INFONET, GIST
Journal Club (2013. 03. 21)

Scaling Up MIMO: Opportunities and challenges with very large arrays

Authors: Fredrik Rusek, Thomas L. Marzetta, et al.
Publication: IEEE Signal Processing Magazine
Speaker: Woongbi Lee

Short summary: Very large MIMO systems is an emerging research area in antenna systems, electronics, and wireless communication systems. A base station with an antenna array serves a multiplicity of single-antenna terminals. In this presentation, the fundamental principle of massive MIMO technology and several issues are introduced.

I. Introduction

Multiple-Input, Multiple-Output (MIMO) technology is becoming mature, and incorporated into emerging wireless broadband standard like LTE. Basically, the more antennas the transmitter/receiver is equipped with, and the more degrees of freedom that the propagation channel can provide, the better performance in terms of data rate or link reliability. However, MIMO technology requires increased complexity of the hardware, the complexity and energy consumption of the signal processing, and the physical space for accommodating antennas including rents of real estate.

Today, as mobile data traffic exponentially increases, further capacity enhancement is needed. As a solution for the high capacity demand, Massive MIMO (very large MIMO, Large-Scale Antenna System, Full Dimension MIMO) technology has been widely studied for last few years. Massive MIMO adopts hundreds of antennas at base station (BS) serving a much smaller number of terminals. The number of terminals that can be simultaneously served is limited, not by the number of antennas, but rather by inability to acquire channel-state information for an unlimited number of terminals. With an unlimited number of antennas, the transmit power can be made arbitrarily small and the uncorrelated interference and noise can be vanished. But, the performance is limited by pilot contamination.

This paper approaches to Massive MIMO according to three directions: Information-theoretic performance limit, and antennas and propagation aspects of large MIMO, and transmit and receive schemes.

II. Information Theory for Very Large MIMO Arrays

According to the noisy-channel coding theorem in information theory, for any communication link, there is a capacity or achievable rate, such that for any transmission rate less than the capacity, there exists a coding scheme that makes the error-rate arbitrarily small.

A. Point-to-point MIMO

1) Channel model

Transmitter has an array of n_t antennas and a receiver has an array of n_r antennas. The simplest narrowband memoryless channel has the following mathematical description,

$$\mathbf{x} = \sqrt{\rho}\mathbf{G}\mathbf{s} + \mathbf{w}$$

where $\boldsymbol{s}$ is the n_t component vector of transmitted signals, $\boldsymbol{x}$ is the n_r component vector of received signals, $\boldsymbol{G}$ is the $n_r \times n_t$ propagation matrix of complex-valued channel coefficients, and $\boldsymbol{w}$ is the n_r component vector of receiver noise. The components of the additive noise vector are i.i.d. zero mean and unit-variance circular-symmetric complex-Gaussian random variables ($CN(0,1)$). The scalar ρ is a measure of the Signal-to-Noise Ratio (SNR) of the link.

2) Achievable rate

With the assumption that the receiver has perfect knowledge of the channel matrix, $\boldsymbol{G}$, the mutual information between the input and the output of the point-to-point MIMO channel is

$$C = I(x;s) = \log_2 \det\left(\mathbf{I}_{n_r} + \frac{\rho}{n_t}\mathbf{G}\mathbf{G}^H\right)$$

where $\mathbf{I}_{n_r}$ denotes the $n_r \times n_r$ identity matrix. The propagation matrix can be decomposed by

$$\mathbf{G} = \mathbf{\Phi}\mathbf{D}_\nu\mathbf{\Psi}^H,$$

where $\mathbf{\Phi}$ and $\mathbf{\Psi}$ are unitary matrices of dimension $n_r \times n_r$ and $n_t \times n_t$ respectively, and $\mathbf{D}_\nu$ is a $n_r \times n_t$ diagonal matrix whose diagonal elements are the singular values, $\left\{\nu_1, \nu_2, \cdots, \nu_{\min(n_t, n_r)}\right\}$. The achievable rate can be written as

$$C = \sum_{l=1}^{\min(n_t, n_r)} \log_2\left(1 + \frac{\rho \nu_l^2}{n_t}\right)$$

