

## Resource Allocation in Cognitive Radio Relay Networks

Authors: J.C Liang et. al.

Publication: IEEE JSAC, Mar. 2013

Speaker: Asif Raza

**Short summary:** In this paper authors formulate the problem of Resource Allocation (RA) in Cognitive Radio (CR) networks with relay stations. The problem takes into account the issues like: fluctuations of usable spectrum resource, channel quality variations caused by frequency selectivity, and interference caused by different transmit power levels. They propose easy to implement heuristic algorithms. The simulation results reveal that presented solutions show good proportional fairness among CR users and improvement in system throughput by power control.

### I. INTRODUCTION

Resource allocation in CR relay networks is considered in the paper. The relay nodes are simply Medium Access Control (MAC) repeaters.

Main challenges in designing RA algorithm are:

- Fluctuations in available spectrum resource: The number of usable resource may differ in each area. It may cause great variations in available band-width and jitters between packets. Therefore they adopt proportional fair scheduling which allocates resource to CR MSs proportionally to their capabilities such as transmission rates determined by channel quality.
- Instability of wireless channels: The frequency selectivity would cause variations of Signal-to-Noise Ratio (SNR). As a result, the quality of each channel would differ on each node. The RA algorithm needs to take into account the instability of channel quality.
- Power control and interference among nodes: the number of usable channels are determined by the activity of PR networks. With proper power control, spatial reuse can be achieved and two adjacent CR RSs can transmit over the same channel.

### II. SYSTEM DESCRIPTION

The network design considered is shown in the following figure.

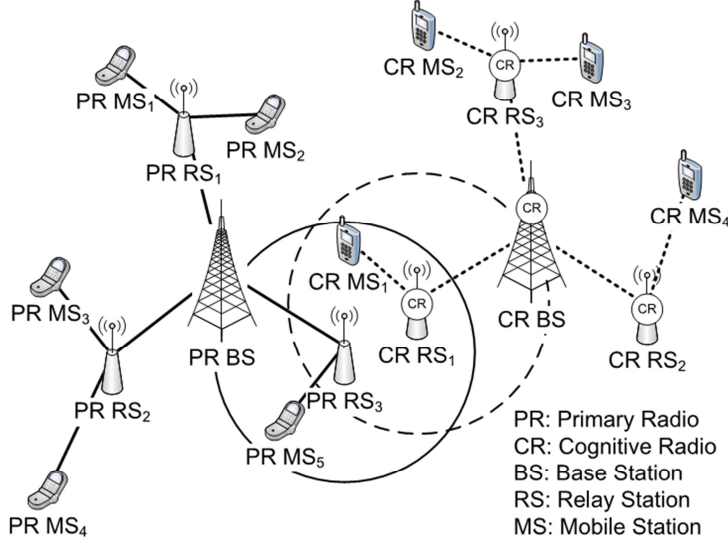


Fig. 1. An illustration of a cognitive radio relay network.

The network drawn with solid and dashed lines corresponds to Primary Radio (PR) network and Cognitive Radio (CR) network. The RA is performed by the Cognitive Radio Base Station (CR BS) in a centralized manner. The transmission of CR network is time-divided into frames and synchronized with the PR network. In each frame there are multiple time-slots not used by PR. The CR BS gathers available channels within the vicinity of each node and the quality of each channel. The CR BS then schedules the usage of frequency bands and time-slots for its downstream Cognitive Radio Relay Stations (CR RSs) and Cognitive Radio Mobile Stations (CR MSs). Each CR MS can connect to CR BS directly or through CR RS. The RA is done in downlink transmission only. The notations used are shown in table 1. The authors starts RA problem formulation by considering fixed transmit power and then extend the formulation to incorporate adjustable transmit power.

### III. FIXED TRANSMIT POWER

A. *Problem Formulation:* proportional fair scheduling for cognitive relay networks with fixed transmit power in relay stations is considered. (Given parameters same as in system description section)

$$O(\lambda) = \max \sum_{m \in \mathcal{M}} \frac{\lambda_m(t)}{\rho_m(t)} \quad \text{where} \quad \lambda_m = \sum_{c \in \mathcal{C}} \sum_{t=0}^{N-1} r_{1_m^c}^p(c, t) \quad (1)$$

Where  $\lambda_m(t)$  is scheduled rate of CR MS  $m$  in frame  $t$ , and  $\rho_m(t)$  is the long-term average rate of CR MS  $m$  until frame  $t$ .

