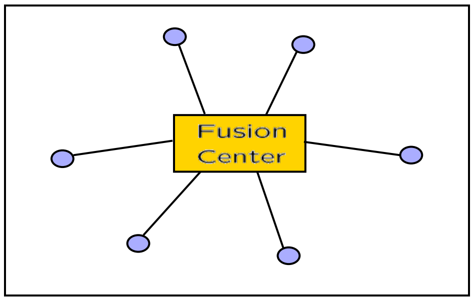
- Take incoherent measurements at each CR
- Reconstruct independently $\Rightarrow$ **Independent CS**
  - Local CR receiver *j* makes local decision on the sparse spectrum
  - FC makes global decision via averaging all local decisions
- Reconstruct jointly $\Rightarrow$ **Joint CS**
  - FC acquires all local compressive measurements
  - FC performs joint sparse spectrum reconstruction:
    - FC needs to know all *measurement matrices* and *channel info*

$$\boxed{\min_{\mathbf{s}_{p,f}} \; \|\mathbf{s}_{p,f}\|_1 + \sum_{j=1}^{J} \lambda_j \left\| \mathbf{x}_t^{(j)} - \underbrace{\mathbf{S}_c^{(j)} \mathbf{F}_M^{-1} \mathbf{H}_{p,f}^{(j)}}_{\mathbf{A}^{(j)}} \mathbf{s}_{p,f} \right\|_2^2}$$

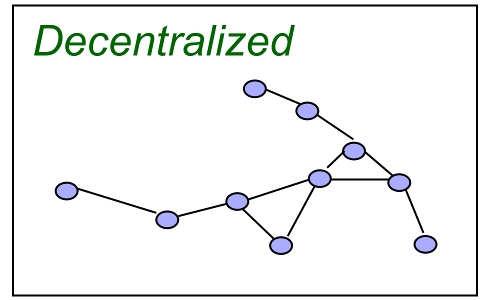
globally optimal; but, issues in robustness, complexity & power costs

Consensus-based Distributed Sensing

- Centralized Lasso: $\hat{\mathbf{s}}_{p,f} : \min_{\mathbf{s}_{p,f}} \left[\sum_{j=1}^{J} \lambda \left\| \mathbf{x}_t^{(j)} - \mathbf{A}^{(j)} \mathbf{s}_{p,f} \right\|_2^2 + \frac{1}{J} \left\| \mathbf{s}_{p,f} \right\|_1 \right]$



Scalability
Robustness
Lack of infrastructure



- Decentralized equivalence

solvable locally

$$\hat{\mathbf{s}}_{p,f}^{(j)} : \min_{\mathbf{s}_{p,f}^{(j)}} \left[\cancel{\sum_{j=1}^{J}} \lambda \left\| \mathbf{x}_t^{(j)} - \mathbf{A}^{(j)} \mathbf{s}_{p,f}^{(j)} \right\|_2^2 + \frac{1}{J} \left\| \mathbf{s}_{p,f}^{(j)} \right\|_1 \right]$$

$$s.t. \quad \mathbf{s}_{p,f}^{(j)} = \mathbf{s}_{p,f}^{(k)} \qquad \forall k \in \mathcal{N}_j$$

Exchange of local estimates $\hat{\mathbf{s}}_{p,f}^{(j)}$

$$s.t. \quad \mathbf{s}_{p,f}^{(j)} = \sum_{k \in \mathcal{N}_j} \alpha_{jk} \mathbf{s}_{p,f}^{(k)}$$

- Constraints impose consensus across the network $\hat{\mathbf{s}}_{p,f}^{(j)} = \hat{\mathbf{s}}_{p,f}, \ \forall j$

Decentralized Joint CS Algorithm

- Alternating-direction method of multipliers (ADMoM)
  - Augmented Lagrange function

$$\mathcal{L}\left(\mathbf{s}_{p,f}^{(j)}; \lambda, \mathbf{z}_j, c, \{\bar{\mathbf{s}}_{p,f}^{(k)}\}_{k\in N^{(j)}}\right) = \left\|\mathbf{s}_{p,f}^{(j)}\right\|_1 + \lambda \left\|\mathbf{x}_t^{(j)} - \mathbf{A}^{(j)}\mathbf{s}_{p,f}^{(j)}\right\|_2^2 + \mathbf{z}_j^T \mathbf{s}_{p,f}^{(j)} + \frac{c}{2}\left\|\mathbf{s}_{p,f}^{(j)} - \sum_{k\in N^{(j)}} w_{jk}\bar{\mathbf{s}}_{p,f}^{(k)}\right\|_2^2$$

  - Iterative implementation
    - Each CR j reconstructs locally:

$$\mathbf{s}_{p,f}^{(j)}(t+1) = \arg\min_{\mathbf{s}_{p,f}^{(j)}} \mathcal{L}\left(\mathbf{s}_{p,f}^{(j)}; \lambda, \mathbf{z}_j(t), c, \left\{\mathbf{s}_{p,f}^{(k)}(t)\right\}_{k\in N^{(j)}}\right)$$

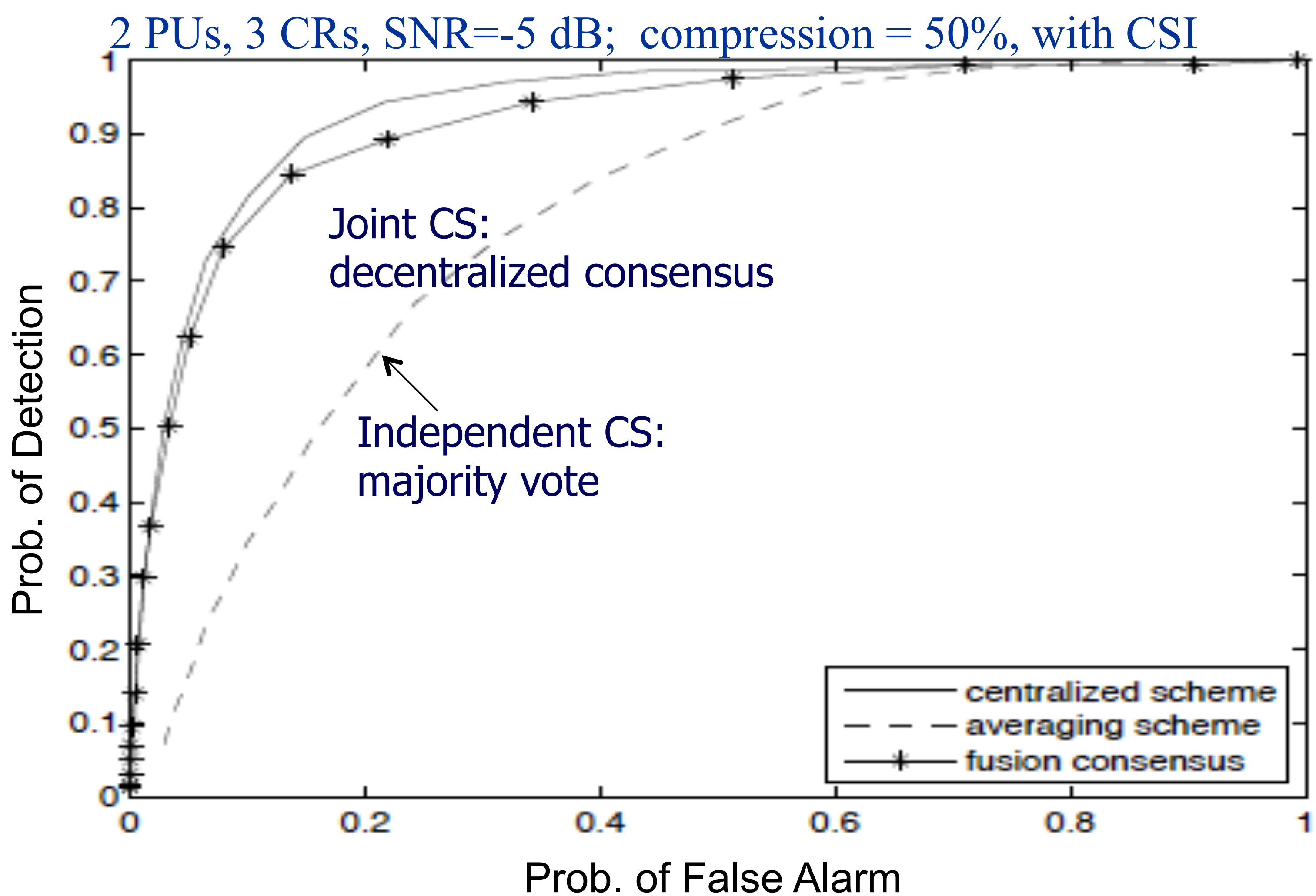
    - Each CR j updates multipliers:

$$\mathbf{z}_j(t+1) = \mathbf{z}_j(t) + \frac{c}{2}\left(\mathbf{s}_{p,f}^{(j)}(t+1) - \sum_{k\in N^{(j)}} w_{jk}\mathbf{s}_{p,f}^{(k)}(t+1)\right)$$

and broadcasts one-hop: $\mathbf{s}_{p,f}^{(j)}(t+1) \longrightarrow \text{CR } k,\ \forall k \in N^{(j)}$

- **Scalable**: one-hop communication, local computation
- **Globally optimal**: guaranteed if the network is connected

Cooperative Compressed Sensing



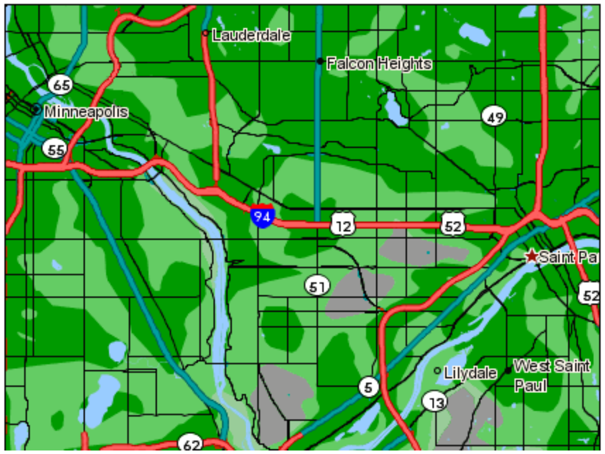
- Performance gain by decentralized fusion over majority vote

❹ Spectrum Cartography

- Idea: CRs collaborate to form a spatial map of the spectrum

given the PSD $\Phi_r(f) = \Phi(f; v_r)$ at position v_r, find $\Phi(v, f)$, $\forall v$

- Goal: $\Phi(v, f), \quad \forall v$
- Specifications: coarse approx. suffices
- Approach: basis expansion of $\Phi(v, f)$
- *Compressive Sampling* possible to form the PSD data



[Bazerque-Giannakis etal.; Asilomar'2008, T-SP'2010, ICASSP'2011]

Modeling

- Transmitters

  $\mathrm{Tx}_s, \ s = 1, \ldots, N_s$

- Sensing CRs

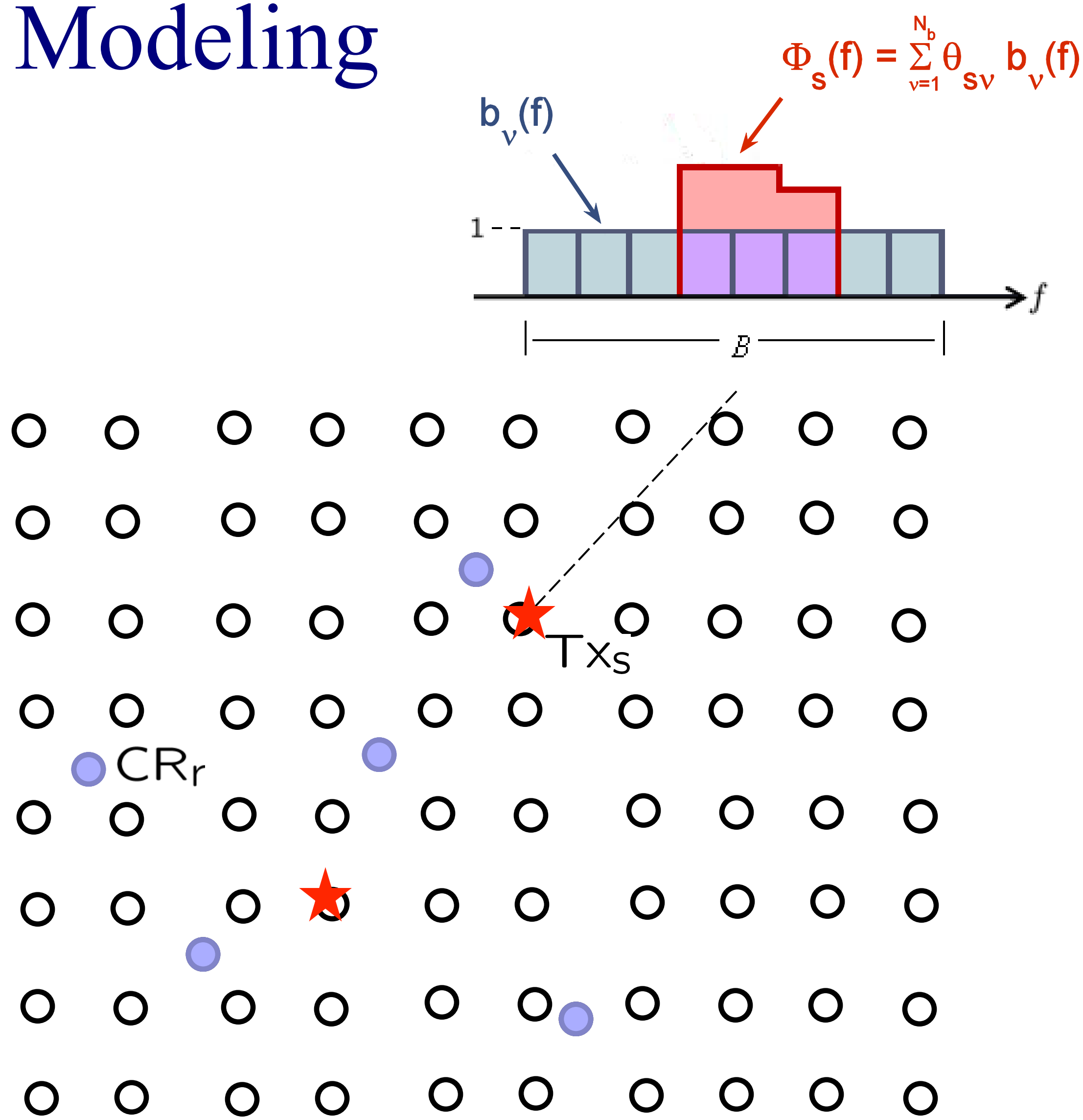
  $\mathrm{CR}_r, \ r = 1 : N_r$

- Frequency bases

  $b_\nu(f), \ \nu = 1 : N_b$

- Sensed frequencies

  $f_k, \ k = 1 : K$



Sparsity present in space and frequency

Space-Frequency Basis Expansion

- Superimposed Tx spectra measured at CR r

$$\boxed{\Phi_r(f) = \sum_{s=1}^{N_s} \gamma_{sr}\Phi_s(f) + \sigma_r^2 = \sum_{s=1}^{N_s} \gamma_{sr} \sum_{\nu=1}^{N_b} \theta_{s\nu} b_\nu(f) + \sigma_r^2}$$

  - Average path-loss $\gamma_{sr} = \mathbb{E}(|H_{sr}(f)|^2) = \gamma_0 \left(\frac{d_0}{\|v_s - v_r\|}\right)^{-\alpha}, \ \alpha \in [2, 5)$
  - Frequency bases $b_\nu(f) = \mathrm{rect}(f - f_\nu)$

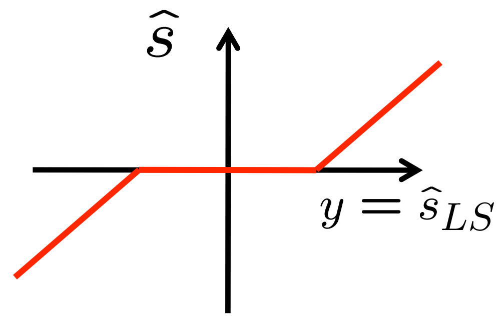
- Linear model in $\theta_{s\nu}$ and σ_r^2

$$\mathbf{y} = \begin{pmatrix} \Phi_1(f_1) \\ \vdots \\ \Phi_1(f_K) \\ \hdashline \Phi_2(f_1) \\ \vdots \\ \Phi_2(f_K) \\ \hdashline \vdots \\ \vdots \\ \hdashline \Phi_{N_r}(f_1) \\ \vdots \\ \Phi_{N_r}(f_K) \end{pmatrix} \begin{matrix} \\ \\ \\ \\ \\ \\ \mathbf{y}_i \\ \\ \\ \\ \\ \end{matrix} = \begin{pmatrix} b_1(f_1)\gamma_{11} & \cdots & b_{N_b}(f_1)\gamma_{N_s 1} & 1 & 0 & \cdot & 0 \\ \hdashline \vdots & & \vdots & \vdots & & & \vdots \\ \hdashline & & \mathbf{H}_i & & & & \\ \hdashline b_1(f_K)\gamma_{1N_r} & \cdots & b_{N_b}(f_K)\gamma_{N_s N_r} & 0 & \cdot & 0 & 1 \end{pmatrix} \begin{pmatrix} \theta_{11} \\ \theta_{21} \\ \vdots \\ \theta_{N_s 1} \\ \theta_{12} \\ \vdots \\ \theta_{N_s 2} \\ \vdots \\ \vdots \\ \theta_{N_s N_b} \\ \sigma_1^2 \\ \vdots \\ \sigma_{N_r}^2 \end{pmatrix} = \mathbf{H}\mathbf{s}$$

Sparse Regression

- Seek a space ***s*** to capture the spectrum measured at all CR_r
  - Lasso: $\hat{\mathbf{s}} = \arg\min \frac{1}{2}\|\mathbf{y} - \mathbf{H}\mathbf{s}\|_2^2 + \lambda\|\mathbf{s}\|_1$

    Soft threshold shrinks noisy estimates to zero
    Similar to Akaike's Information Criterion,
    it penalizes the number of parameters
    spectrum selection + estimation via $\|\,.\,\|_1$ penalty



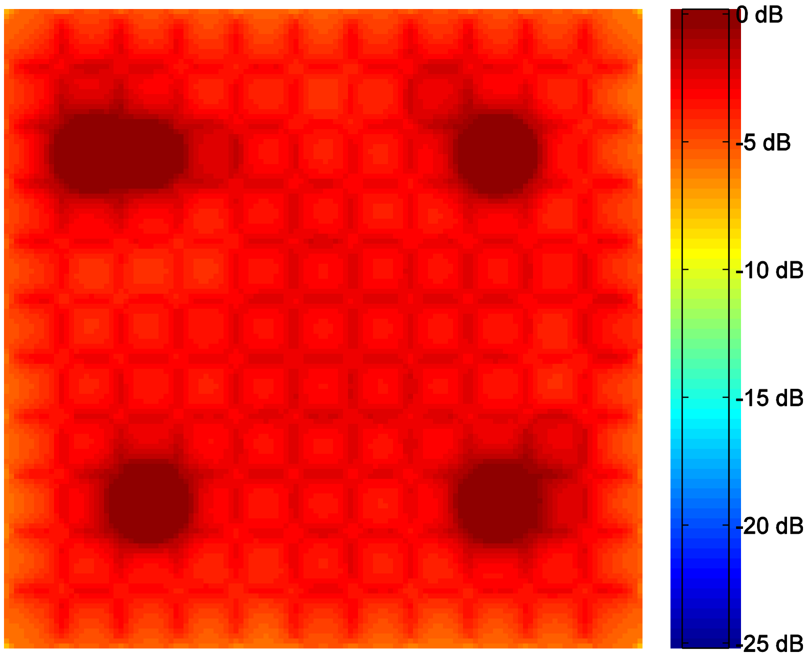
- Power spectrum is non-negative $\Longrightarrow$ non-negativity constraints

$$\hat{\mathbf{s}}^* : \min_{\mathbf{s}} \frac{1}{2}\sum_{i=1}^{N_r} \|\mathbf{y}_i - \mathbf{H}_i\mathbf{s}\|_2^2 + \lambda\sum_{j=1}^{N} s_j$$
$$\text{s.t.} \quad s_j \geq 0, \quad j = 1\ldots N := N_s N_b + N_r$$