With the decomposed propagation matrix,

$$\sum_{l=1}^{\min(n_t, n_r)} \nu_l^2 = \mathrm{Tr}\left(\mathbf{G}\mathbf{G}^H\right)$$

where "Tr" denotes "trace". There can exist two extreme cases: the worst case when all except one of the singular values are equal to zero and the best case when all of the $\min(n_t, n_r)$ singular values are equal. The two cases bound the achievable rate as follows,

$$\log_2\left(1 + \frac{\rho \cdot \mathrm{Tr}\left(\mathbf{G}\mathbf{G}^H\right)}{n_t}\right) \le C \le \min(n_t, n_r) \cdot \log_2\left(1 + \frac{\rho \cdot \mathrm{Tr}\left(\mathbf{G}\mathbf{G}^H\right)}{n_t \min(n_t, n_r)}\right)$$

The rank-1 (worst) case occurs either for compact arrays under Line-of-Sight (LOS) propagation conditions such that the transmit array cannot resolve individual elements of the receive array and vice-versa, or under extreme keyhole propagation conditions. The equal singular value (best) case is approached when the entries of the propagation matrix are IID random variables. Under favorable propagation conditions and a high SNR, the achievable rate is proportional to the smaller of the number of transmit and receive antennas.

3) Limiting cases

a) Low SNRs

Low SNRs can be experience by terminals at the edge of a cell. For low SNRs, only beamforming gains are important and the achievable rate becomes

$$C_{\rho \to 0} \approx \frac{\rho \cdot \mathrm{Tr}\left(\mathbf{G}\mathbf{G}^H\right)}{n_t \ln 2}$$
$$\approx \frac{\rho n_r}{\ln 2}$$

which is independent of n_t, and thus, even under the most favorable propagation conditions the multiplexing gains are lost, and multiple transmit antennas are of no value under low SNRs.

b) Number of transmit antennas grow large

It is assumed that the row-vectors of the propagation matrix are asymptotically orthogonal. Then,

$$\left(\frac{\mathbf{G}\mathbf{G}^H}{n_t}\right)_{n_t >> n_r} \approx \mathbf{I}_{n_r}$$

and the achievable rate becomes

$$\begin{aligned} C_{n_t >> n_r} &\approx \log_2 \det\left(\mathbf{I}_{n_r} + \rho \cdot \mathbf{I}_{n_r}\right) \\ &= n_r \cdot \log_2 (1+\rho) \end{aligned}$$

c) Number of receive antennas grow large

It is also assumed that the column-vectors of the propagation matrix are asymptotically orthogonal,

$$\left(\frac{\mathbf{G}^H\mathbf{G}}{n_r}\right)_{n_r >> n_t} \approx \mathbf{I}_{n_t}$$

and the achievable rate becomes

$$\begin{aligned} C_{n_r >> n_t} &= \log_2 \det\left(\mathbf{I}_{n_t} + \frac{\rho}{n_t} \cdot \mathbf{G}^H\mathbf{G}\right) \\ &\approx n_t \cdot \log_2\left(1 + \frac{\rho n_r}{n_t}\right) \end{aligned}$$

Thus, a large number of transmit or receive antennas, combined with asymptotic orthogonality of the propagation vectors (i.i.d. complex Gaussian), can increase the achievable rate. Extra receive antennas can compensate for a low SNR and restore multiplexing gains.

B. Multi-user MIMO

Multi-user MIMO consists of an array of M antennas and K autonomous terminals. We assume that each terminal has only one antenna. Multi-user MIMO differs from point-to-point MIMO in two respects: first, the terminals are typically separated by many wavelengths, and second, the terminals cannot collaborate among themselves.

1) Propagation

We assume Time Division Duplex (TDD), so the reverse link propagation matrix is merely the transpose of the forward link propagation matrix. Assumption on the TDD comes from the need to acquire channel state-information between extreme numbers of service antennas and much smaller numbers of terminals. The propagation matrix, $\mathbf{G} \in \mathbb{R}^{M \times K}$, can be decomposed by

$$\mathbf{G} = \mathbf{H}\mathbf{D}_{\beta}^{1/2}$$

where $\mathbf{H} \in \mathbb{R}^{M \times K}$ represents small scale fading and $\mathbf{D}_{\beta}^{1/2} \in \mathbb{R}^{K \times K}$ whose diagonal elements constitute a $K \times 1$ vector, and β is large scale fading coefficients. By assumption, the antenna array is sufficiently compact that all of the propagation paths for a particular terminal are subject to the same large scale fading.