TABLE I  
LIST OF NOTATIONS

Notation	Description
$\mathcal{M}$	Set of CR Mobile Stations
$\mathcal{R}$	Set of CR Relay Stations
$\mathcal{R}^+$	Set of CR Relay Stations and the CR Base Station
$\mathcal{L}$	Set of all links
$\mathcal{C}$	Set of sub-channels
$\mathcal{P}$	Set of available transmit power levels
$N$	Number of slots in one frame
$\mathcal{I}_i^c$	The set of links connecting to node $i$ 's children.
$\mathcal{I}_i^p$	The link connecting to node $i$ 's parent.
$R_l(c)$	The maximum sustainable rate of sub-channel $c$ on link $l$ in bits/sec, $l \in \mathcal{L}, c \in \mathcal{C}$
$r_l(c, t)$	The actual data transmitted on link $l$ over sub-channel $c$ at time-slot $t$ in bits, $l \in \mathcal{L}, c \in \mathcal{C}$
$\rho_m$	Long-term average rate of MS $m$ in bits/frame
$p_i$	The transmit power level of CR RS $i$
$I_l(c, t)$	Indicator variable as defined in (6)
$e_{(i,j)}$	Indicator variable as defined in (7)
$v_{(i,c)}$	Indicator variable as defined in (8)

$$\sum_{c \in \mathcal{C}} \sum_{t=0}^{\tau} r_{\mathcal{I}_i^p}^c(c, t) \geq \sum_{k \in \mathcal{I}_i^c} \sum_{c \in \mathcal{C}} \sum_{t=0}^{\tau} r_k(c, t) \quad \forall i \in \mathcal{R}, 0 \leq \tau \leq N-1 \quad (2)$$

$$\sum_{c \in \mathcal{C}} \sum_{t=0}^{N-1} r_{\mathcal{I}_i^p}^c(c, t) = \sum_{k \in \mathcal{I}_i^c} \sum_{c \in \mathcal{C}} \sum_{t=0}^{N-1} r_k(c, t) \quad \forall i \in \mathcal{R}$$

$$\max_{c \in \mathcal{C}} I_{\mathcal{I}_i^p}^c(c, t) + \max_{k \in \mathcal{I}_i^c, c \in \mathcal{C}} I_k(c, t) \leq 1 \quad \forall i \in \mathcal{R}, 0 \leq t \leq N \quad (3)$$

$$\sum_{k \in \mathcal{I}_i^c} I_k(c, t) + \sum_{j \in \{\mathcal{R}^+ \setminus i\}} \sum_{m \in \mathcal{I}_j^c} e_{(i,j)} I_m(c, t) \leq 1 \quad \forall i \in \mathcal{R}^+, \forall c \in \mathcal{C}, 0 \leq t \leq N \quad (4)$$

$$r_j(c, t) \leq I_j(c, t) \mathcal{R}_j(c) \quad \forall j \in \mathcal{L}, \forall c \in \mathcal{C}, 0 \leq t \leq N \quad (5)$$

$$\max_{k \in \mathcal{I}_i^c, 0 \leq t \leq N} I_k(c, t) \leq v_{(i,c)} \quad \forall i \in \mathcal{R}^+, \forall c \in \mathcal{C} \quad (6)$$

The problem is integer programming problem which is NP-hard and computationally infeasible to solve. By relaxing the integrality constraint the computationally lighter upper bound

of original problem is defined as: let  $w_l(c) = \frac{1}{N} \sum_{t=0}^{N-1} I_l(c, t)$

$$\text{Maximize: } O(\lambda) = \sum_{m \in \mathcal{M}} \frac{\lambda_m^*}{\rho_m} \quad (1)$$

$$\text{Subject to: } \lambda_m^* = N \sum_{c \in \mathcal{C}} w_{1_m^p}(c) R_{1_m^p}(c) \quad (2)$$

$$\sum_{c \in \mathcal{C}} w_{1_m^p}(c) R_{1_m^p}(c) \geq \sum_{k \in \mathcal{I}_i^c} \sum_{c \in \mathcal{C}} w_k(c) R_k(c) \quad \forall i \in \mathcal{R} \quad (3)$$

$$\sum_{k \in \mathcal{I}_i^c} w_k(c) + \sum_{j \in \{\mathcal{R}^+ \setminus i\}} e_{(i,j)} \sum_{m \in \mathcal{I}_j^c} w_m(c) \leq \frac{2}{R+1} \quad \forall i \in \mathcal{R}^+, \forall c \in \mathcal{C} \quad (4)$$

$$\max_{k \in \mathcal{I}_i^c} w_k(c) \leq v_{(i,c)} \quad \forall i \in \mathcal{R}^+, \forall c \in \mathcal{C} \quad (5)$$