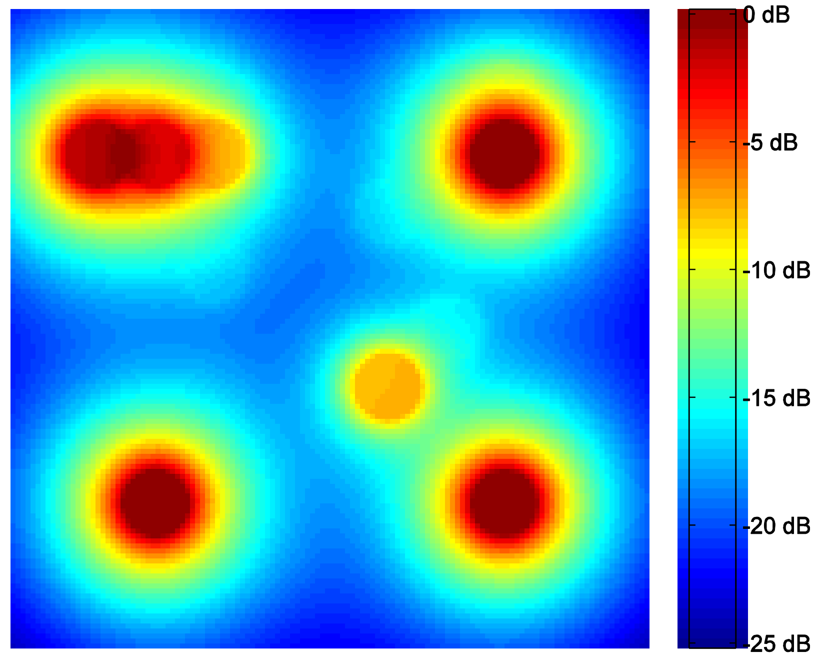
- Decentralized cooperation: distributed consensus optimization

Power Spectrum Cartography

- 5 sources
- $N_s = 121$ candidate locations, $N_r = 50$ CRs



NNLS



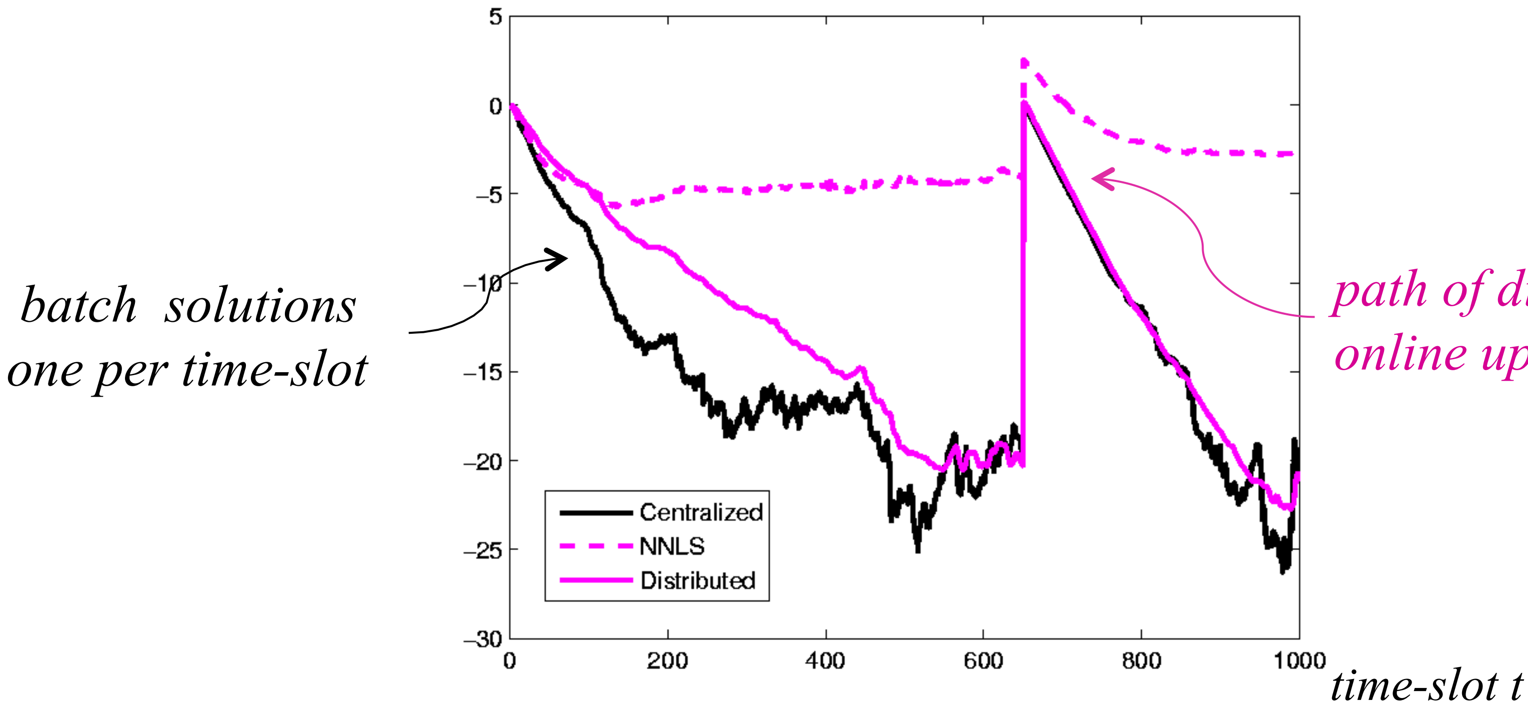
Lasso

- Sparsity-unaware NNLS is prone to false alarms
- As a byproduct, Lasso localizes all sources via variable selection

Tracking Capabilities

- Adaptive implementation via recursive Lasso
  [Giannakis etal.; ICASSP'09, TSP'12]

  - Normalized error $\|\hat{\mathbf{s}} - \mathbf{s}\| / \|\mathbf{s}\|$



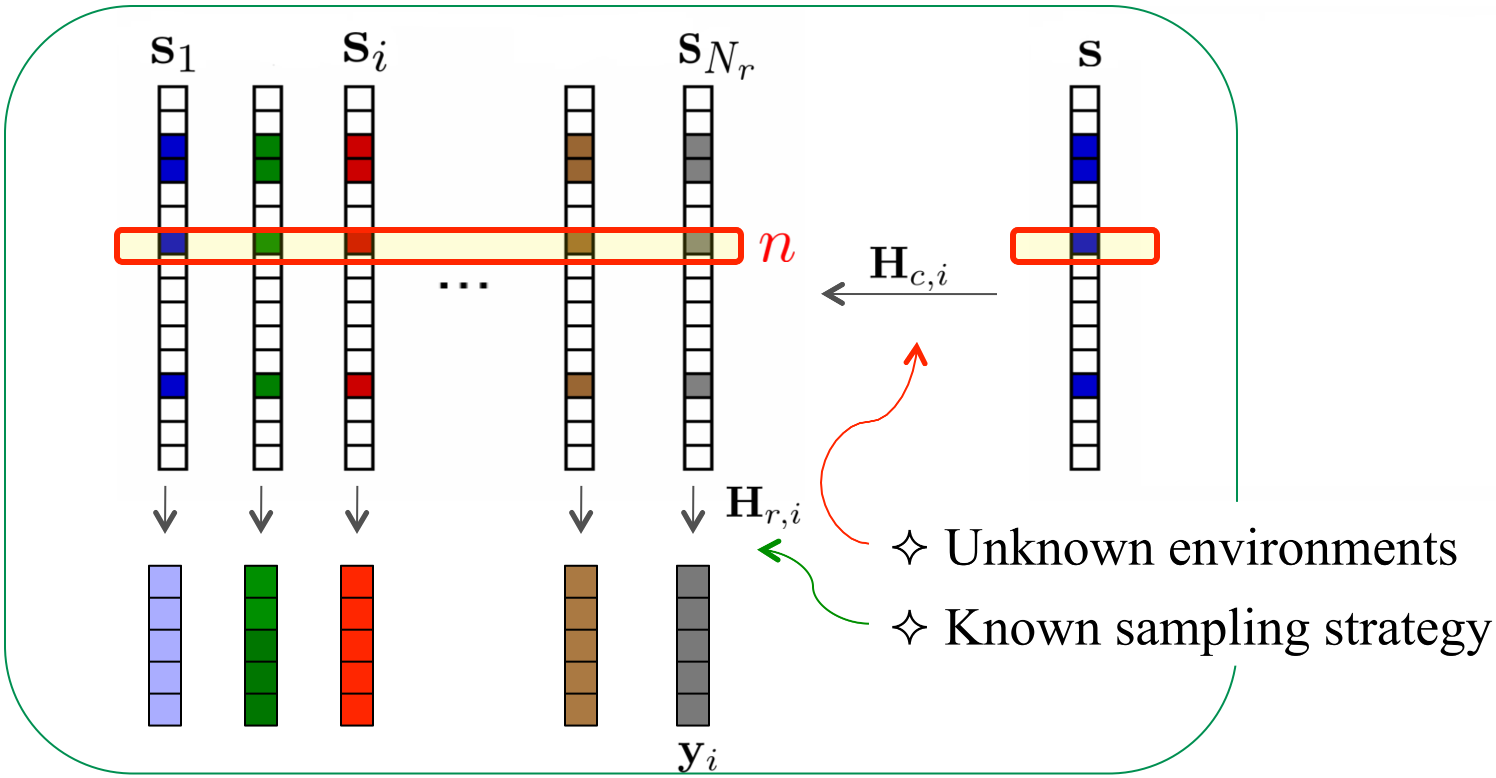
  - Non-stationarity: one Tx exits at time-slot t=650

5 Cooperative Spectrum Support Detection

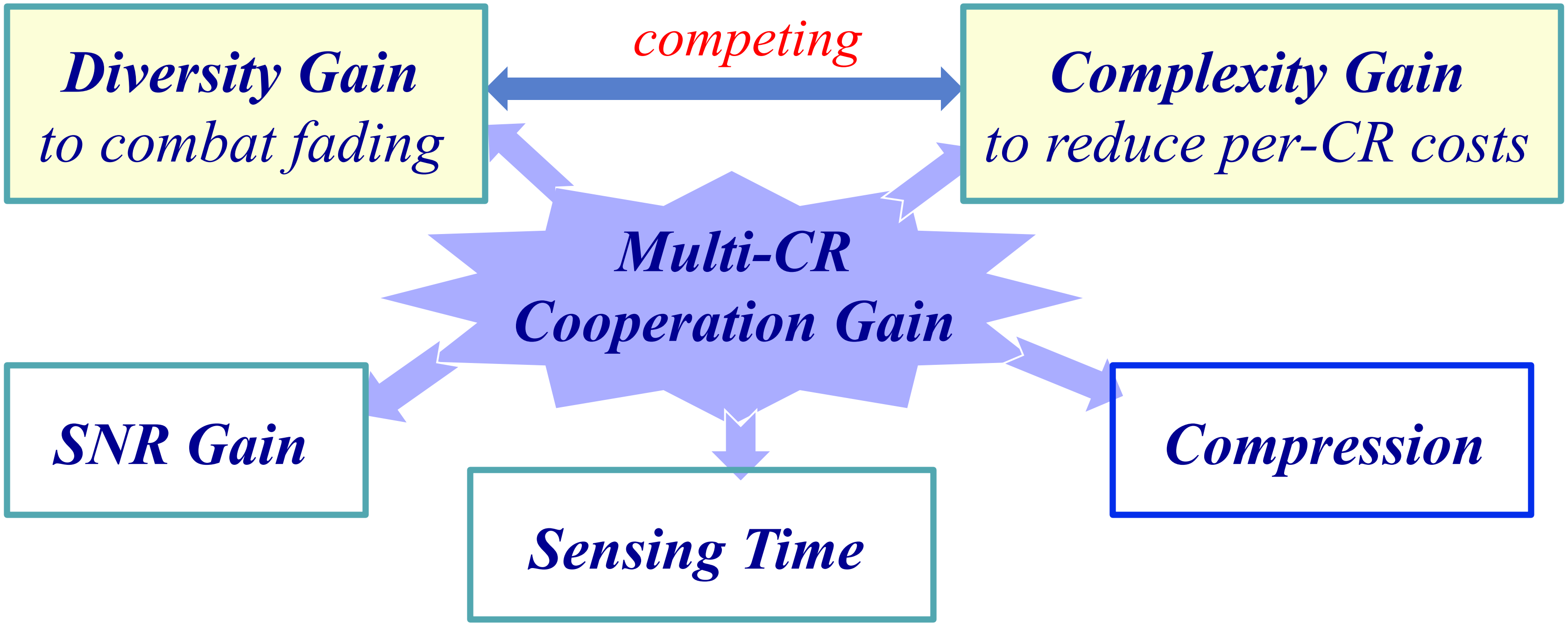
Cooperative spectrum sensing without CSI

- CRs recover signals of different amplitudes, but common support
- No need for channel or location information $\mathbf{H}_i = \mathbf{H}_{r,i}\mathbf{H}_{c,i}$

$$\mathbf{y}_i = \mathbf{H}_i\mathbf{s} = \mathbf{H}_{r,i}\mathbf{s}_i,\ \ i = 1, \ldots, N_r$$



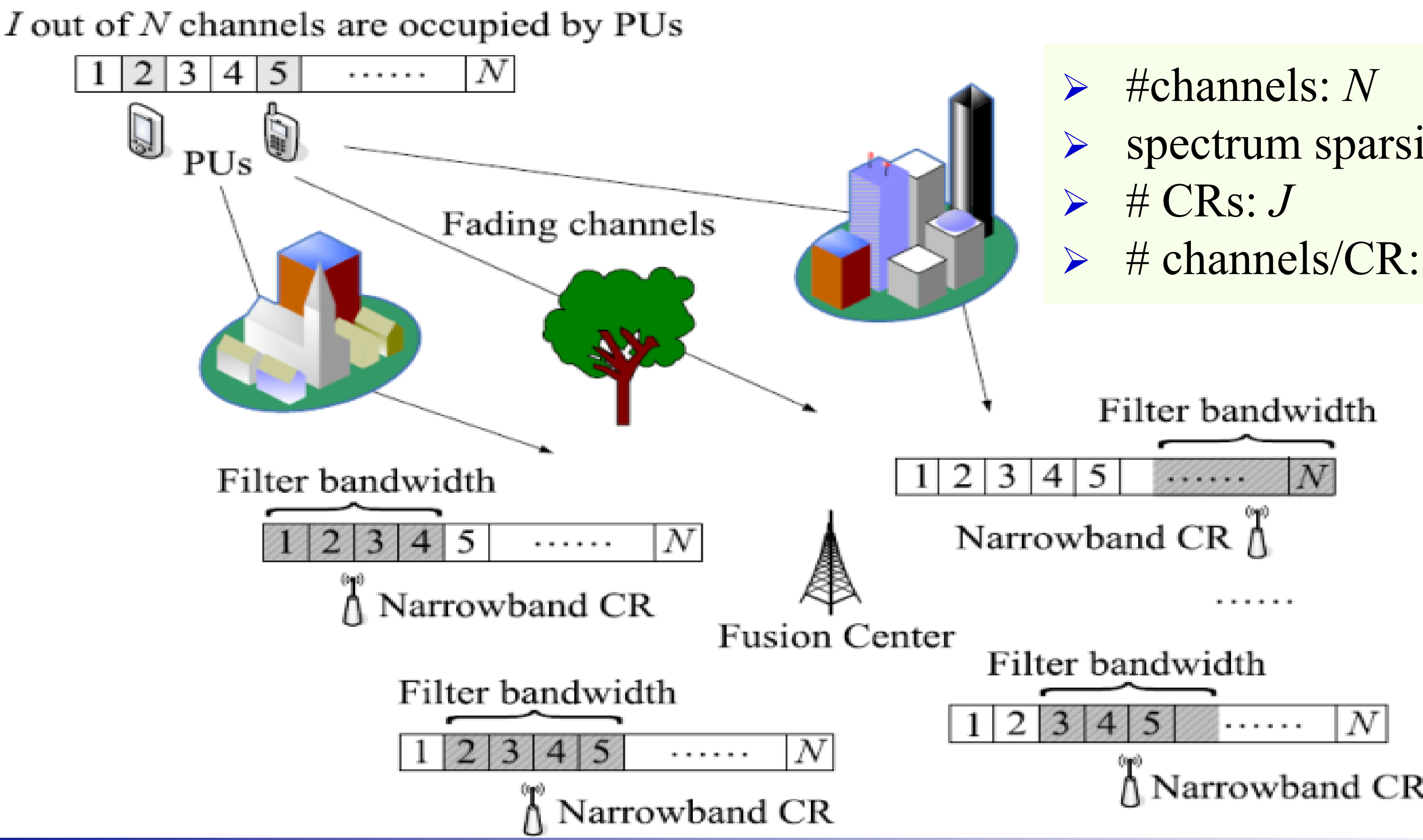
Objective of Cooperation



- Tradeoff between diversity vs. complexity gains
  - To find cooperative sensing solutions with desired tradeoff
  - To delineate the tradeoffs in cooperative CR sensing

System Model

- Wideband spectrum: freq. selective in wide band, flat per channel
- Channel assignment: M out of N channels per CR, uniformly assigned

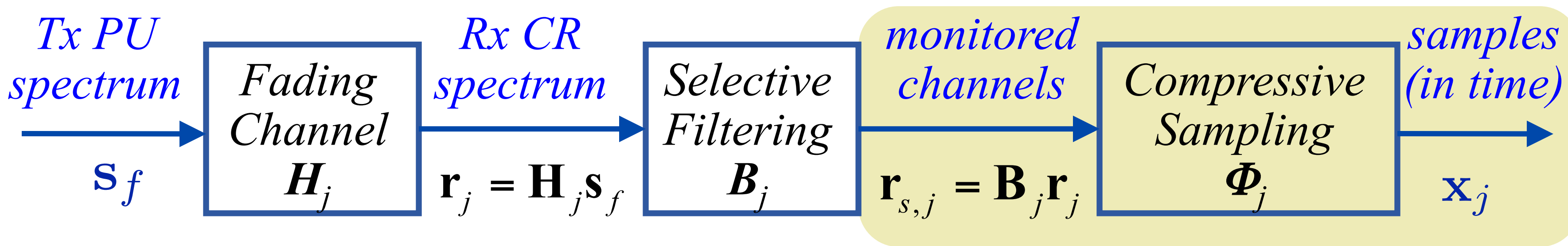


- #channels: N
- spectrum sparsity: $I < N$
- # CRs: J
- # channels/CR: $M<N$

Signal Model

- Spectrum perceived at individual CR $\mathbf{r}_j = \mathbf{H}_j \mathbf{s}_f$
  - fading channel matrix (CSI): diagonal, unknown $\mathbf{H}_j \in \mathcal{C}^{N \times N}$
- Received spectrum after selective filtering $\mathbf{r}_{s,j} = \mathbf{B}_j \mathbf{r}_j$
  - channel selection matrix: binary-valued ($M<N$) $\mathbf{B}_j \in \{0,1\}^{M \times N}$
- Discrete-time samples $\boxed{\mathbf{x}_j = \mathbf{\Phi}_j \mathbf{F}^{-1} \mathbf{B}_j \mathbf{r}_j + \mathbf{w}_j}$
  - (random) sampling matrix ($K \leq M$): $\mathbf{\Phi}_j \in \mathcal{C}^{K \times M}$ ***F:** DFT matrix*
- Cooperative spectrum sensing (CSS): decides PU freq. occupancy
  - cooperative estimation → cooperative support detection

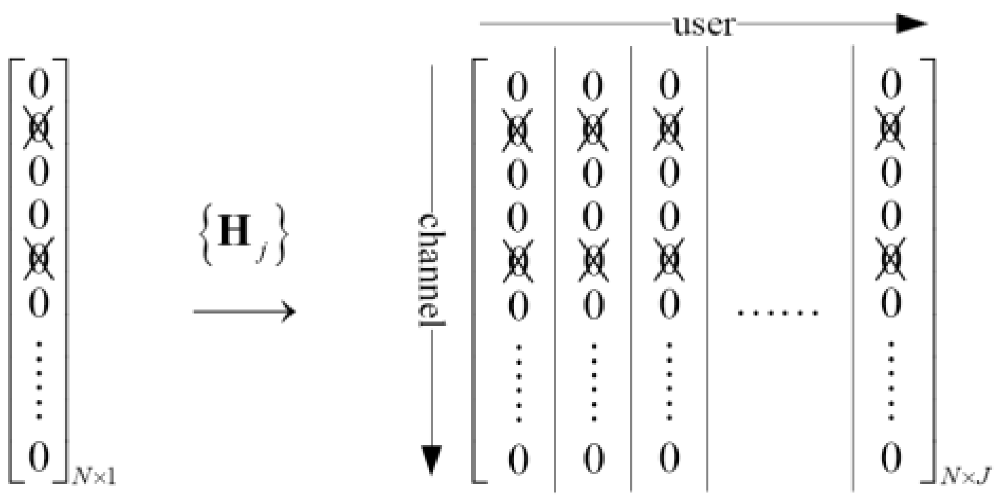
CSS via a Separate Approach (SA)



- Step 1: (sparse) spectrum recovery per-CR
  - recover partial received spectrum: $\mathbf{x}_j \longrightarrow \mathbf{r}_{s,j}$
  - make local (binary) decisions for the monitored *M* channels
    - Per-CR samples are *incomplete*: only captures partial spectrum
    - CRs are *separable*: no cooperation during recovery
- Step 2: decision fusion at Fusion Center
  - do majority vote: each channel is monitored by avg. *J*(*M*/*N*) CRs
    - reduction in sampling costs: *M*/*N*; additional *K*/*M* compression
    - detection diversity: *J(M/N)*

Joint CSS: Low Rank Property

- Key observation: received spectrum matrix is low rank
  - Rank order = size of the nonzero support of the wide spectrum



$$\mathbf{R}_f = [\mathbf{r}_1, \ldots, \mathbf{r}_J]$$

$\mathbf{s}_f$ *is sparse* $\Rightarrow$ $\mathbf{R}_f$ *is low rank*

CSS based on MRM

- Task: joint spectrum recovery prior to decision making
  - capitalize on the low rank property
  - recover $\mathbf{R}_f$ of all J CRs: $\mathbf{x}_j = \underbrace{\mathbf{\Phi}_j \mathbf{F}^{-1} \mathbf{B}_j}_{\mathbf{A}_j} \mathbf{r}_j + \mathbf{w}_j$
- Matrix rank minimization (MRM)

$$\boxed{\min_{\mathbf{R}_f} \ \mathrm{rank}(\mathbf{R}_f) + \lambda \|\mathbf{x}_t - \mathbf{A}\mathrm{vec}(\mathbf{R}_f)\|_2^2}$$

$$\sum_{j=1}^{J} \|\mathbf{x}_j - \mathbf{A}_j \mathbf{r}_j\|_2^2$$

$$\mathbf{A} = \mathrm{diag}\{\mathbf{A}_1, \ldots, \mathbf{A}_J\} \qquad \tilde{\mathbf{w}} = \left[\mathbf{w}_1^T, \ldots, \mathbf{w}_J^T\right]^T$$

$$\mathbf{x}_t = \left[\mathbf{x}_1^T, \ldots, \mathbf{x}_J^T\right]^T = \mathbf{A}\mathrm{vec}(\mathbf{R}_f) + \tilde{\mathbf{w}}$$

  - Rank function **rank(·)**: # of nonzero singular values of matrix

CSS based on MRM (Cont.)