For multi-user MIMO with large arrays, the number of antennas greatly exceeds the number of terminals. Under the most favorable propagation conditions the column-vectors of the propagation matrix are asymptotically orthogonal,

$$\begin{aligned}\left(\frac{\mathbf{G}^{H}\mathbf{G}}{M}\right)_{M>>K} &= \mathbf{D}_{\beta}^{1/2}\left(\frac{\mathbf{H}^{H}\mathbf{H}}{M}\right)_{M>>K}\mathbf{D}_{\beta}^{1/2} \\ &\approx \mathbf{D}_{\beta}\end{aligned}$$

2) Reverse link

On the reverse link, for each channel use, the K terminals collectively transmit a $K \times 1$ vector of QAM symbols, $\mathbf{q}_r$, and the antenna array receives a $M \times 1$ vector, $\mathbf{x}_r$,

$$\mathbf{x}_r = \sqrt{\rho_r}\mathbf{G}\mathbf{q}_r + \mathbf{w}_r$$

Under the assumption that the columns of the propagation matrix are nearly orthogonal, i.e., $\mathbf{G}^{H}\mathbf{G} = M \cdot \mathbf{D}_{\beta}$, the base station could process its received signal by a matched-filter (MF),

$$\begin{aligned}\mathbf{G}^{H}\mathbf{x}_r &= \sqrt{\rho_r}\mathbf{G}^{H}\mathbf{G}\mathbf{q}_r + \mathbf{G}^{H}\mathbf{w}_r \\ &\approx M\sqrt{\rho_r}\mathbf{D}_{\beta}\mathbf{q}_r + \mathbf{G}^{H}\mathbf{w}_r\end{aligned}$$

3) Forward link

For each use of the channel the base station transmits a $M \times 1$ vector, $\mathbf{s}_f$, through its M antennas, and the K terminals collectively receive a $K \times 1$, $\mathbf{x}_f$,

$$\mathbf{x}_f = \sqrt{\rho_f}\mathbf{G}^{T}\mathbf{s}_f + \mathbf{w}_f$$

III. ANTENNA AND PROPAGATION ASPECTS OF VERY LARGE MIMO

The performance of all types of MIMO systems strongly depends on properties of the antenna arrays and the propagation environment in which the system is operating. With well separated ideal antenna elements, in a sufficiently complex propagation environment and without directivity and mutual coupling, each additional antenna element in the array adds another degree of freedom that can be used by the system. But, in reality, the antenna elements are not ideal, they are not always well separated, and the propagation environment may not be complex enough to offer the large number of degrees of freedom that a large antenna array could exploit. These practical issues are presented in this section.

A. Spatial focus with more antennas

The field strength is not necessarily focused in the direction of the intended receiver, but rather to a geographical point where the incoming multipath components add up constructively. As a technique for focusing transmitted energy to a specific location, Time Reversal (TR) has drawn attention, where the transmitted signal is a time-reversed replica of the channel impulse response. In this paper, the Time-Reversal Beam Forming (TRBF) is considered.

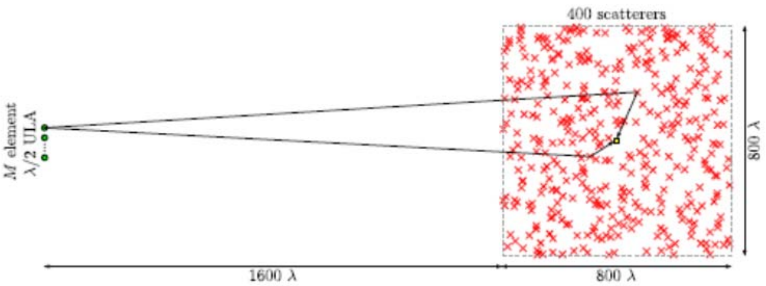


Figure 1. Geometry of the simulated dense scattering environment.

Figure 1 shows a simple geometrical channel model. The channel is composed of 400 uniformly distributed scatterers in a square of dimension $800\lambda \times 800\lambda$, where λ is the signal wavelength. The broadside direction of the *M*-element Uniform Linear Array (ULA) with adjacent element spacing $d = \lambda / 2$ is pointing towards the center of the scatterer area. This model creates a field strength that varies rapidly over the geographical area, typical of small-scale fading. With a complex enough scattering environment and a sufficiently large element spacing in the transmit array, the filed strength resulting from different elements in the transmit array can be seen as independent.