---

**Algorithm 1** Greedy Algorithm for PFSCRN-FTP

- 1: Let  $T$  be the set of CR RSs that have been scheduled.
  - 2: Let  $U$  be the set of CR RSs that interfere with each other.
  - 3: Let  $M$  be the candidate CR MSs to be scheduled.
  - 4:  $M \leftarrow \phi$
  - 5: **for all**  $c \in \mathcal{C}$  **do**
  - 6:    $T \leftarrow \phi$
  - 7:   **for all**  $r \in \mathcal{R}$  such that  $v_{(r,c)} = 1$  and  $r \notin T$  **do**
  - 8:      $U \leftarrow \phi$
  - 9:     **for all**  $i \in \mathcal{R}$  **do**
  - 10:       **if**  $e_{(r,i)} = 1$  and  $v_{(i,c)} = 1$  **then**
  - 11:           $T \leftarrow i, \quad U \leftarrow i$
  - 12:       **end if**
  - 13:     **end for**
  - 14:     **for all**  $u \in U$  **do**
  - 15:        $m_{(u,c)} = \arg \max_{m \in u's \text{ children}} \frac{R_{1_m^p}(c)}{\rho_m}$
  - 16:        $M \leftarrow M \cup \{m_{(u,c)}\}$
  - 17:     **end for**
  - 18:   **end for**
  - 19: **end for**
  - 20: Sort elements in  $M$  in descending order of contribution to the objective function
  - 21: Denote by  $n_{(l,c)}$  the available time-slots on link  $l$  over sub-channel  $c$
  - 22: **for all**  $m_{(u,c)} \in M$  **do**
  - 23:    $D_m = R_{m_{(u,c)}}(c) n_{(l_{m_{(u,c)}}, c)}$
  - 24:    $SchedData \leftarrow 0$
  - 25:   **while**  $SchedData < D_m$  and CR BS has available sub-channels **do**
  - 26:      $c' \leftarrow \arg \max_{c \in \mathcal{C} \text{ and } v_{(BS,c)}=1} R_{1_u^p}(c)$
  - 27:     Allocate available time-slots of sub-channel  $c'$  on link  $l_u^p$  to CR RS  $u$
  - 28:      $SchedData \leftarrow SchedData + R_{1_u^p}(c')$
  - 29:   **end while**
  - 30:   Allocate time-slots of sub-channel  $c$  on link  $l_{m_{(u,c)}}^p$  to  $m_{(u,c)}$
  - 31: **end for**
-

The algorithm has two parts:

- Find and schedule available sub-channels for each CR RS.
- Allocate sub-channels and time-slots of all hops in a greedy-based approach.

Resolve conflicts among interfering CR RSs and schedule their available sub-channels: The algorithm scans through all the sub-channels. For each sub-channel  $c$ , it scans through each CR RS  $r$ , and record all CR RSs that interfere with  $r$  in the set  $U$ . All of the CR RSs in the set  $U$  have access to sub-channel  $c$ , but are interfered with each other. The CR RS  $m$  with maximum value of  $\frac{R_{1_m^p}(c)}{\rho_m}$  in  $U$  can access the sub-channel  $c$  at a given time.

Allocate the resource between the CR BS and the CR RSs: the CR MSs are sorted in descending order of their contributions to the objective function. For each  $m$  in the sorted order and the CR RS  $u$  that  $m$  attaches to, the amount of service that  $m$  receives is initialized to the number of bits that can be transmitted using all the available time-slots between  $m$  and  $u$  over the scheduled sub-channel  $c$  (line 23). The sub-channels between the CR BS and  $u$  are allocated in the order of transmission rate. That is, the sub-channel  $c'$  with highest rate  $R_{1_u^p}(c')$  is allocated first, and then the second highest sub-channel is allocated, and so on.

#### IV. VARIABLE TRANSMIT POWER

Better spectrum utilization and system throughput can be achieved with proper control of CR RS transmit power. A scheduler decides power level of a CR RS from pool of discrete power levels. The decision normally involves the interference indicator between two CR RSs,  $e(i, j)$ , and the maximum sustainable rate  $R_l(c)$  on an CR RS-CR MS link  $l$  over a sub-channel  $c$  become functions of the transmit power  $p$ .