- Reformulation of MRM
  - the function rank(·) is combinational, NP hard
  - rank(.) can be relaxed by nuclear norm
    - Nuclear norm $\|\cdot\|_*$: sum of singular values of the matrix
- CSS via Nuclear norm minimization

$$\min_{\mathbf{R}_f} \ \operatorname{rank}(\mathbf{R}_f) + \lambda \|\mathbf{x}_t - \mathbf{A}\operatorname{vec}(\mathbf{R}_f)\|_2^2$$

$$\Big\Downarrow \ \textit{convex}$$

$$\boxed{\min_{\mathbf{R}_f} \ \|\mathbf{R}_f\|_* + \lambda \|\mathbf{x}_t - \mathbf{A}\operatorname{vec}(\mathbf{R}_f)\|_2^2}$$

user cooperation during reconstruction

- Sensing decision

$$\hat{\mathbf{d}}_f[n] = \left(\sum_{j=1}^{J} |\mathbf{r}_j[n]|^2 \geq \eta\right), \quad \forall n \in [1, N]$$

The Role of Low-Rank Property

- *If the nuclear norm term is absent ....*
  - MRM reduces to conventional least-squares (LS)

$$\min_{\mathbf{R}_f} \left\{ \|\mathbf{x}_t - \mathbf{A}\mathrm{vec}(\mathbf{R}_f)\|_2^2 = \sum_{j=1}^{J} \|\mathbf{x}_j - \mathbf{A}_j \mathbf{r}_j\|_2^2 \right\}$$

$$\Leftrightarrow \quad \min_{\mathbf{r}_{s,j}} \|\mathbf{x}_j - \mathbf{\Phi}_j \mathbf{F}^{-1} \mathbf{r}_{s,j}\|_2^2, \quad j = 1, \ldots, J$$

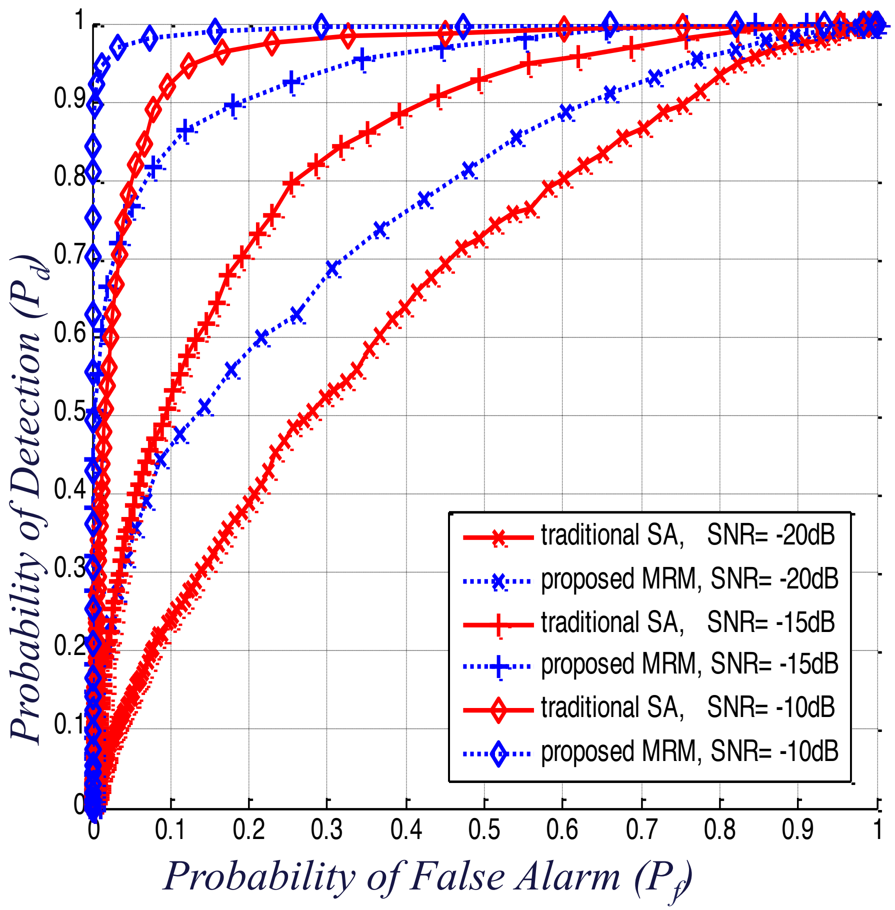
- error penalty terms are completely separable

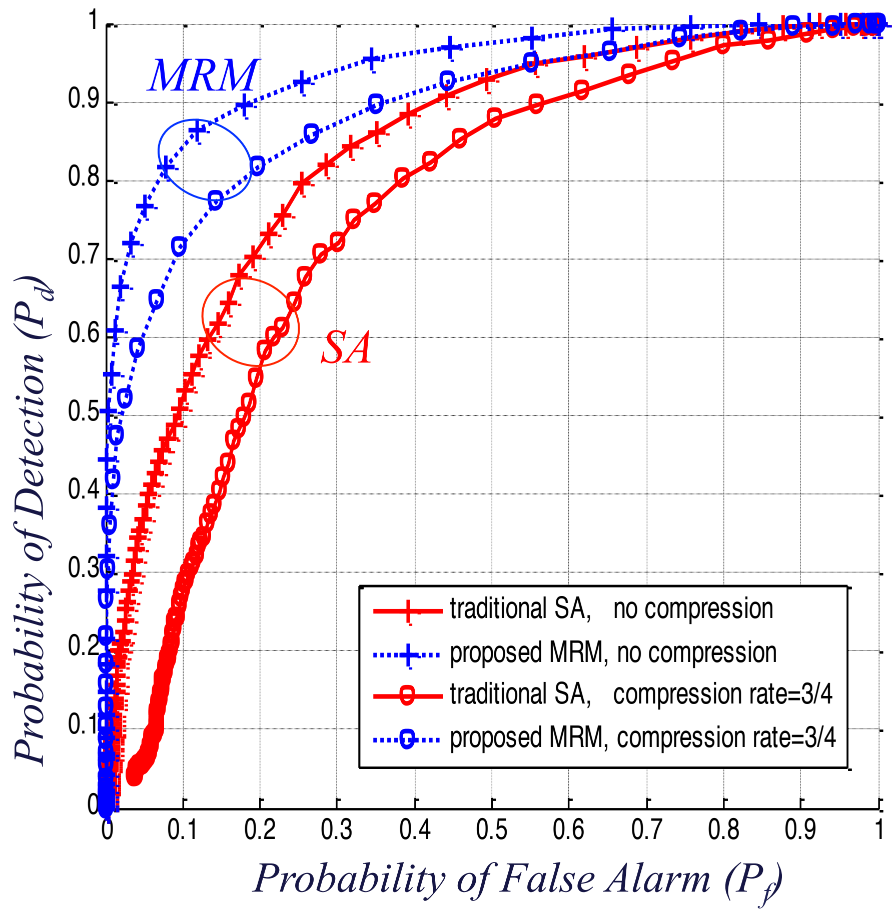
  → no mechanism to enforce user cooperation
- The wideband spectrum $\mathbf{r}_j$ is partially unobservable from $\mathbf{x}_j$

- Low rank property enables cooperation from measurements that are otherwise *non-coupling* and *incomplete*

Simulations – Cooperative Support Detection

- ROC curves (N=20, I=2, J=20, M=4, SNR = -20, -15, -10dB)





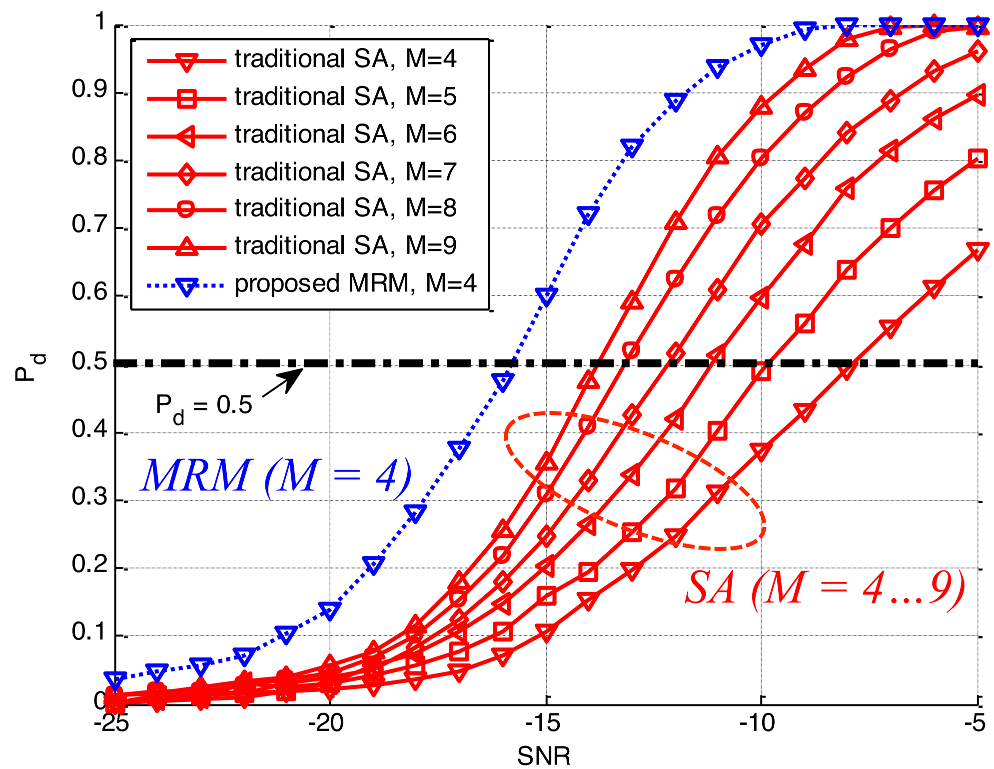
- *MRM makes better use of user diversity*
- *MRM is robust to compression*

Tradeoff Analysis

- **Detection diversity**
  [Daher-Adve, T-AES'10]

$$D = \left.\frac{\partial\ P_d}{\partial\ \mathrm{SNR}}\right|_{P_d=0.5}$$

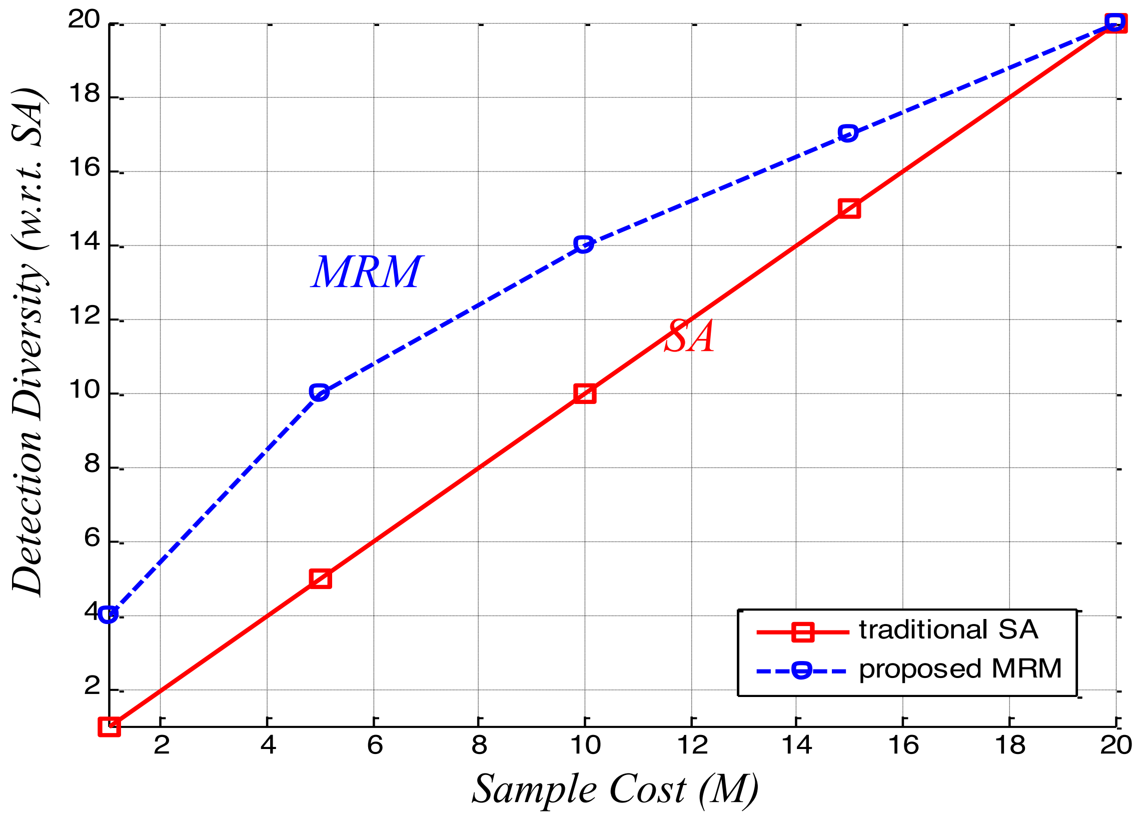
MRM-based approach has better capability in collecting effective detection diversity than separate approach (SA), given the same user diversity



- **Sampling cost**
  - Given user diversity, hardware complexity is measured by *M*
  - Smaller *M* results in lower sampling cost per CR

Tradeoff Results

- Tradeoff in diversity gain vs. complexity gain
  - Detection diversity & sampling cost are competing elements
  - Given same sampling costs, MRM attains higher diversity gain

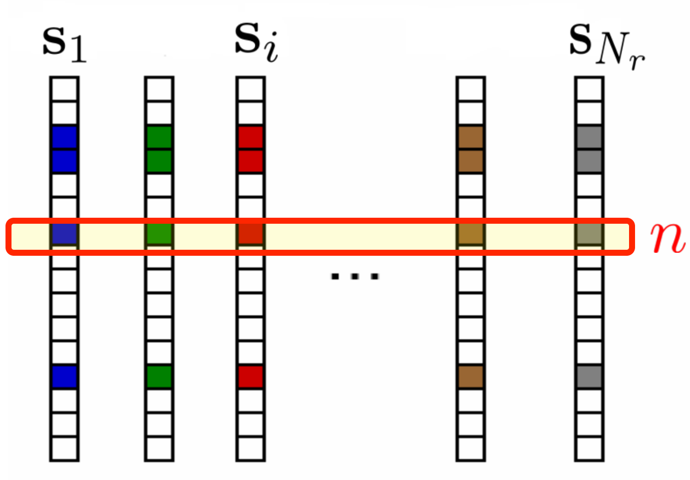


❻ Decentralized Support Detection

- Cooperation as a multiple measurement vector (MMV) problem
- Row Lasso for the MMV problem

$$\min_{\{\mathbf{s}_1,\ldots,\mathbf{s}_{N_r}\}} \frac{1}{2}\sum_{i=1}^{N_r} \|\mathbf{y}_i - \mathbf{H}_{r,i}\mathbf{s}_i\|_2^2 + \lambda N \sum_{n=1}^{N} \sqrt{\sum_{i=1}^{N_r} s_i^2[n]}$$

  - Similar to Group Lasso in centralized form [Yuan-Lin' 06]
  - Coupled variables in mixed-norm



- Distributed Implementation

Q: ***What to consent on?***

[Ling-Tian; ICASSP'2011]

Consensus-based Support Detection

Centralized R-Lasso:

$$\min_{\{\mathbf{s}_1,\ldots,\mathbf{s}_{N_r}\}} \frac{1}{2}\sum_{i=1}^{N_r}\|\mathbf{y}_i - \mathbf{H}_{r,i}\mathbf{s}_i\|_2^2 + \lambda N \sum_{n=1}^{N}\sqrt{\sum_{i=1}^{N_r} s_i^2[n]}$$

- Energy-based Consensus
  - Energy vector $\mathbf{r} \in \mathcal{R}^N : r_n = \sqrt{\sum_{i=1}^{N_r} s_i^2[n]}$

$$\min \frac{1}{2}\sum_{i=1}^{N_r}\|\mathbf{y}_i - \mathbf{H}_{r,i}\mathbf{s}_i\|_2^2 + \lambda N \mathbf{1}^T\mathbf{r}$$

  - Consensus optimization formulation

$$\hat{\mathbf{s}}_i : \min \frac{1}{2}\cancel{\sum_{i=1}^{N_r}}\|\mathbf{y}_i - \mathbf{H}_{r,i}\mathbf{s}_i\|_2^2 + \lambda \cancel{\sum_{i=1}^{N_r}} \mathbf{1}^T\mathbf{r}^{(i)}$$

$$s.t. \quad \mathbf{r}^{(i)} = \bar{\mathbf{r}}^{(i)}, \quad \cancel{\forall i}$$

solved locally

exchange $\mathbf{r}^{(i)}$ in one-hop

with $\bar{\mathbf{r}}^{(i)} = \sum_{i' \in \mathcal{N}_i} w_{ii'}\mathbf{r}^{(i')}$

Decentralized Algorithm

- Alternating-direction method of multipliers (ADMoM)
  - Augmented Lagrange function

$$\mathcal{L}\left(\mathbf{s}_i; \lambda, \boldsymbol{\beta}_i, c, \{\bar{\mathbf{r}}^{(i')}\}_{i' \in \mathcal{N}_i}\right) = \frac{1}{2}\|\mathbf{y}_i - \mathbf{H}_{r,i}\mathbf{s}_i\|_2^2 + \lambda \mathbf{1}^T \mathbf{r}^{(i)}(\mathbf{s}_i) + \boldsymbol{\beta}_i^T\left(\mathbf{r}^{(i)}(\mathbf{s}_i) - \bar{\mathbf{r}}^{(i)}\right) + \frac{c}{2}\left\|\mathbf{r}^{(i)}(\mathbf{s}_i) - \bar{\mathbf{r}}^{(i)}\right\|_2^2$$

  - Iterative implementation
    - each CR i reconstructs locally:

$$\mathbf{s}_i(t+1) = \arg\min_{\mathbf{s}_i} \mathcal{L}\left(\mathbf{s}_i; \lambda, \boldsymbol{\beta}_i(t), c, \{\bar{\mathbf{r}}^{(i')}(t)\}_{i' \in \mathcal{N}_i}\right)$$

$$r_n^{(i)}(t+1) = \sqrt{\left[r_n^{(i)}(t)^2 - s_i[n]^2(t) + s_i[n]^2(t+1)\right]^+}, \quad \forall n$$

    - each CR i updates multipliers:

$$\boldsymbol{\beta}_i(t+1) = \boldsymbol{\beta}_i(t) + c\left(\mathbf{r}^{(i)}(t+1) - \bar{\mathbf{r}}^{(i)}(t)\right)$$