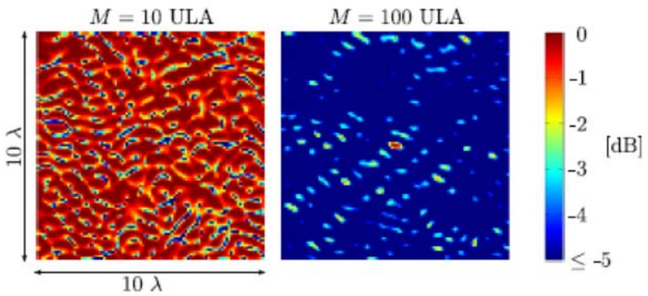
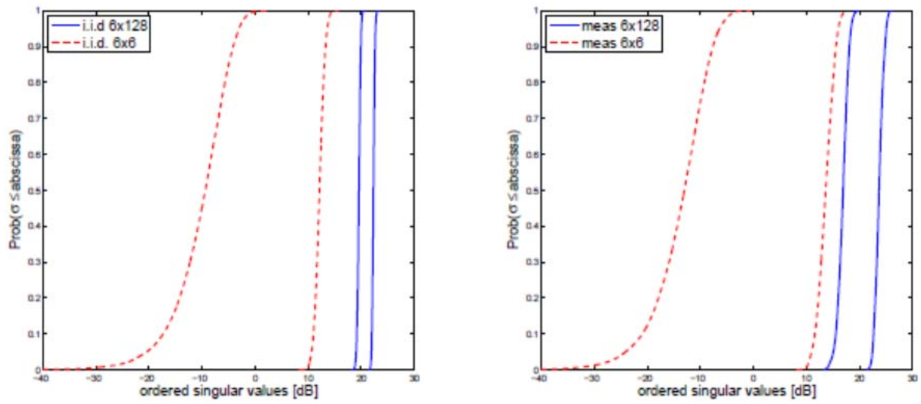


Figure 2. Normalized filed strength in a $10\lambda \times 10\lambda$ area

Figure 2 shows the resulting normalized field strength in a small $10\lambda \times 10\lambda$ environment around the receiver to which we focus the transmitted signal (using MF precoding), for ULAs with $d = \lambda / 2$ of size $M = 10$ and $M = 100$ elements. Figure 2 illustrates two important properties of the spatial MF precoding: (i) that the field strength can be focused to a point rather than in a certain direction and (ii) that more antennas improve the ability to focus energy to a certain point, which leads to less interference between spatially separated users.

B. Antenna aspects

Massive MIMO relies to a large extent on a property of the radio environment called *favorable propagation*. Favorable propagation means that propagation channel responses from the base station to different terminals are sufficiently different. One way of quantifying how different the channel responses to different terminals are, is to look at the spread between the smallest and largest singular value of the channel. Figure 3(a) shows this for a computer simulated "i.i.d." channel. The figure shows the cumulative density function for the smallest respectively the largest singular value for two cases: A conventional array of 6 elements serving 6 terminals (red curves), and a massive array of 128 elements serving 6 terminals (blue curves)



(a) Theory. (b) Measured results.

Figure 3. Singular value spread of massive MIMO channels

For the 6-element array, the singular value spread is about 30 dB, meaning that 1000 times more power would be required to serve all six terminals, as compared to the power required to serve just one of them. With the massive array, the gap is less than 3 dB.

In real channel implementation, measurements were conducted using an indoor 128-antenna base station consisting of four stacked double polarized 16 element circular patch arrays as shown in Fig. 4. Three of the terminals are indoors at various positions and 3 users are outdoors. The measurements were performed at 2.6 GHz with a bandwidth of 50 MHz, and the results were averaged over this bandwidth and over a physical displacement of 10 meters.

Figure 4. Massive MIMO antenna used in measurements

The blue curves in Fig 3. (b) show the corresponding singular value distributions. It is striking how well reality resembles the ideal case in Fig. 3. (a). The spread between the smallest and the largest singular value is a bit larger than for the ideal case, but the probability that the spread exceeds 10 dB is negligible. As a reference, Fig. 3(b) also shows the result when only 6 of the 128 elements are activates (red curves). Overall, there is compelling evidence that the assumption on favorable propagation that underpin massive MIMO are substantially valid in practice.

IV. DISCUSSION

After meeting, please write discussion in the meeting and update your presentation file.

In the antenna implementation, they compared system of 6-antenna array and 6 terminals with single antenna and system of 128-antenna array and 6 terminals with single antenna. They used $f_c[\text{cycle/sec}] = 2.6\times10^9 \approx 3.0\times10^9$, $c\left[\text{m/sec}\right] = 3.0\times10^8$. Thus, the wavelength $\lambda\left[\text{m/cycle}\right] = \frac{c}{f_c} = 0.1$. According to the wavelength, if they used $d = \lambda/2$, then the distance between patched antennas is $d = \lambda/2 = 50\text{cm}$.

Appendix

Reference

[1]