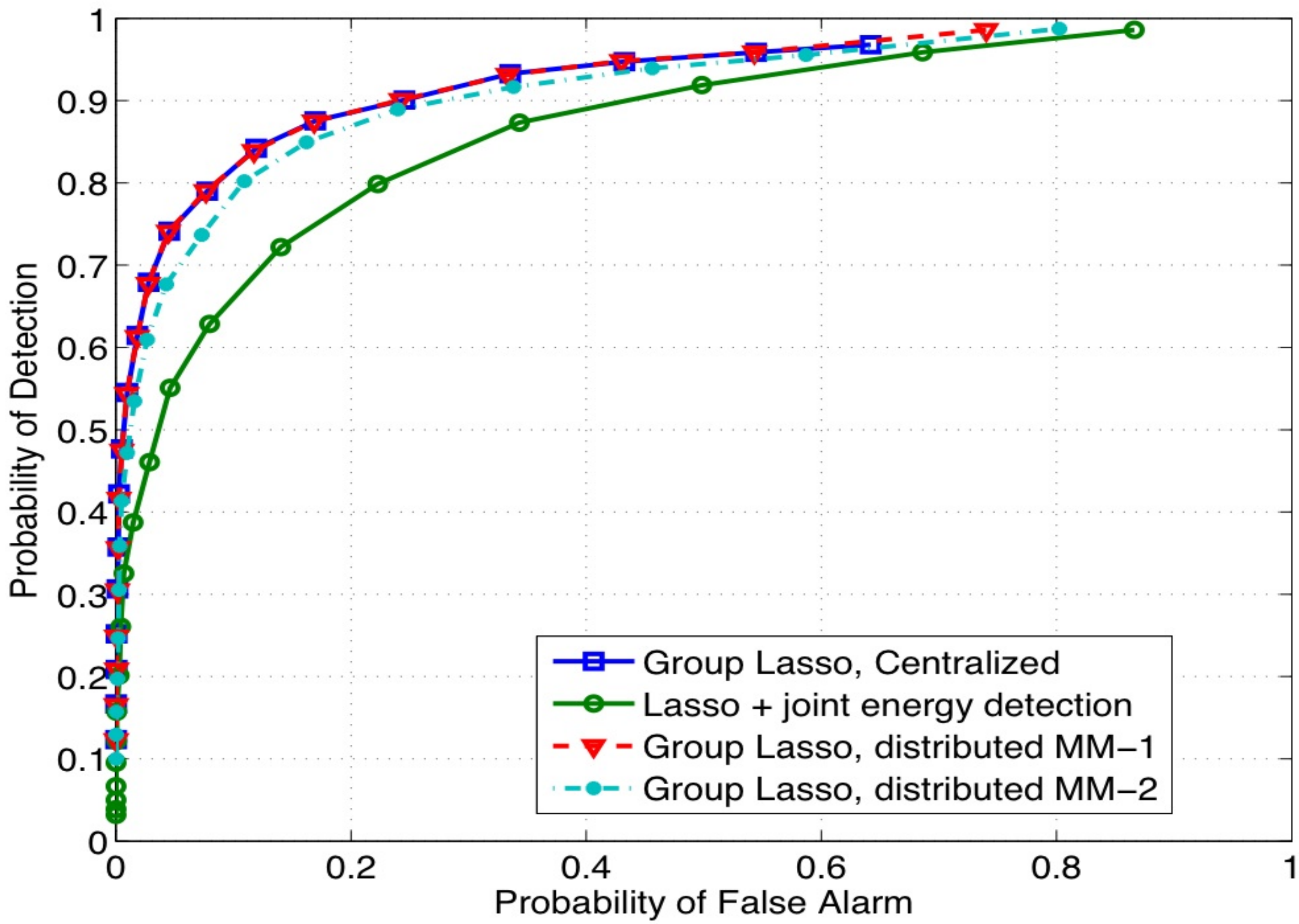
    - broadcasts local decision one-hop:

$$\mathbf{r}^{(i)}(t+1) \longrightarrow \text{neighbors } i', \ \forall i' \in \mathcal{N}_i$$

fast convergence w/ thresholding

Cooperative Support Detection

- 20 channels, 5 PUs, 6 cooperative CRs, SNR = 5dB, 25% compression



7 Compressive Cyclic Feature Detection

- Cyclostationarity-based approach for detection
  - ✔ insensitive to unknown signal parameters
  - ✔ cyclic statistics robust to multipath
  - ✔ resilient against Gaussian noise
  - ✔ can differentiate modulation types and separate interferences
- Issues with cyclic feature detection
  - ✘ Cyclostationarity is induced by OVER-sampling

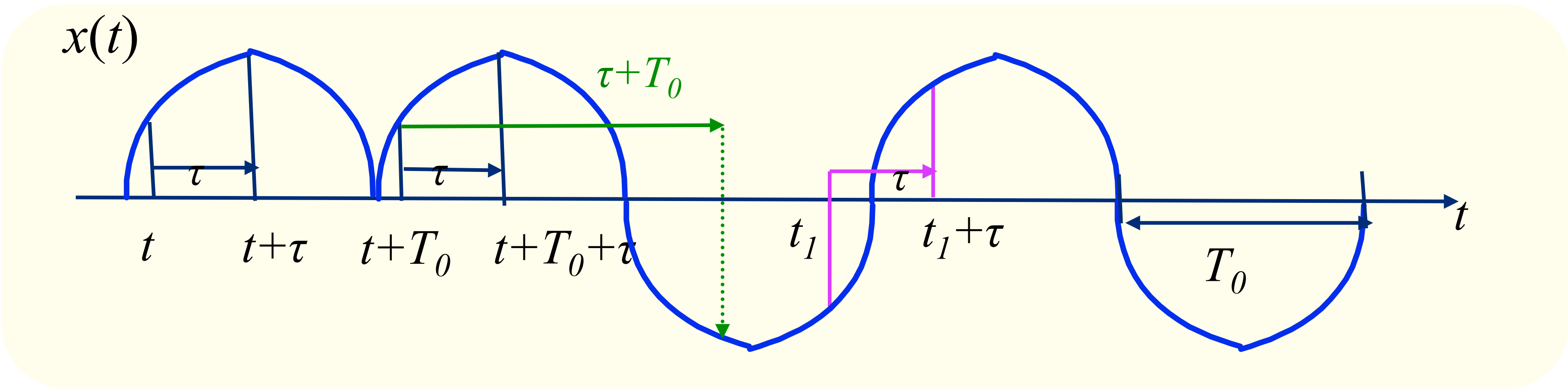
    → excessive sampling-rate requirements
  - ✘ Cyclic statistics converge slowly with finite samples

    → long sensing time

➡ Cyclic Feature Detection and Classification using Compressive Sampling

Cyclostationarity in Modulated Signals

- Modulated signals are cyclostationary processes
  - Cyclic features reveal critical signal parameters:
    - carrier frequency
    - symbol rate
    - modulation type
    - timing, phase etc.
  - Non-cyclic signals (e.g. noise) do not possess cycle frequencies



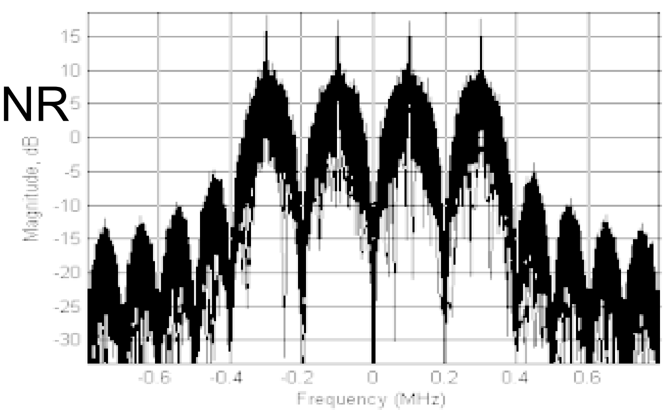
$$R_x(t,\tau) = R_x(t+T_0,\tau)$$

Why Cyclic Statistics (1)

- Energy detection vs. feature detection [Sahai-Cabric'05]

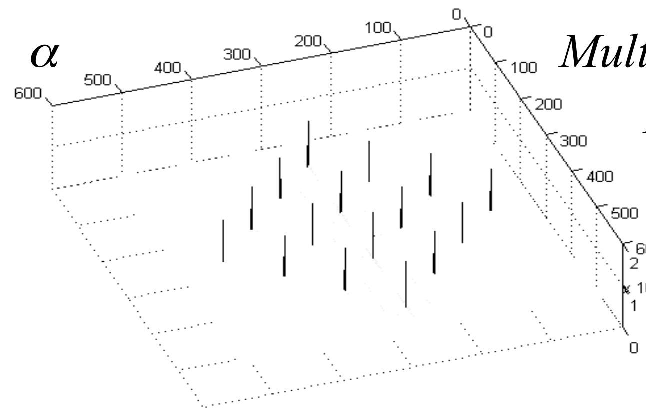
spectrum density ($\alpha = 0$) spectral correlation density (SCD)

High SNR



f

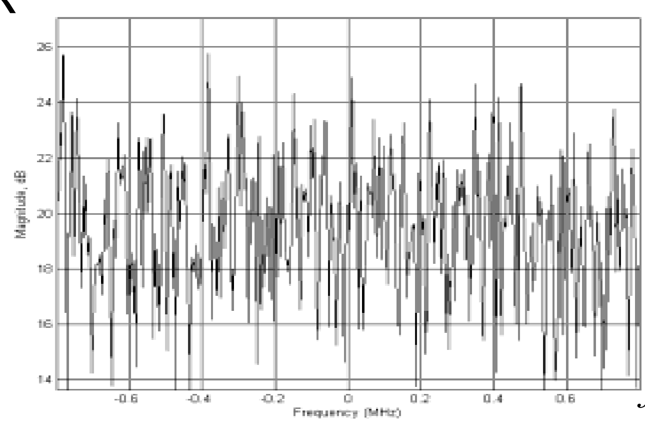
α



Multi-harmonics peaks at $\alpha = kf_m$

f

Low SNR



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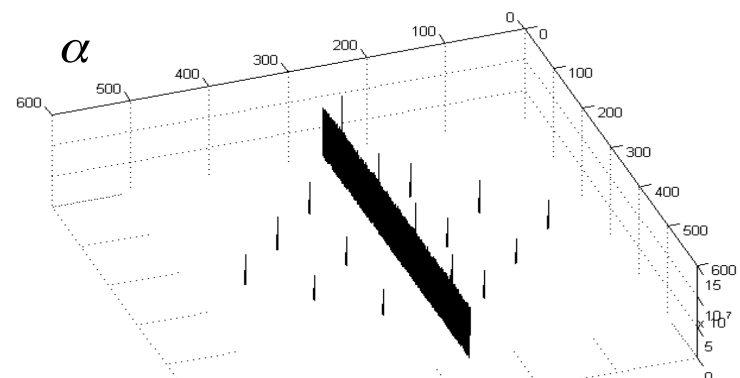
α



f

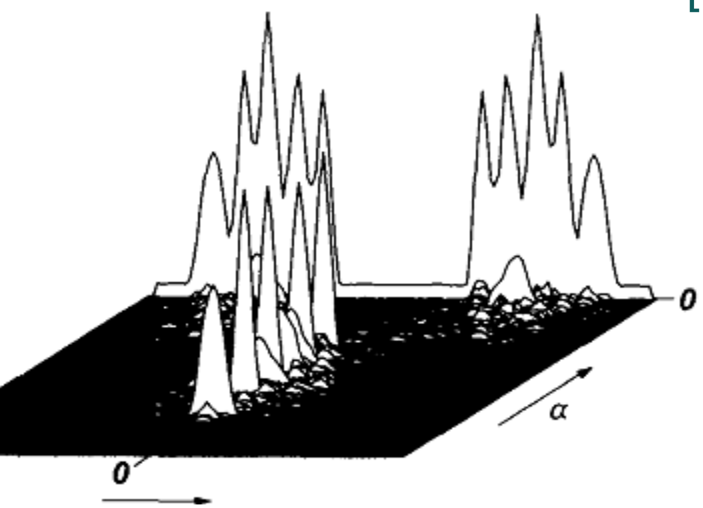
no noise components when $\alpha \neq 0$

Why Cyclic Statistics (2)

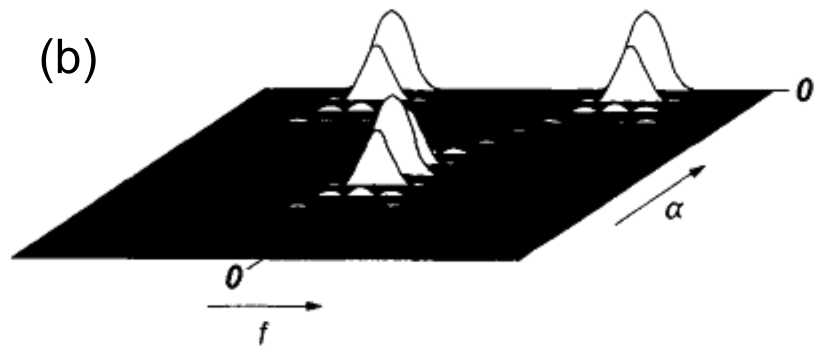
Spectral Correlation Density (SCD)

[Gardner'88]

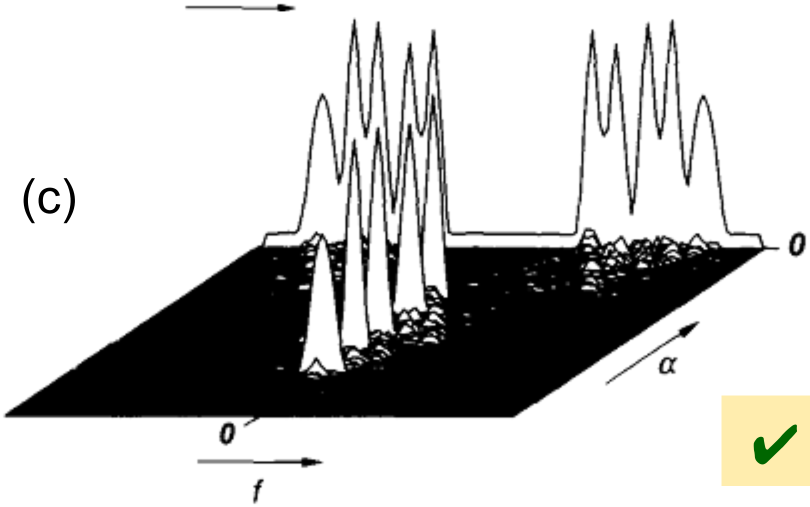
(a)



(b)



(c)



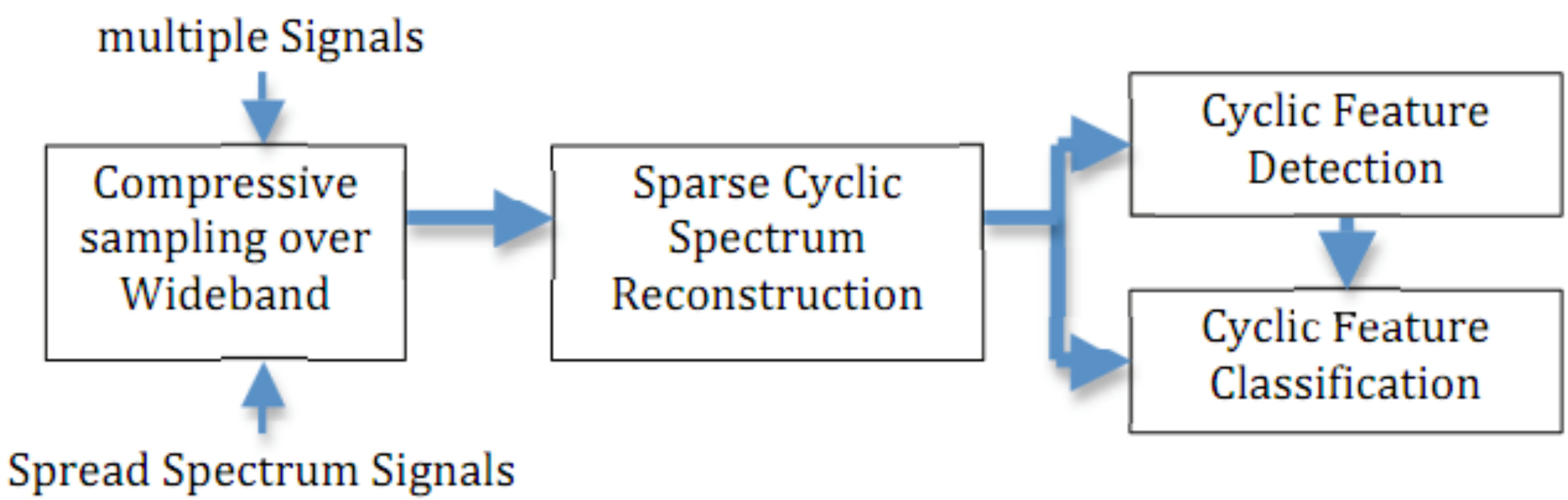
Magnitudes of estimated SCD

a) a BPSK signal corrupted by white noise and five AM interferences
b) the BPSK signal alone
c) the white noise and five AM interferences

✔ overlapping in PSD, separable in SCD

Wideband Cyclic Feature Detection

- Cyclic feature detection over a wide band
  - Goal is to perform simultaneous detection of multiple sources
  - Need to alleviate the sampling rates and sensing time
- Exploiting signal sparsity in two dimensions
  - Sparsity in frequency domain ← low spectrum utilization
  - Sparsity in cyclic-freq. domain ← modulation-dependent cycles



Signal Model

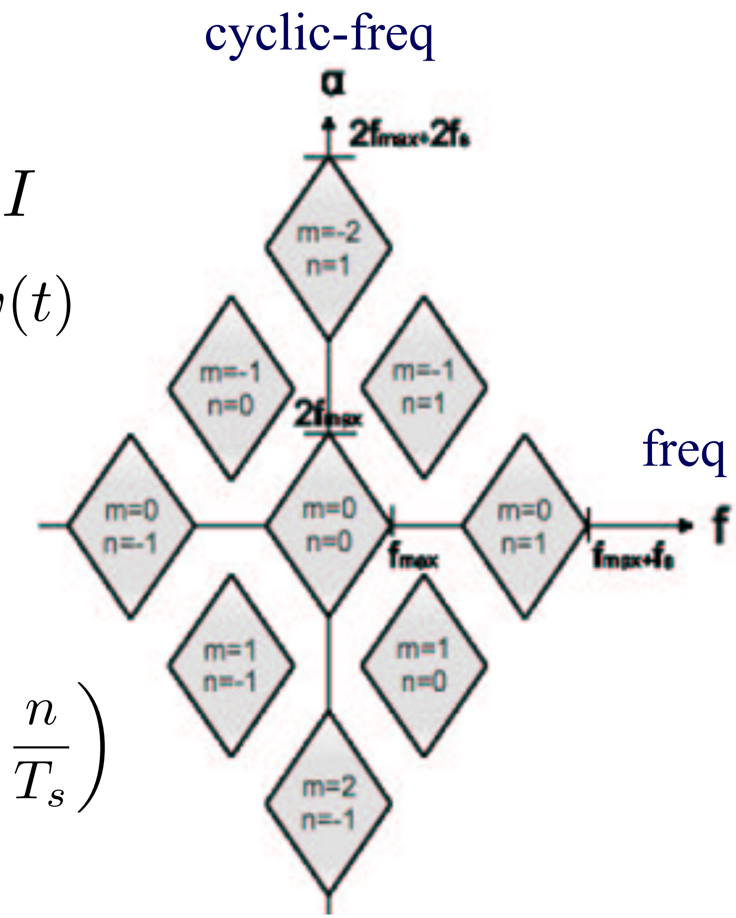
- Wide band of interest: $[-f_{\max}, f_{\max}]$
- Multiple PU signals: $x_i(t), \quad i = 1, \ldots, I$
- Received signal: $x(t) = \sum_{i=1}^{I} x_i(t) + w(t)$
- Cyclic spectrum (SCD):

  $S(\alpha, f)$ nonzero for $|f| + \frac{\alpha}{2} \leq f_{\max}$
- Folded SCD of sampled signal:

$$S(\alpha, f) = \frac{1}{T_s} \sum_{m,n=-\infty}^{\infty} S\left(\alpha + \frac{m}{T_s}, f - \frac{m}{2T_s} - \frac{n}{T_s}\right)$$

  - Aliasing-free condition:

$$f_s = 1/T_s \geq 2 f_{\max}$$



Cyclic spectrum $\tilde{S}(\alpha, f)$ of digital samples. The central diamond region is the non-zero support [Gardner'91]

Problem Setup

$x(t) \leftrightarrow \mathbf{x}_t$ → CS-ADC (sub-Nyquist) → $\mathbf{z}_t : \{z[n]\}$ → Sparse Signal Recovery → $S_x(\alpha, f)$

$S_x(\alpha,f)$ sparse

$\mathbf{z}_t = \mathbf{A}\mathbf{x}_t$

compressive samples

recovered SCD

- Cyclostationarity in communication signals
  - time-varying (TV) covariance is period in time

$$r_x(n, \nu) = \mathrm{E}\{x(nT_s)x(nT_s + \nu T_s)\} = \mathrm{E}\{\mathbf{x}_t(n)\mathbf{x}_t(n+\nu)\}$$
$$r_x(n, \nu) = r_x(n + kP, \nu), \ \forall n, k, \nu$$

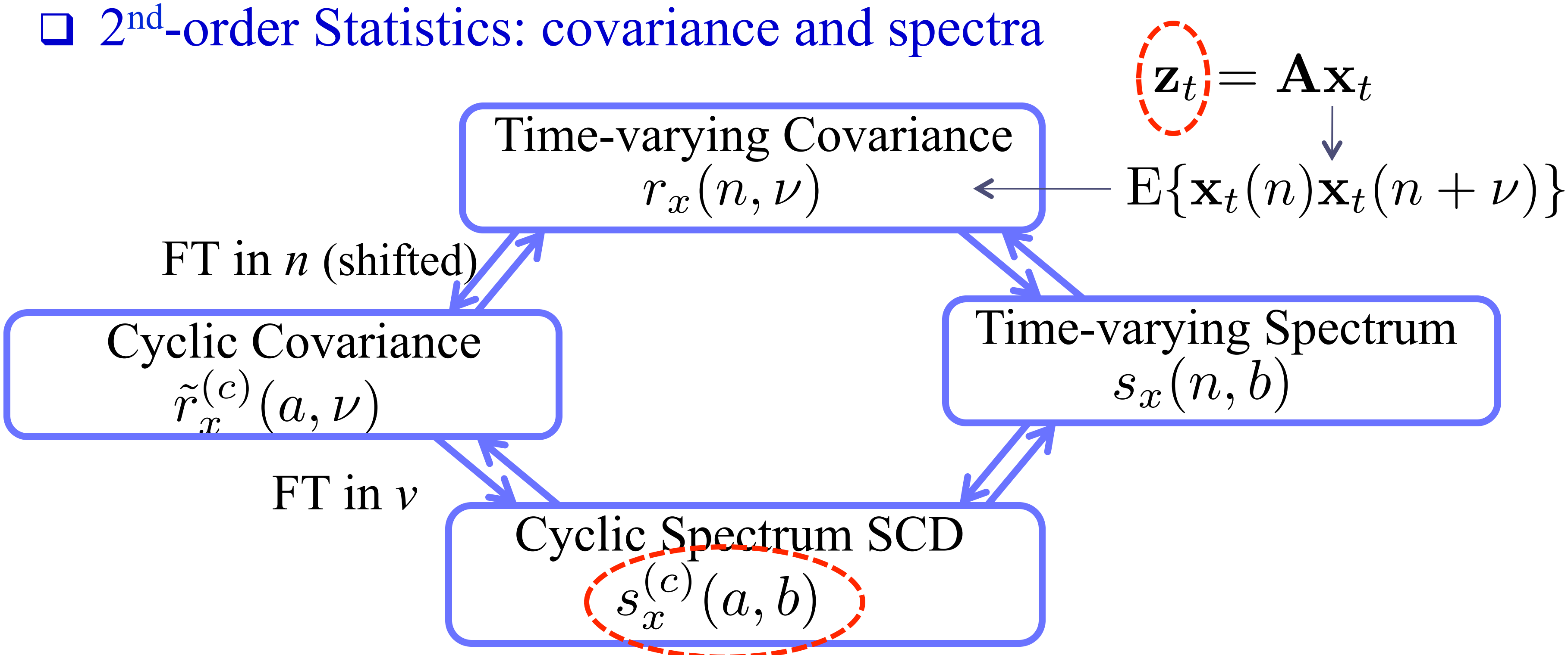
- Sparse signal recovery
  - to reconstruct $S_x(\alpha,f)$ from samples z[n] at sub-Nyquist rate $\frac{M}{N} f_s$

✘ 2D cyclic spectrum is NOT LINEAR in the time-domain samples

→ CS framework not immediately applicable

Defining Cyclic Spectrum

- 2nd-order Statistics: covariance and spectra



- Cyclic-frequency: $a \in [0, N-1] \longleftrightarrow \alpha = \dfrac{1}{NT_s} a$
- Frequency: $b \in [0, N-1] \longleftrightarrow f = \dfrac{1}{NT_s}(b - \dfrac{N-1}{2}) \in (-\dfrac{f_s}{2}, \dfrac{f_s}{2})$

Q: How can we relate sub-Nyquist data and sparse SCD linearly?

Vector-form Relationship (1)

- Linking time-varying covariance matrix with cyclic spectrum
  - TV covariance matrix: $\mathbf{R}_x = \mathbf{E}\{\mathbf{x}_t \mathbf{x}_t^T\}$

$$\mathbf{R}_x = \begin{bmatrix} r_x(0,0) & r_x(0,1) & r_x(0,2) & \cdots & r_x(0,N-1) \\ r_x(0,1) & r_x(1,0) & r_x(1,1) & \cdots & r_x(1,N-2) \\ r_x(0,2) & r_x(1,1) & r_x(2,0) & \cdots & r_x(2,N-3) \\ \vdots & & & \ddots & \vdots \\ r_x(0,N-1) & \cdots & \cdots & \cdots & r_x(N-1,0) \end{bmatrix}$$

  - Degree of freedom: $N(N+1)/2$

$$\mathbf{r}_x = [r_x(0,0), r_x(1,0), \cdots, r_x(N-1,0), r_x(0,1), r_x(1,1), \\ \cdots, r_x(N-2,1), \cdots\cdots, r_x(0,N-1)]^T \in \mathcal{R}^{\frac{N(N+1)}{2}}.$$

  - Vectorized cyclic spectrum

$$\mathbf{s}_x^{(c)} = \mathrm{vec}\{\mathbf{S}_x^{(c)}\} = \underbrace{(\mathbf{I} \otimes \mathbf{F}) \sum_{\nu=0}^{N-1} (\mathbf{D}_\nu^T \otimes \mathbf{G}_\nu) \mathbf{B}^T}_{:=\mathbf{T}} \mathbf{r}_x$$

Vector-form Relationship (2)

- Linking time-varying covariance matrices
  - TV covariance of compressed data $\mathbf{R}_z = \mathrm{E}\{\mathbf{z}_t \mathbf{z}_t^T\} \in \mathcal{R}^{M \times M}$
    - Finite-sample estimate: $\hat{\mathbf{R}}_z = \frac{1}{L} \sum_l \mathbf{z}_{t,l} \mathbf{z}_{t,l}^T$
  - Degree of freedom: $M(M+1)/2$

$$\mathbf{r}_z = [r_z(0,0), r_z(1,0), \cdots, r_z(M-1,0), r_x(0,1), r_z(1,1), \cdots, r_z(M-2,1), \cdots\cdots, r_z(0,M-1)]^T.$$

  - Relationship: $\mathbf{z}_t = \mathbf{A}\mathbf{r}_t \longrightarrow \mathbf{R}_z = \mathbf{A}\mathbf{R}_x\mathbf{A}^T$

- Linear representation for compressed covariance

$$\underbrace{\mathbf{r}_z}_{\frac{M(M+1)}{2} \times 1} = \mathbf{Q}_M \mathrm{vec}\{\mathbf{A}\mathbf{R}_x\mathbf{A}^T\} = \mathbf{Q}_M(\mathbf{A} \otimes \mathbf{A})\mathrm{vec}\{\mathbf{R}_x\} = \mathbf{\Phi}\underbrace{\mathbf{r}_x}_{\frac{N(N+1)}{2} \times 1}$$

Sparse Cyclic Spectrum Recovery

- Reformulated linear relationship

$$\mathbf{r}_z = \mathbf{\Phi}\mathbf{r}_x \qquad \mathbf{s}_x^{(c)} = \mathbf{T}\mathbf{r}_x$$

  - $\mathbf{\Phi} : \frac{M(M+1)}{2} \times \frac{N(N+1)}{2}$ under-determined

- Prior Information
  - $\mathbf{s}_x^{(c)}$ is highly sparse
  - $\mathbf{R}_x$ is positive semi-definite (psd)

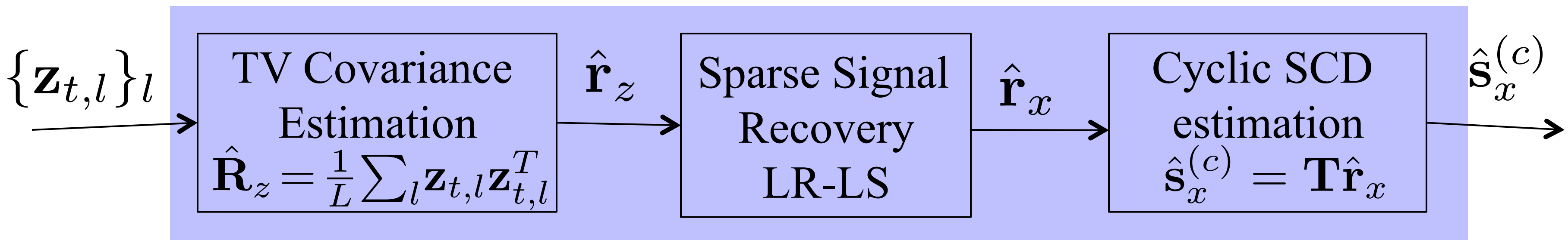
- L_1-norm regularized LS (LR-LS)

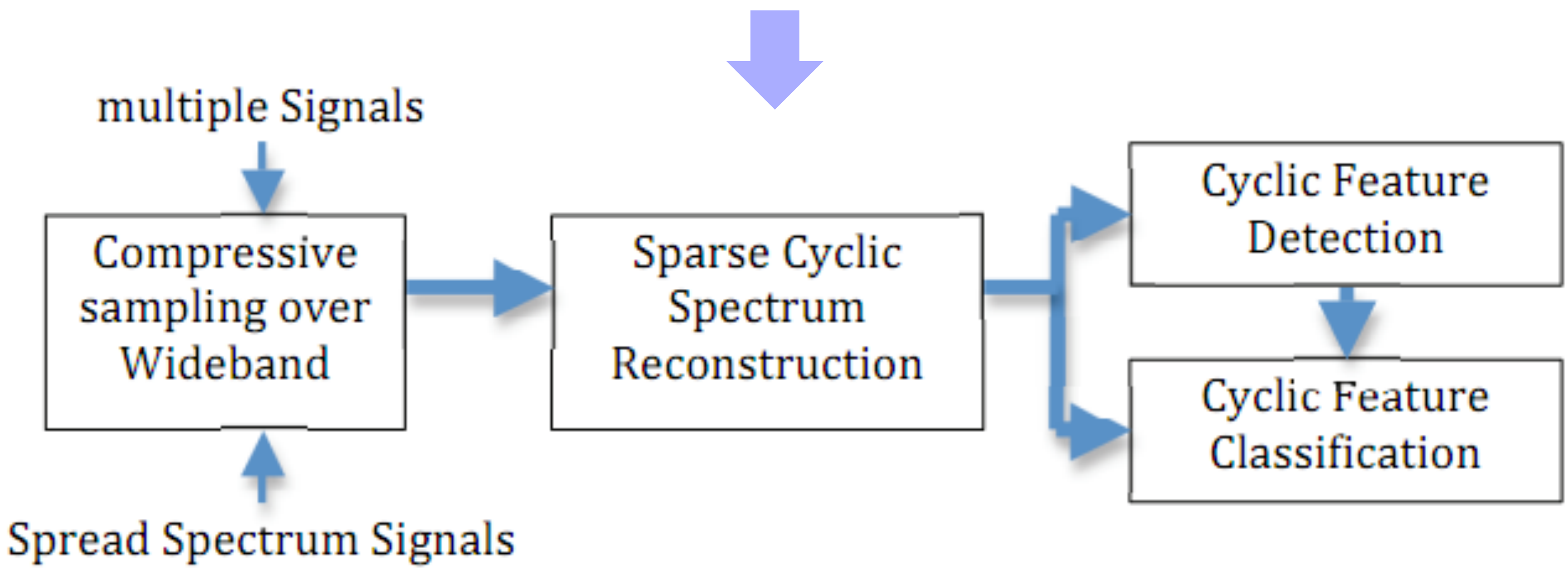
$$\boxed{\begin{aligned} \min_{\mathbf{r}_x} \quad & \|\mathbf{T}\mathbf{r}_x\|_1 + \lambda \|\mathbf{r}_z - \mathbf{\Phi}\mathbf{r}_x\|_2^2 \\ s.t. \quad & \mathbf{R}_x \text{ is psd, with } \mathrm{vec}\{\mathbf{R}_x\} = \mathbf{P}_N \mathbf{r}_x. \end{aligned}}$$

Convex!

$$\boxed{\min_{\mathbf{s}_x} \|\mathbf{s}_x\|_2^2 + \lambda\|\mathbf{r}_z - \mathbf{\Phi}\mathbf{T}^{-1}\mathbf{s}_x\|_2^2}$$

Summary of Reconstruction Steps





Spectrum Occupancy Estimation

- Band-by-band estimation

Is $f^{(n)}$ occupied or not?

$$f^{(n)} = \frac{n - \frac{N-1}{2}}{N} f_s \in \left[-\frac{f_s}{2}, \frac{f_s}{2}\right]$$

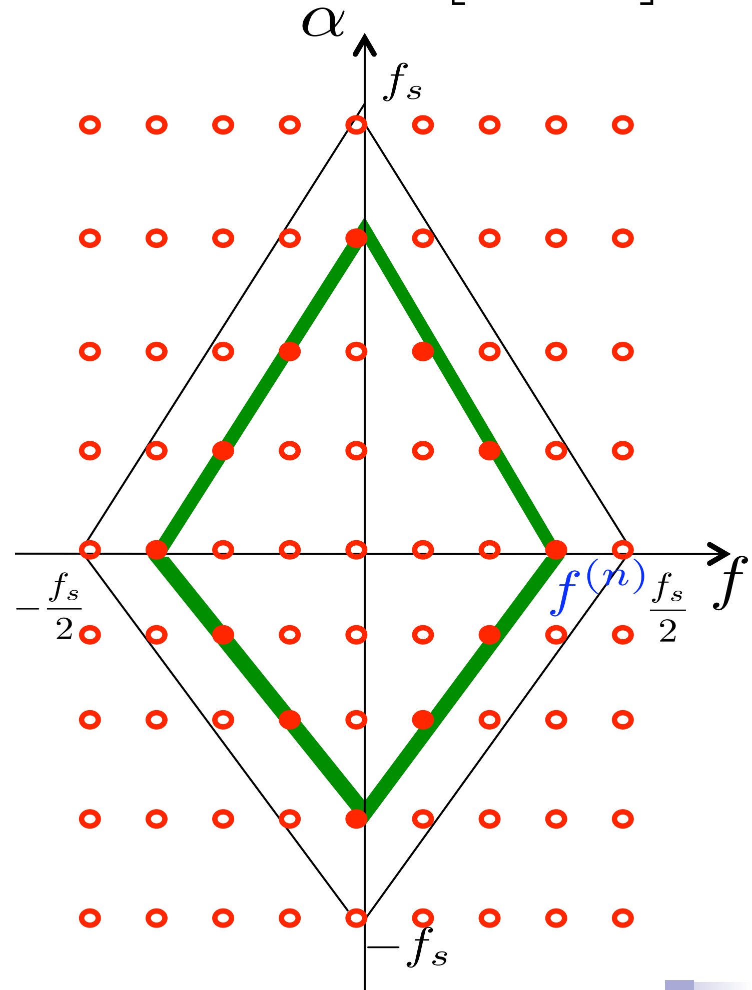
- Region of relevance

$$(\alpha, f) : \begin{cases} f + \dfrac{\alpha}{2} = f^{(n)} \\ |f| + \dfrac{|\alpha|}{2} \le f_{\max} \end{cases}$$

$$(a_i, b_i) : \begin{cases} b_i + \dfrac{a_i}{2} = n \\ \left|b_i - \frac{N-1}{2}\right| + \dfrac{|a_i|}{2} \le \dfrac{f_{\max} N}{f_s} \le \dfrac{N}{2} \end{cases}$$

- Relevant SCD vector for band n

$$\hat{\mathbf{c}}^{(n)} : \left\{ \hat{s}_x^{(c)}(a_i, b_i) \right\}_i$$



Multi-Cycle GLRT

- Binary hypothesis test on band *n*

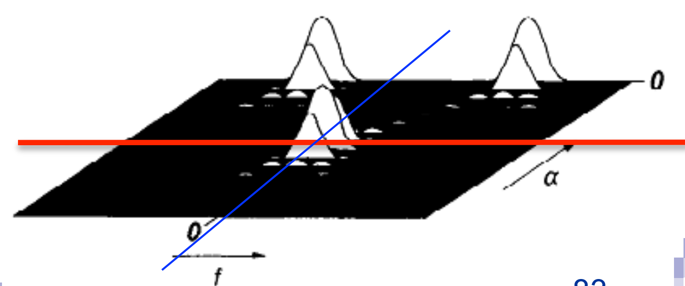
$$\begin{cases} H_1 : & \hat{\mathbf{c}}^{(n)} = \mathbf{c}^{(n)} + \epsilon \\ H_0 : & \hat{\mathbf{c}}^{(n)} = \epsilon \end{cases}$$

  - $\mathbf{c}^{(n)}$: $\left\{ s_x^{(c)}(a_i, b_i) \right\}_i$: unknown true SCD; multiple cyclic freq.
  - $\epsilon : \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_\epsilon)$: noise statistics determined mainly by finite-sample effects, not ambient noise

- GLRT formulation
  - Test statistics: $\mathcal{T}^{(n)} = (\hat{\mathbf{c}}^{(n)})^H \mathbf{\Sigma}_\epsilon{}^{-1} \hat{\mathbf{c}}^{(n)}$
  - Binary decisions by thresholding

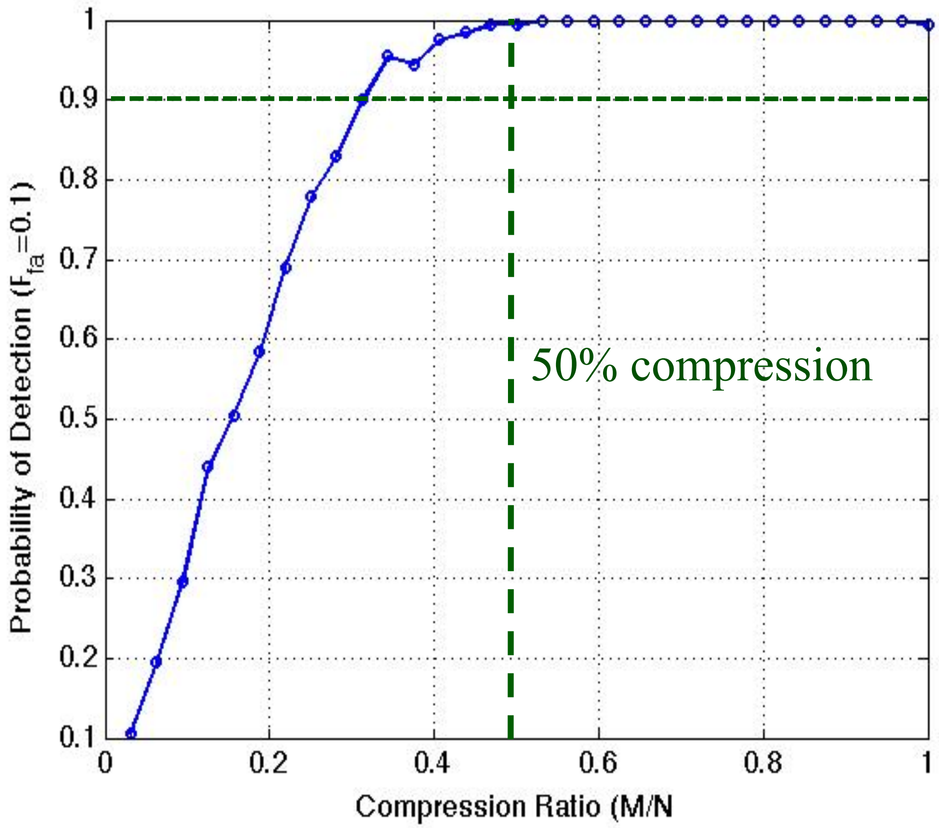
- A single wideband DSP, as opposed to multiple NB filters

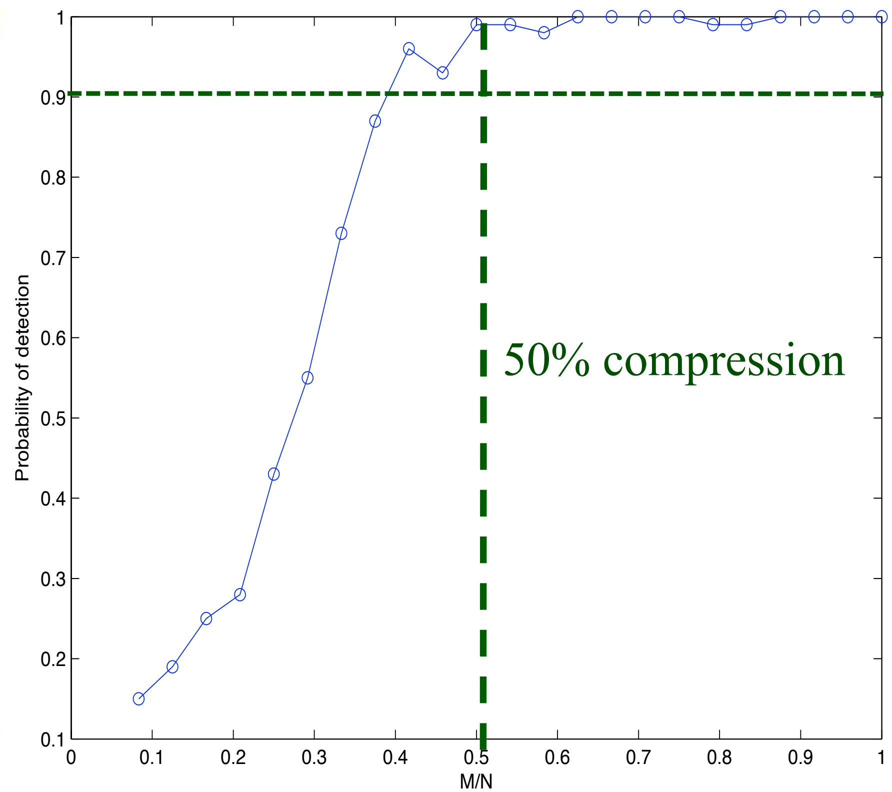
- Fast algorithms possible based on modulation type, say, for BPSK



Simulation: Robustness to Rate Reduction

Probability of Detection vs. Compression Ratio (P_{FA}= 0.1, N=32, L=200 blocks)

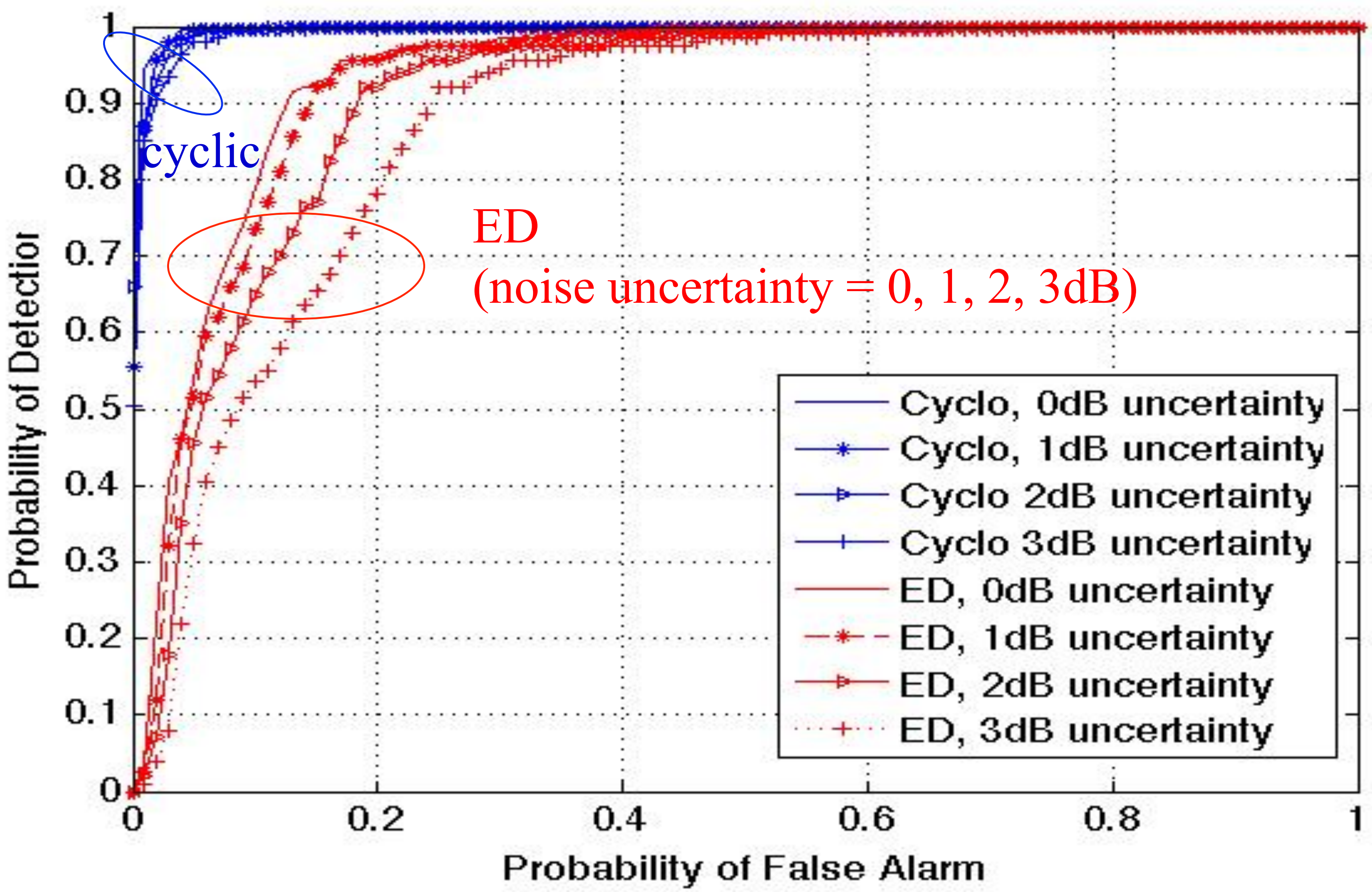




- Monitored band $|f_{max}|$ < 300 MHz
- 2 sources (noise-free): PU_1 - BPSK at 150MHz; PU_2 - QPSK at 225MHz; Ts=0.02667μs
- Cisco 802.11 DSSS Spread spectrum

Simulation: Robustness to Noise Uncertainty

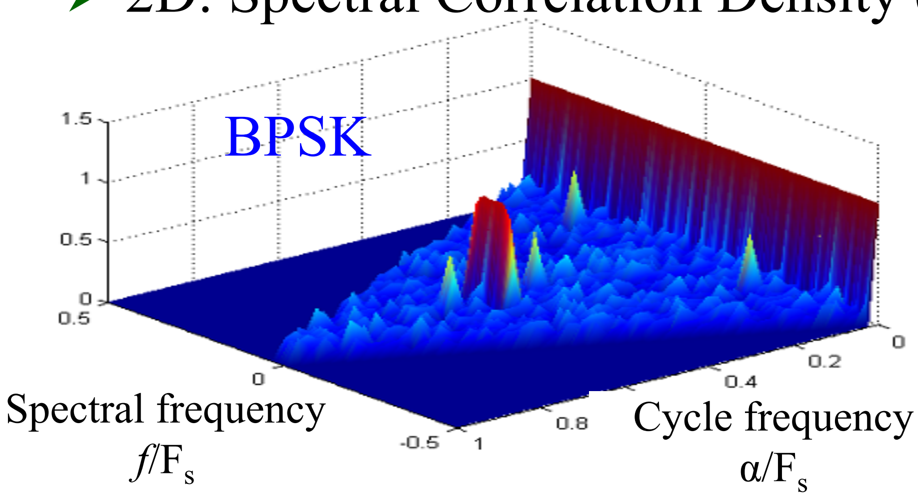
Receiver Operating Characteristic (ROC): P_D vs P_{FA} (SNR=5dB, 50% compression)

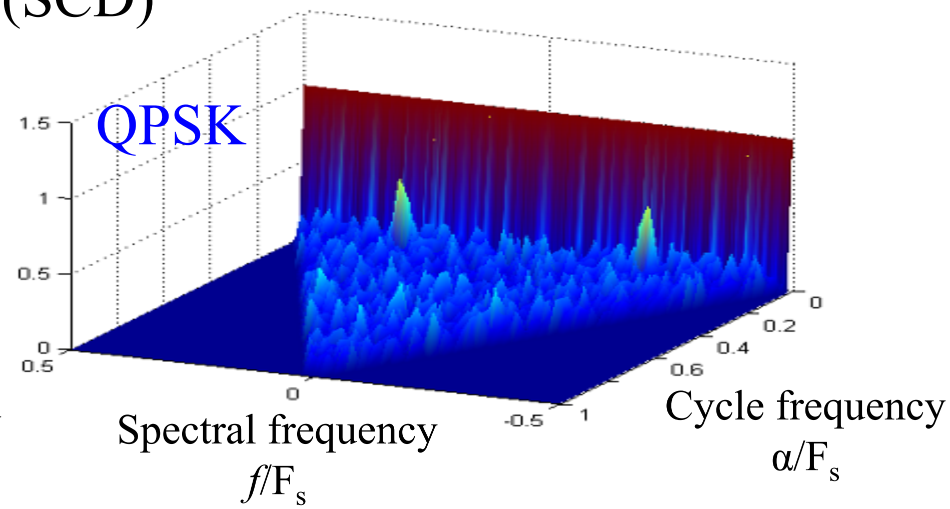


- outperforms energy detection (ED)
- insensitive to noise uncertainty

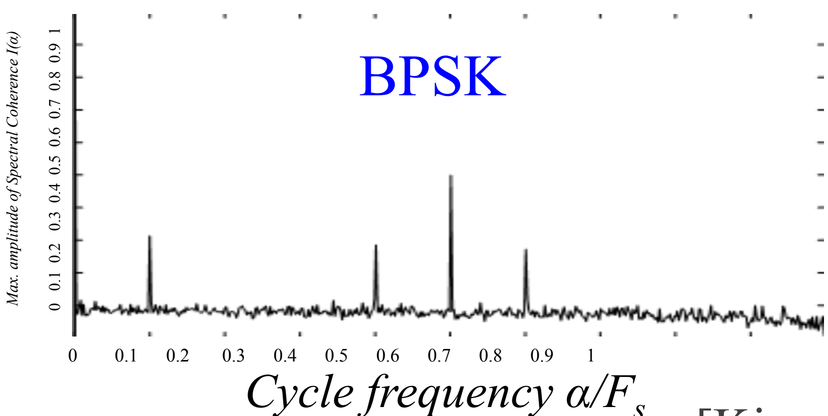
Classification using Cyclic Statistics

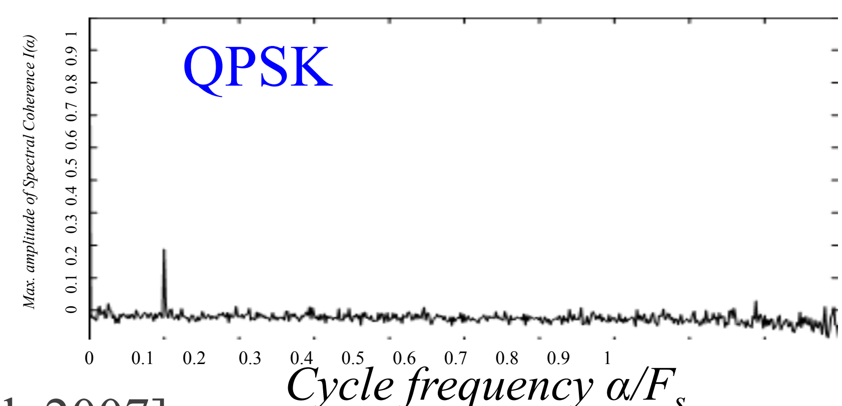
- 2D: Spectral Correlation Density (SCD)





- 1D: cyclic-frequency domain profile (CDP) $I(\alpha) \triangleq \max_{f} \left| c_x^{\alpha}(f) \right|$





[Kim etal. 2007]

Simulations: Classification

Confusion Matrix (SVM Classifier)

	BPSK	QPSK	DS-BPSK	DS-QPSK
BPSK	95.45%	0%	4.55%	0%
QPSK	0%	90.9%	9.09%	0%
DS-BPSK	9.09%	0%	59.09%	31.82%
DS-QPSK	4.5%	4.5%	36.46%	54.54%

- When compression ratio is adequate for detection, classification accuracy is comparable to non-compression
- Good separation of narrowband from spread spectrum
- Considerable confusion among spread spectrum signals

8 Power Spectrum Recovery

- Stationary processes as a special case of cyclostationary ones

  [Tian etal. (JSTSP'2012); Leus etal. (SPL'2011)]

  - 2D cyclic spectrum reduces to 1D power spectrum

$$r_x(n,\nu) = \bar{r}_x(\nu), \quad \forall n. \qquad \bar{\mathbf{r}}_x \overset{\mathcal{F}}{\longleftrightarrow} \bar{\mathbf{s}}_f \quad \text{(PSD)}$$

$$\mathbf{R}_x = \begin{bmatrix} \bar{r}_x(0) & \bar{r}_x(1) & \bar{r}_x(2) & \cdots & \bar{r}_x(N-1) \\ \bar{r}_x(1) & \bar{r}_x(0) & \bar{r}_x(1) & \cdots & \bar{r}_x(N-2) \\ \bar{r}_x(2) & \bar{r}_x(1) & \bar{r}_x(0) & \cdots & \bar{r}_x(N-3) \\ \vdots & & & \ddots & \vdots \\ \bar{r}_x(N-1) & \cdots & \cdots & \cdots & \bar{r}_x(0) \end{bmatrix}$$

  - # measurements $\mathbf{r}_z$ generated by cross-correlations: $M(M+1)/2$
  - #unknowns $\{r_x(v)\}$ in power spectrum $\mathbf{R}_x$: N

Power Spectrum Blind Sampling

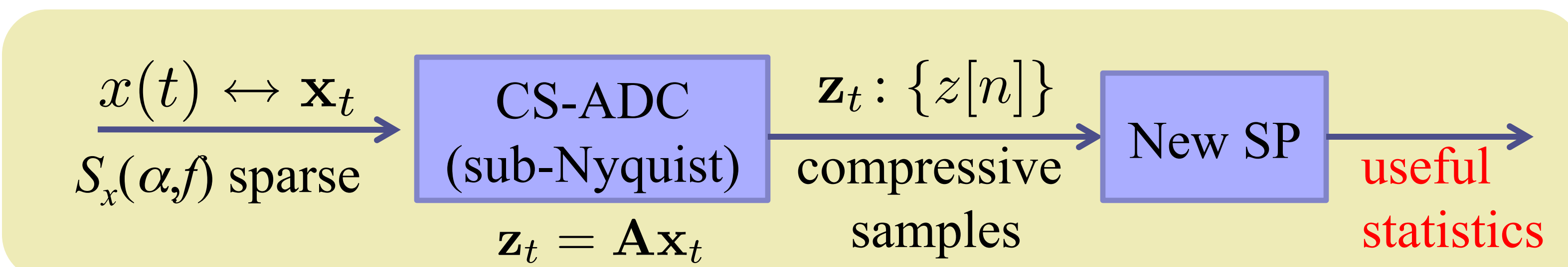
$$\underbrace{\mathbf{r}_z}_{\frac{M(M+1)}{2} \times 1} = \bar{\mathbf{\Phi}}\bar{\mathbf{r}}_x = \bar{\mathbf{\Phi}}\mathbf{F}^{-1}\underbrace{\bar{\mathbf{s}}_f}_{N}$$

- Minima sampling rates for non-sparse signals
  - Lossless recovery of power spectrum as long as $M(M+1)/2 \geq N$
  - Asymptotic compression ratio $\left(\frac{K}{N}\right)_{\min} \xrightarrow{N\to\infty} \sqrt{\frac{1}{N}} \ll 1$
- Sampler design [DSP'2011]
  - minimal sparse rulers [Leech'1956]
- Stronger compression allowed for sparse signals

N-1	M	M/N
1	2	1
5	4	0.667
9	5	0.5
49	12	0.24
128	20	0.156

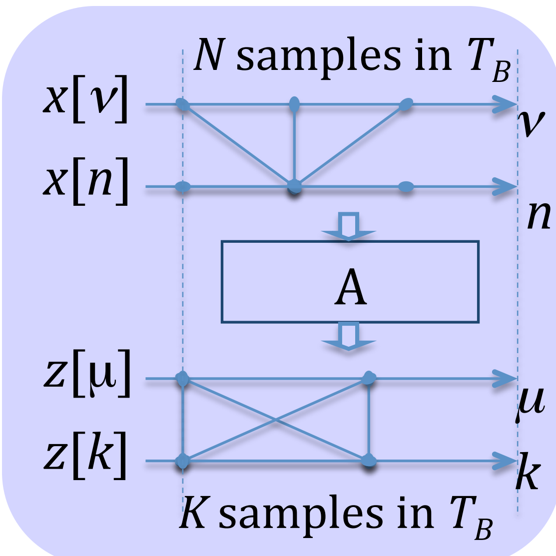
9 CS Framework for Random Processes

- CS for linear deterministic systems
  - Goal is perfect signal reconstruction [Venkataramani-Bresler'2001; Donoho etal.; Candes etal.; Mishali-Eldar'2010]
- CS for random processes
  - Perfect recovery of original signals using existing CS is over-kill
    - ✗ high computation costs ✗ wasteful of sampling resources
  - Goal: direct extraction of useful (2nd-order) statistics, which has less degrees of freedom than the random signal itself
    - ✔ stronger compression allowed for sparse signals
    - ✔ enables compression for non-sparse signals
    - ✔ reduced computational load, bypassing signal recovery



Intuition for Stationary Signals

- Linear measurement systems
  - Measurements: cross-correlation of compressive samples $z[k]$
  - Unknowns: cross-correlation or cyclic statistics of input $x[n]$



Input signal: stationary
- #cross-correlation = N

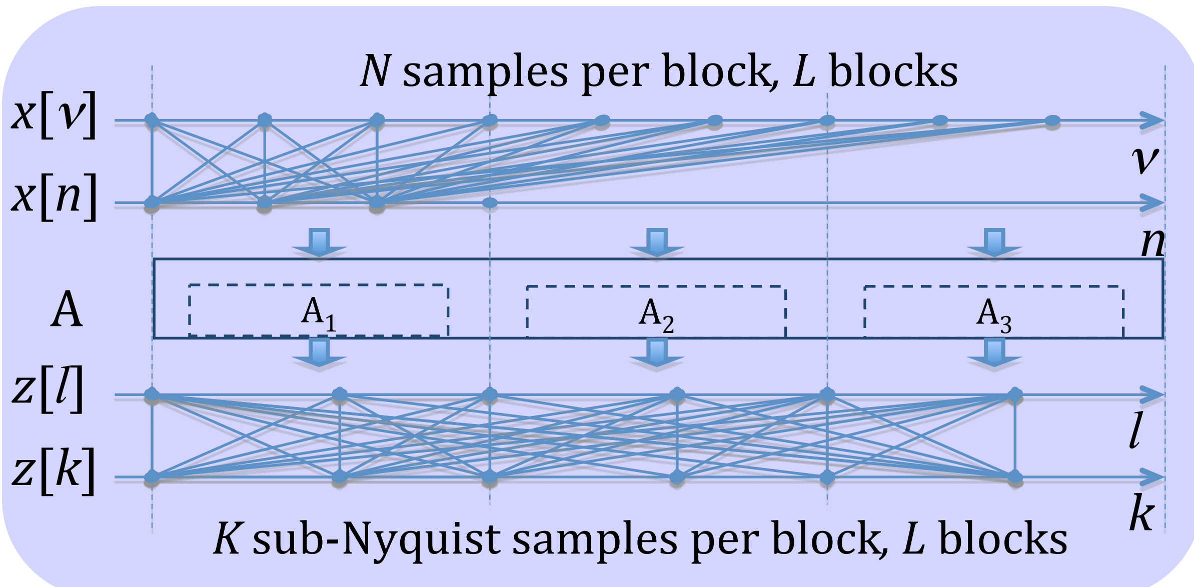
CS-ADC output: cyclostationary
- Compression ratio: K/N
- #cross-correlation = K^2

Over-determined when $N \leq K^2$ even when $x(t)$ is non-sparse

CS for random processes: extract 2nd-order statistics directly!
→ sub-Nyquist-rate sampling is feasible for non-sparse signals

Intuition for Cyclostationary Signals

- Compressed cyclic feature based wideband sensing [ICC'2011, JSTSP'2012]
  - CS with time span over multiple cyclic periods [CAMSAP' 11]
  - Simple reconstruction of cyclic power spectrum



Cyclostationary, with period N

- $N(NL)$

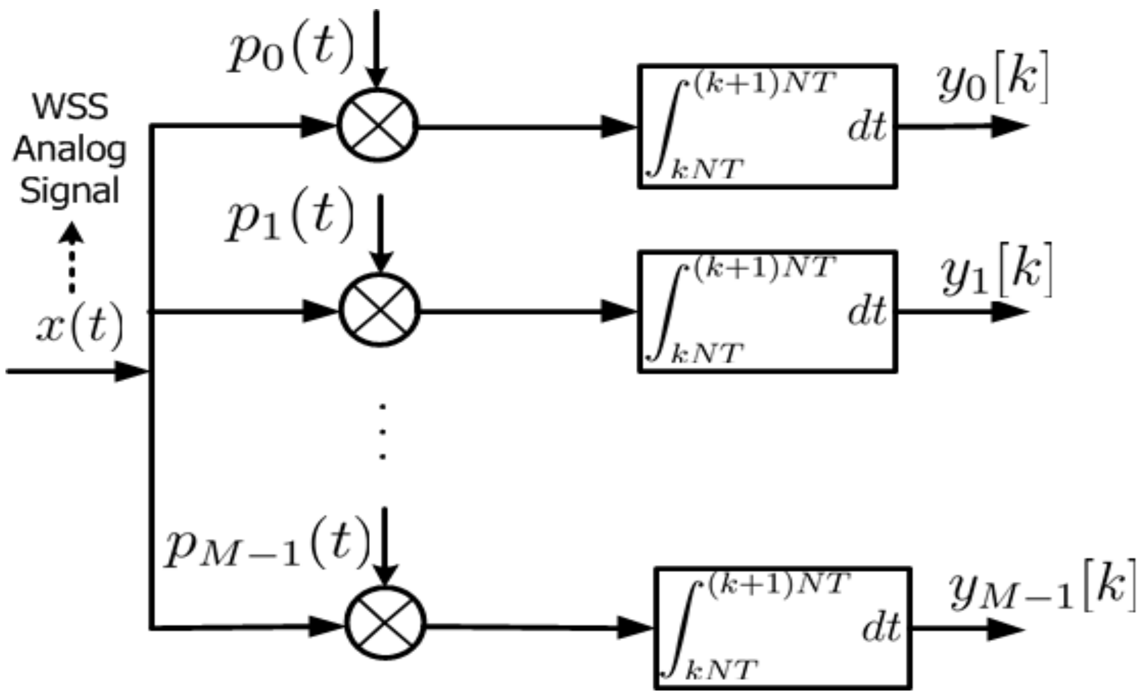
Cyclostationary, with period KL

- $(KL)^2$

Overdetermined when $N^2 \leq K^2L$, even when cyclic spectrum is non-sparse

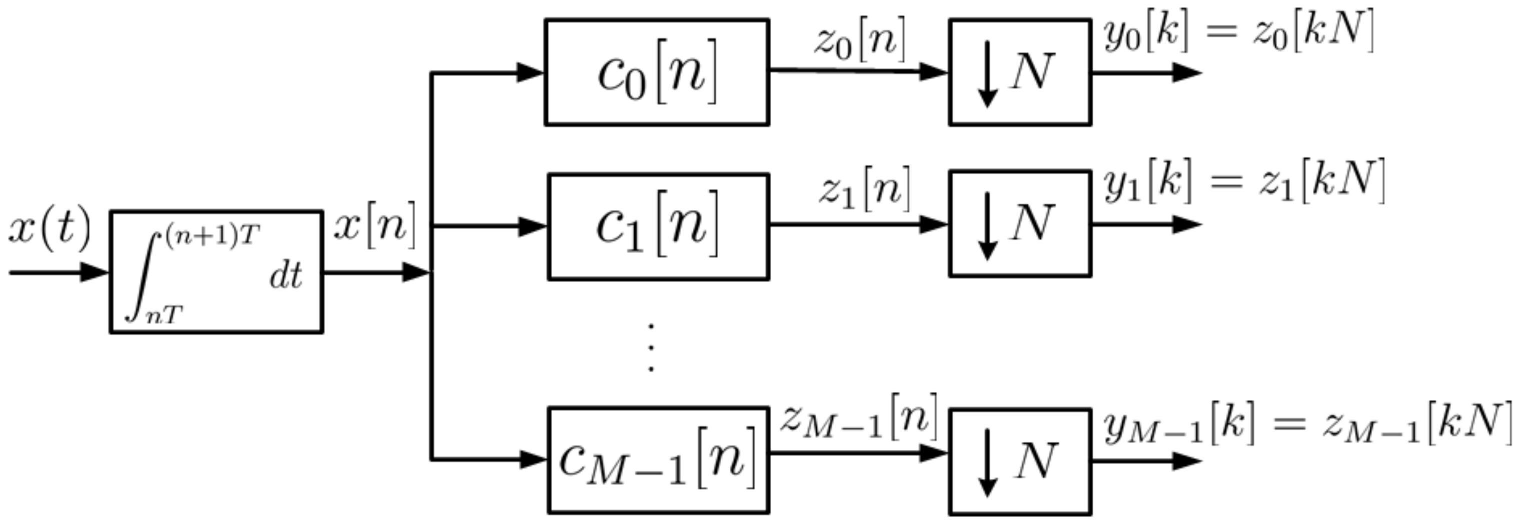
10 Sampler Design

- Sampler structure for periodic sampling
  - Sampling devices with *M* branches [Mishali etal.; Hoyos etal.]



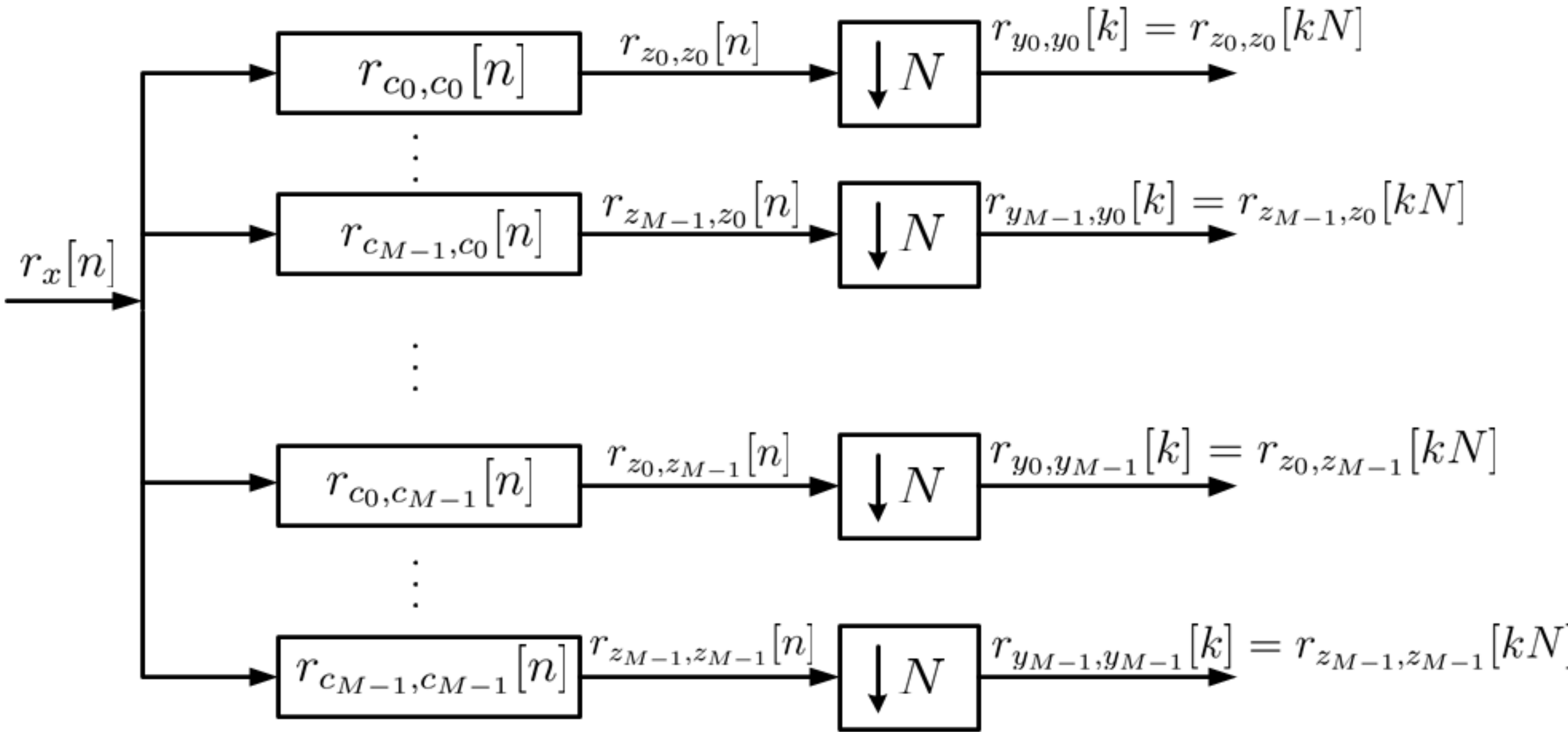
Alternative View of Periodic Sampling

- The sampling device can also be viewed as [Leus etal.'2011]



2^{nd}-order Statistics in Periodic Sampling

- The relationship between cross-correlations $r_{y_i,y_j}[k]$ and auto-correlation $r_x[n]$ can be perceived as:



- Every block of $N r_x[n]$ values leads to $M^2 r_{y_i,y_j}[k]$ values!

Reconstruction

- Collecting all the correlation values $r_{y_i y_j}[k]$ and $r_x[n]$ into vectors $\mathbf{r}_y$ and $\mathbf{r}_x$, it holds that [Leus etal.'2011]

$$\boxed{\mathbf{r}_y = \mathbf{R}_c \mathbf{r}_x}$$

  - $\mathbf{R}_c$: a collection of the deterministic correlation values $r_{c_i c_j}[n]$
- Allow rate compression without sparsity constraint on $x(t)$
  - If $\boldsymbol{R}_c$ has full column rank → solvable using least squares (LS)
    - Necessary condition: $M^2 \geq N$
- Sparse power spectrum
  - exploiting the sparsity will lead to even further compression

Multi-Coset Sampling Problem

- Goal: select M rows of identity matrix $\mathbf{I}_N$ to form the sampler coefficients $c_i[n]$ that guarantee the full column rank property of $\boldsymbol{R}_c$

$$c_i[n] = \delta[-n - n_i] \quad \Longrightarrow \quad r_{c_i,c_j}[n] = \delta[n - n_i + n_j]$$

  - Every row of $\boldsymbol{R}_c$ will have only a single one.

- To achieve full column rank $\boldsymbol{R}_c$ → select proper combination of rows of $\mathbf{I}_N$, such that every column of $\boldsymbol{R}_c$ has at least a single one.

Multi-Coset Sampling - Example

$$M = 3, N = 6, L = 1$$

$$\mathbf{c}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\mathbf{c}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\mathbf{c}_2 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{R}_c[0] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

First 4 columns have 1's!

Single one at each row, all columns have one(s)

Automatically, last 3 columns have 1's!

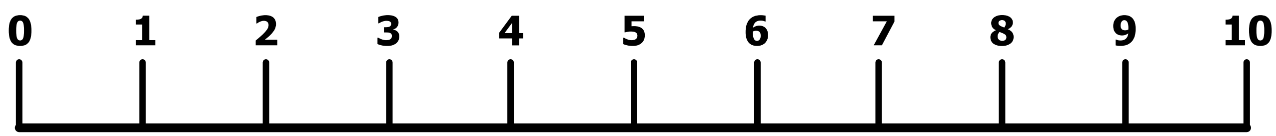
$$\mathbf{R}_c[1] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{R}_c = \begin{bmatrix} \mathbf{R}_c[0] & & \mathbf{R}_c[1] \\ \mathbf{R}_c[1] & \mathbf{R}_c[0] & \\ & \mathbf{R}_c[1] & \mathbf{R}_c[0] \end{bmatrix}$$

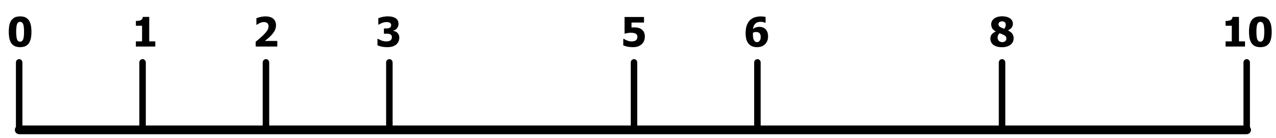
R_c is full rank!

Minimal solution can be approached by minimal length-$\lfloor N/2 \rfloor$ sparse rule problem!

Multi-Coset Sampling using Sparse Ruler



Length-10 sparse ruler



Minimal length-10 sparse ruler

0 1 2 3 6 10

- Connection between multi-coset design and sparse ruler problem guarantees the full rank property of $\boldsymbol{R}_c$

  → uniqueness of the estimates as solutions to simple LS problems

- Adopting minimal length-$\lfloor N/2 \rfloor$ sparse rules

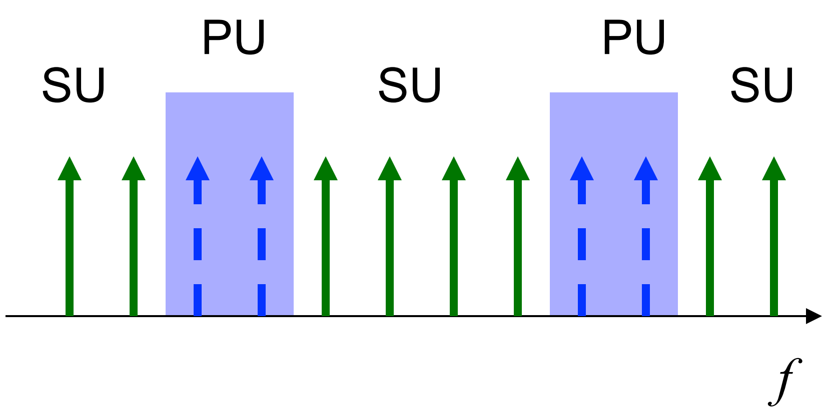
  → reaching the possible minimum compression rate

Outline

- Basis of Compressive Sensing (CS)
- Motivation of CS for Cognitive Radio (CR)
- Compressive Spectrum Sensing for CR
- Sparsity-constrained dynamic resource allocation and waveform design
  - Transceiver structure for joint DRA and waveform adaptation
  - Multi-user DRA game formation
  - Sparse channel estimation and interference sensing
  - Sparsity-constrained DRA games
- References

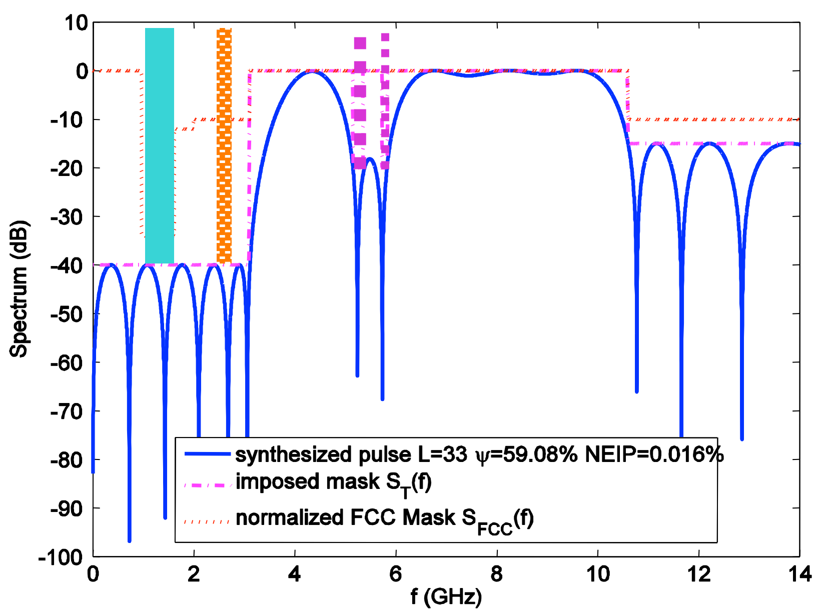
Options for Waveform Adaptation

- Multi-Carrier Methods
  - dynamic subcarrier selection
  - adaptive power loading



- Waveform library

- Digital filterbank pulse shaping for serial transmissions
  - dynamic spectral mask
  - adjustable filter weights

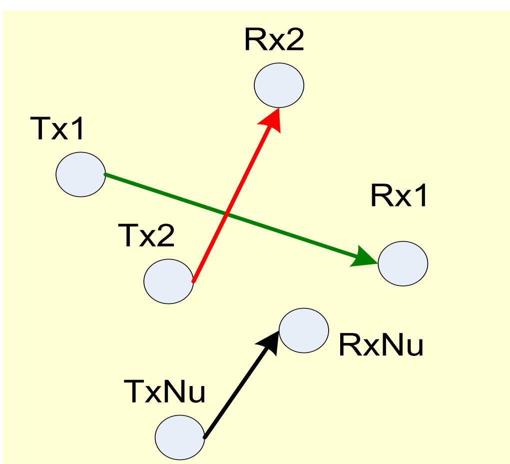


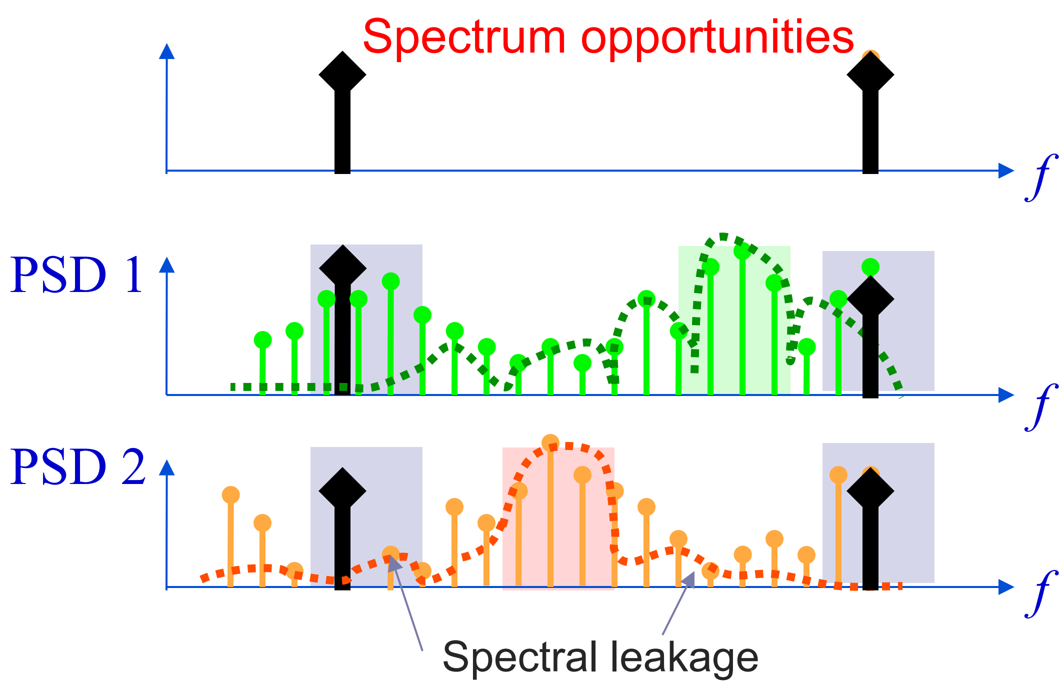
[T-SP' 06]

Setup for Dynamic Resource Management

- A unified treatment of sensing, adaptation and decision at PHY

  crucial for practical QoS guarantees
  - DRA: Dynamic Resource Allocation among multiple CR users
  - FAWA: Frequency Agile Waveform Adaptation in dynamic channels
  - Cognition: identification of RF resources in various domains



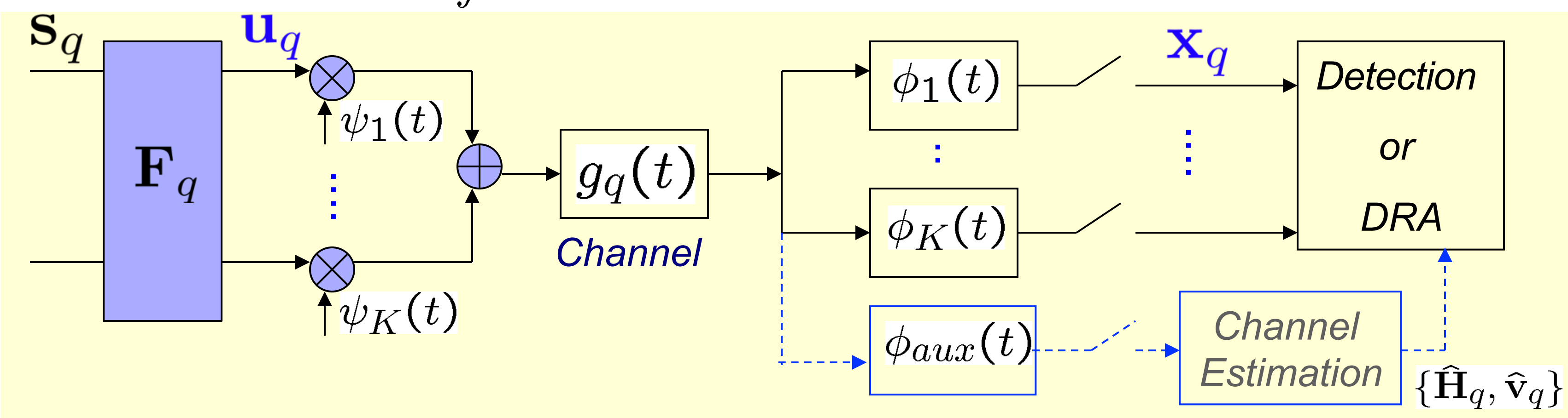


Basic CR Transceiver Model

- Generalized Signal Expansion Framework [JSTSP'2011]
  - OFDM-like: digital subcarriers replaced by expansion functions
    - representation and utilization of radio resources
  - enable diverse radio platforms and combinations
    - FDM (OFDM), TDM (SCCP), CDM (DS-CDMA)

$$\boxed{\mathbf{x}_q = \mathbf{H}_q \mathbf{u}_q + \mathbf{v}_q} \qquad x_{q,l} = \textstyle\sum_{k=1}^{K} u_{q,k} h_{q,k,l} + v_{q,l}$$

$$h_{q,k,l} = \int g_q(t) \star \psi_k(t) \star \phi_l^*(-t) dt, \quad \forall k, l = 1, \ldots, K$$



Goal and Design Parameters per CR

- Design parameters
  - Tx: linear precoding $\mathbf{F}_q : \ \mathbf{u}_q = \mathbf{F}_q \mathbf{s}_q$
  - Tx: power loading $\mathbf{a}_q : \ a_{q,k} = \sqrt{E(|s_{q,k}|^2)}, \quad k = 1, \ldots, K$
  - Rx: linear MMSE, capacity-preserving
- DRA Objective: spectral efficiency

$$\boxed{C(\mathbf{a}_q, \mathbf{F}_q) = \frac{1}{K} \log_2 \left| \mathbf{I}_K + \mathrm{diag}(\mathbf{a}_q) \mathbf{F}_q^H \mathbf{B}_q \mathbf{F}_q \mathrm{diag}(\mathbf{a}_q) \right|}$$

SINR-related: $\mathbf{B}_q = \mathbf{H}_q^H \mathbf{R}_q^{-1} \mathbf{H}_q, \ \mathbf{R}_q = E(\mathbf{v}_q \mathbf{v}_q^H)$

- Transmission Constraints
  - Transmitted PSD: $S_q(f; \mathbf{a}_q, \mathbf{F}_q)$
  - Average power constraint (PC_q): $\int S_q(f; \mathbf{a}_q, \mathbf{F}_q) df \leq P_{q,\max}$
  - Spectral mask constraint (MC_q): $S_q(f; \mathbf{a}_q, \mathbf{F}_q) \leq S_c(f), \quad \forall f$
- Sensing requirements: channel $\mathbf{H}_q$, interference covariance $\mathbf{R}_q$

Multi-User DRA Game

- Centralized global optimization

$$\{(\mathbf{a}_q, \mathbf{F}_q)\}_{q=1}^{Q} : \max_{\{\mathbf{a}_q \succeq 0\}_q, \{\mathbf{F}_q\}_q} \sum_{q=1}^{Q} C(\mathbf{a}_q, \mathbf{F}_q) \quad s.t.\ \{\mathsf{PC}_q, \mathsf{MC}_q\},\ \forall q$$

- Distributed game formulation

$$\boxed{\begin{aligned}(\mathbf{a}_q, \mathbf{F}_q) : \max_{\mathbf{a}_q \succeq 0, \mathbf{F}_q} \quad & C(\mathbf{a}_q, \mathbf{F}_q) \\ s.t. \quad & \mathsf{PC}_q, \mathsf{MC}_q\end{aligned}}$$

  - Per-user basis
    - Self-interested local optimization
    - No knowledge of other CRs' actions $\{(\mathbf{a}_r, \mathbf{F}_r)\}_{r \neq q}$
  - Iterative implementation

    for each CR in its turn to take action:
    - Dynamic sensing: estimate channel & sum interference $\mathbf{H}_q, \mathbf{R}_q$
    - DRA optimization: find best response strategy $(\mathbf{a}_q^\star, \mathbf{F}_q^\star)$
      - Best response via diagonalization and power water-filling
    - Waveform adaptation: update PSD at TX $S_q(f; \mathbf{a}_q^\star, \mathbf{F}_q^\star)$

Sparse Channel Estimation

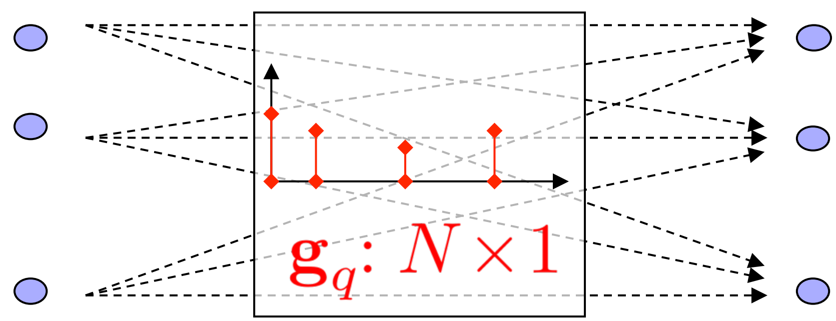
- **Channel Parameters**

$$\mathbf{x}_q = \mathbf{H}_q \mathbf{u}_q + \mathbf{v}_q$$

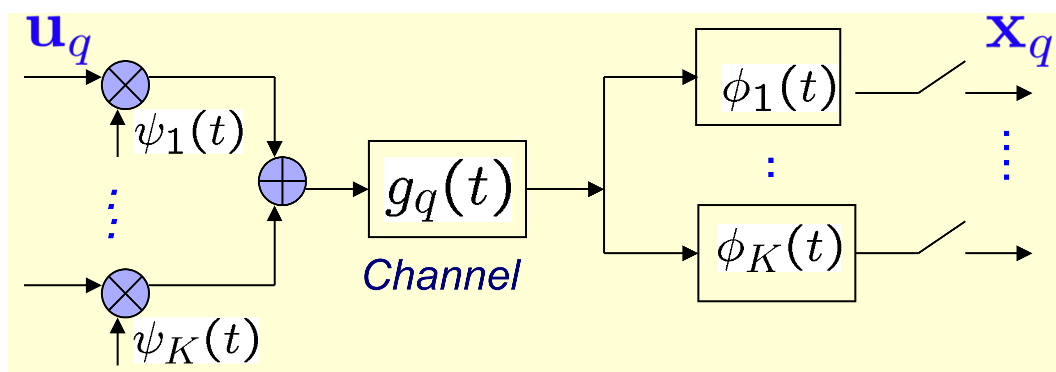
$$\mathbf{H}_q: K \times K \quad h_{q,k,l} = \int g_q(t) \star \psi_k(t) \star \phi_l^*(-t) dt, \quad \forall k, l = 1, \ldots, K$$

$$\boxed{\bar{\mathbf{x}}_q = \underbrace{\left(\mathbf{I}_L \otimes \bar{\mathbf{u}}_q^T\right) \boldsymbol{\Theta}}_{} \mathbf{g}_q + \bar{\mathbf{v}}_q}$$

- **Sparsity**
  - Sparse multipath $\mathbf{g}_q$
  - Oversampling by multiple Rx bases



- **Ideas:**
  - Estimate $\mathbf{g}_q$, then $\mathbf{H}_q$
  - Compress by turning off some basis filters [SPAWC' 10]



Sparse Interference Sensing

- Interference sensing task $\mathbf{R}_v = E\{\mathbf{v}_q \mathbf{v}_q^T\}$ $\hat{\mathbf{v}}_q = \mathbf{x}_q - \hat{\mathbf{H}}_q \mathbf{u}_q$
- Sparsity: representation of the composite interference on sparsifying basis

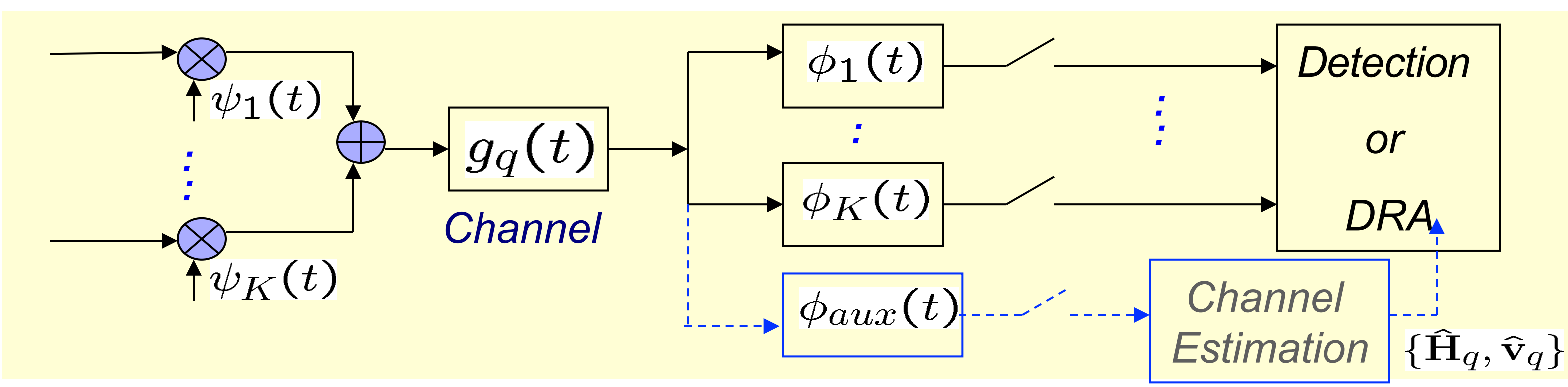
$$\nu_q(t) = \sum_{k=0}^{K-1} \nu_{q,k} \varphi_k(t) \qquad \mathbf{v}_q = \boldsymbol{\Xi} \nu_q$$

- Idea: use of auxiliary filter for compressive sampling

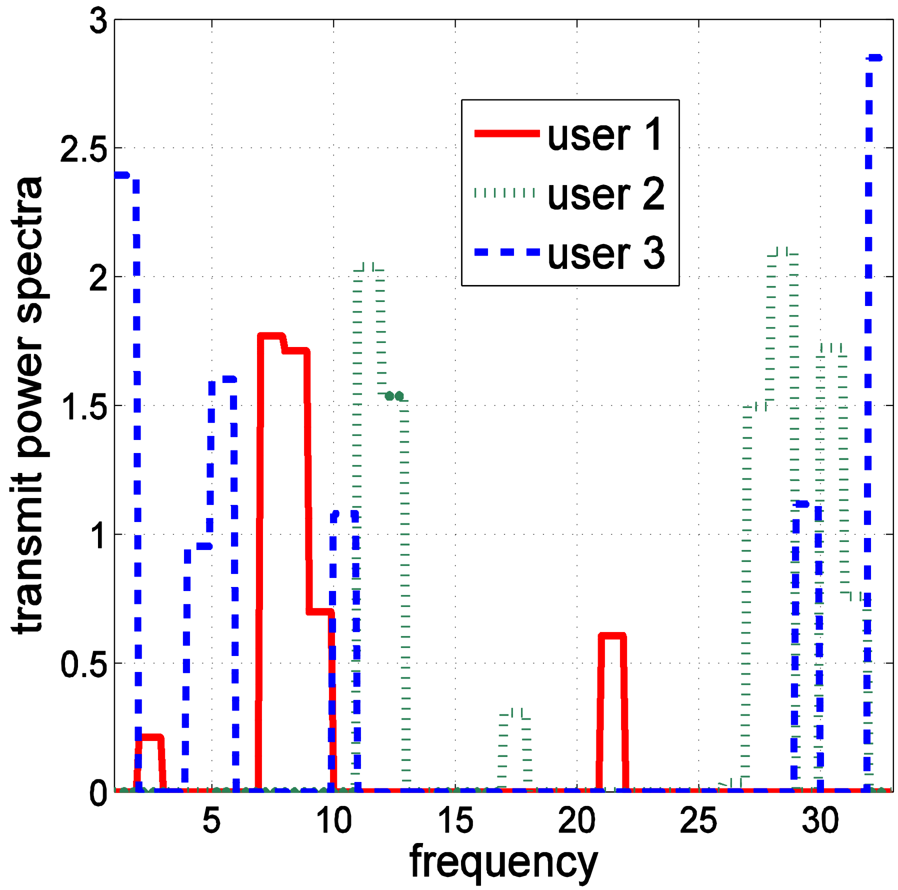
$$\zeta_{q,c} = \zeta_q(t_c) = \sum_{k=0}^{K-1} \nu_{q,k} \left[\varphi_k(t) \star s(t)\right]\Big|_{t=t_c} \qquad \boldsymbol{\zeta}_q = \boldsymbol{\Lambda} \boldsymbol{\nu}_q$$

  - Sparse recovery

$$\boxed{\hat{\boldsymbol{\nu}}_q = \arg\min_{\boldsymbol{\nu}_q} \left\{ \|\boldsymbol{\nu}_q\|_1 + \rho \|\boldsymbol{\zeta}_q - \boldsymbol{\Lambda} \boldsymbol{\nu}_q\|_2^2 \right\}}$$

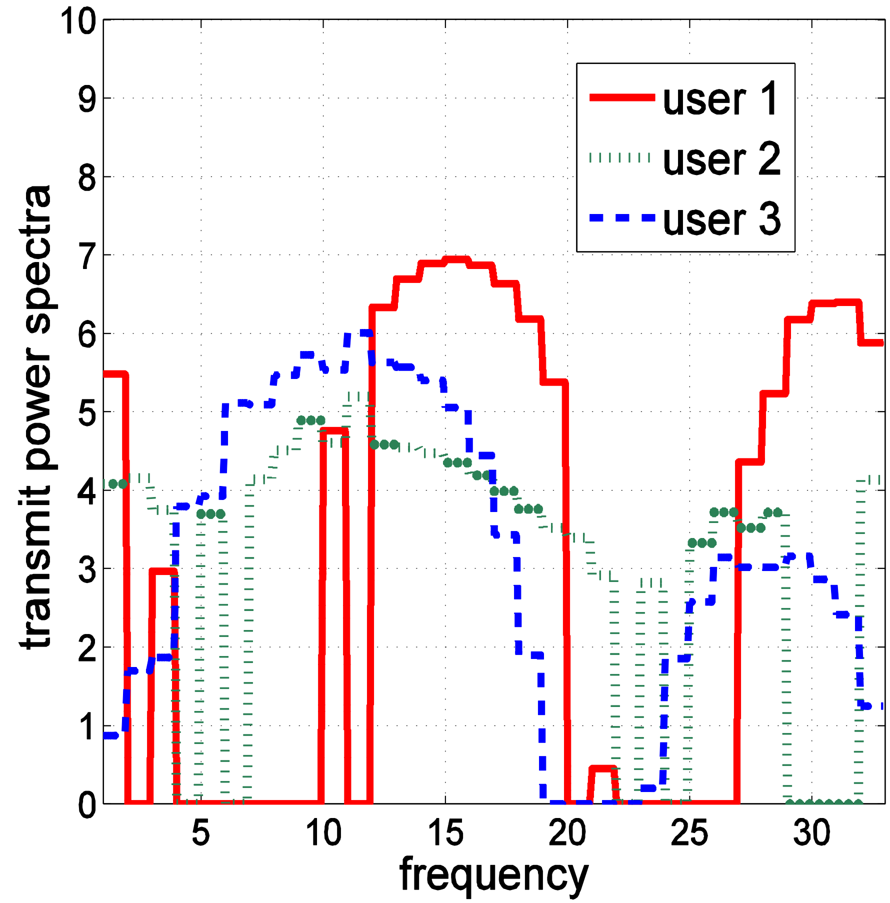


Flexible Waveform Adaptation and DSA



Strong interference case

→ (O)FDMA type; overlay



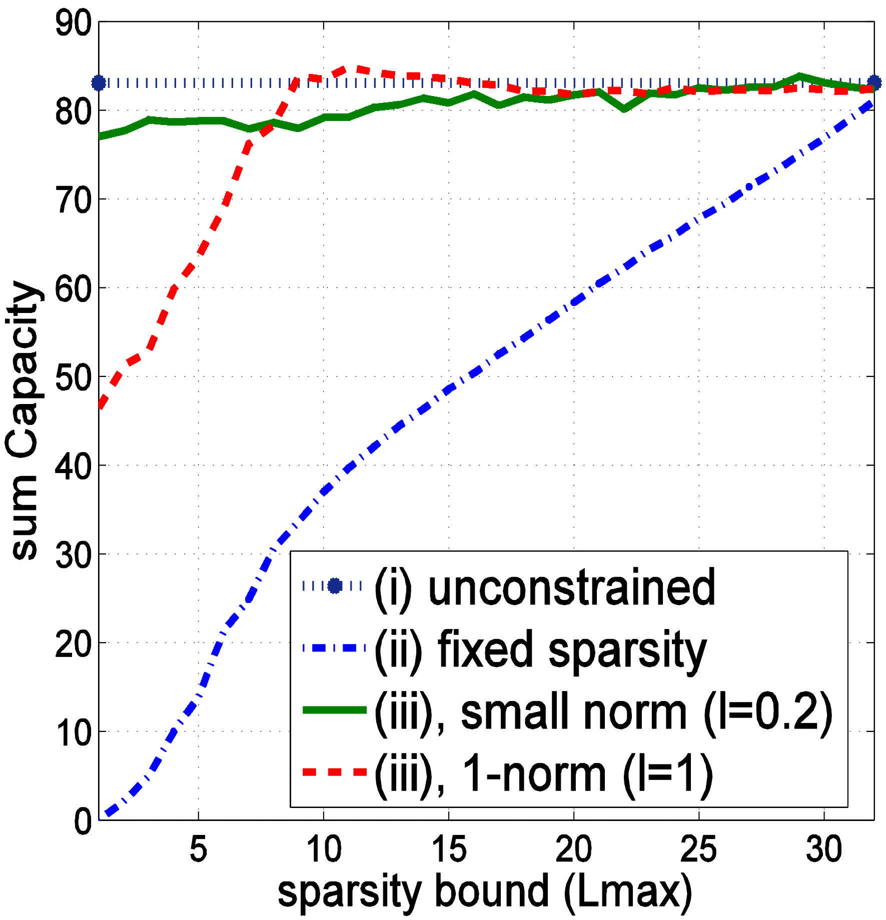
Weak interference case

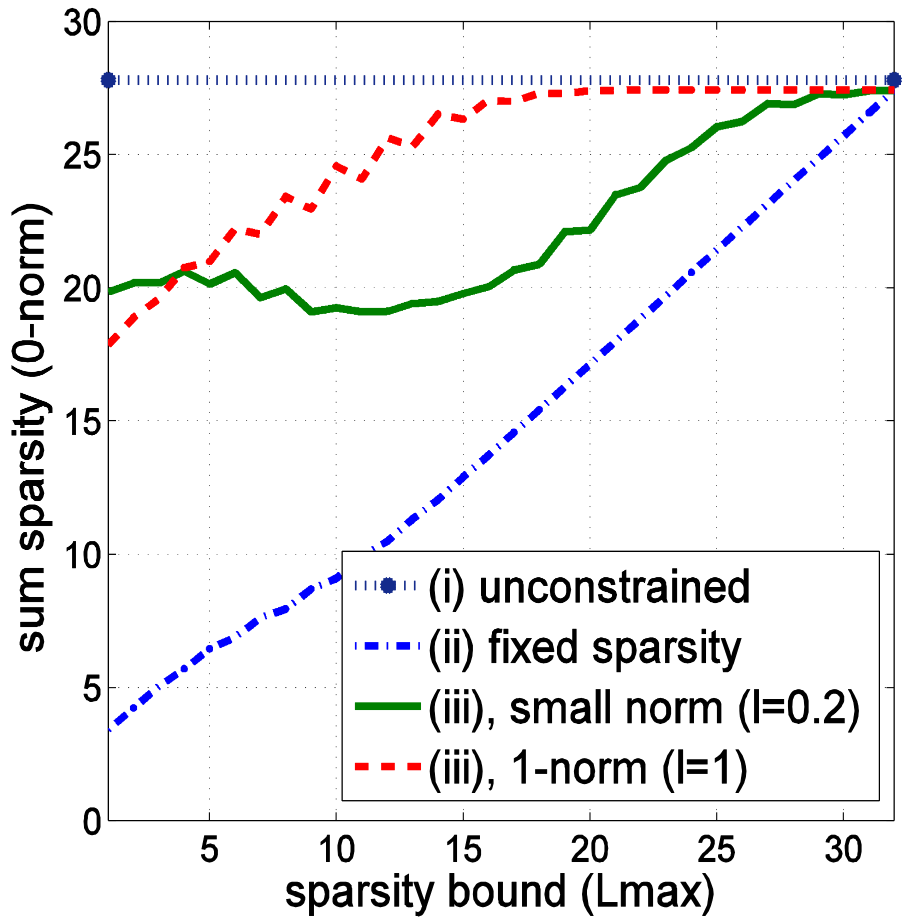
→ SS type; underlay

Sparsity-constrained Waveform Adaptation

- Sparsity in the signal expansion model
  - Available resources are large in a wideband network → large K
  - Effective resources needed per CR are small
    - expansion functions may be redundant to induce flexibility
- Sparsity-constrained optimization
  - Approach: limit the number of active expansion functions
  - Benefits
    - little or no performance loss
    - reduced computation and implementation costs
    - fast convergence of iterative DRA games
    - facilitates limited-rate feedback

Capacity under Sparsity Constraints





lower sparsity → less hardware costs, faster convergence, less feedback

Outline

- Basis of Compressive Sensing (CS)
- Motivation of CS for Cognitive Radio (CR)
- Compressive Spectrum Sensing for CR
- Sparsity-constrained Dynamic Resource Allocation and Waveform Design
- References (Tian etal., Giannakis etal., Leus etal.)

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Multi-CR Cooperative Spectrum Sensing

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