A. *Problem Formulation:* Let  $p_i$  denotes the transmit power level of CR RS  $i$ . (Given parameters are same as in fixed power transmission).

$$\text{Objective: } O(\lambda) = \max \sum_{m \in \mathcal{M}} \frac{\lambda_m(t)}{\rho_m(t)} \quad \text{where } \lambda_m = \sum_{c \in \mathcal{C}} \sum_{t=0}^{N-1} r_{1_m^p}(c, t) \quad (1)$$

$$\text{Subject to: } p_i \in \mathcal{P}, \forall i \in \mathcal{R} \quad (2)$$

$$\begin{aligned} \sum_{c \in \mathcal{C}} \sum_{t=0}^{\tau} r_{1_i^p}(c, t) &\geq \sum_{k \in \mathcal{I}_i^c} \sum_{c \in \mathcal{C}} \sum_{t=0}^{\tau} r_k(c, t) \quad \forall i \in \mathcal{R}, 0 \leq \tau \leq N-1 \\ \sum_{c \in \mathcal{C}} \sum_{t=0}^{N-1} r_{1_i^p}(c, t) &= \sum_{k \in \mathcal{I}_i^c} \sum_{c \in \mathcal{C}} \sum_{t=0}^{N-1} r_k(c, t) \quad \forall i \in \mathcal{R} \end{aligned} \quad (3)$$

$$\max_{c \in \mathcal{C}} I_{l^p}(c, t) + \max_{k \in \mathcal{I}_i^c, c \in \mathcal{C}} I_k(c, t) \leq 1 \quad \forall i \in \mathcal{R}, 0 \leq t \leq N \quad (4)$$

$$\sum_{k \in \mathcal{I}_i^c} I_k(c, t) + \sum_{j \in \{\mathcal{R}^+ \setminus i\}} \sum_{m \in \mathcal{I}_j^c} e_{(i,j)}(p_i) I_m(c, t) \leq 1 \quad \forall i \in \mathcal{R}^+, \forall c \in \mathcal{C}, 0 \leq t \leq N \quad (5)$$

$$r_j(c, t) \leq I_j(c, t) \mathcal{R}_j(c, p_i) \quad \forall i \in \mathcal{R}^+, \forall j \in \mathcal{I}_i^c, \forall c \in \mathcal{C}, 0 \leq t \leq N \quad (6)$$

$$\max_{k \in \mathcal{I}_i^c, 0 \leq t \leq N} I_k(c, t) \leq v_{(i,c)} \quad \forall i \in \mathcal{R}^+, \forall c \in \mathcal{C} \quad (7)$$

The problem formulation same as fixed power transmission case except that interference indicator  $e_{(i,j)}(p)$  between two CR RSs (i, j) and the maximum sustainable rate  $R_i(c, p)$  on an CR RS and CR MS link  $l$  over sub-channel  $c$  are functions of the CR RS transmit power as shown in constraint 5 and 6 respectively.

B. *Proposed Greedy Algorithm for PFSCRN-ATP*: an algorithm for power level selection, shown in Algorithm 2 below, is based upon following criteria:

- Interference to other CR RSs is minimized.
- Spectrum should be fully utilized

---

**Algorithm 2** Arg-Max Algorithm for Determining Power for CR RSs

---

```

1: for all  $r \in \mathcal{R}$  do
2:   for all available power  $p$  do
3:      $u_{(r,p)} \leftarrow 0$ 
4:     for all  $c \in \mathcal{C}$  and  $v_{(r,c)} = 1$  do
5:        $g_{(r,c,p)} \leftarrow \max_{m \in r's \ children} \frac{R_{l_m^p}(c, p)}{\rho_m}$ 
6:        $u_{(r,p)} \leftarrow u_{(r,p)} + g_{(r,c,p)}$ 
7:     end for
8:   end for
9: end for
10: Let  $p(r)$  be the scheduled power level of CR RS  $r$ 
11:  $p(r) \leftarrow \arg \max_{all \ power \ level \ p} u_{(r,p)}$ 
12: If there are multiple maximum  $u_{(r,p)}$ , set  $p(r)$  as the
    lowest  $p$  among them

```

---

A score is calculated for each combination of (CR RS, power level)( $r, p$ ). It is calculated based upon contribution, i.e.,  $\frac{R_{l_m^p}(c, p)}{\rho_m}$ , of  $m^{th}$  CR RM node to (1) on each available sub-channel  $c$  of  $r$ . The contribution of CR MSs is summed up. The power level for CR RS is determined then as: Priority is

given to the  $(r, p)$  combination with highest score while tie is broken by choosing the combination that can achieve the objective function with lowest power level.

After the transmit power of CR RSs are determined by using Algorithm 2, the scheduling problem is solved by using a PFSCRN-ATP as shown in Algorithm 3. It is modified version of Algorithm 1 with selected power levels.

---

**Algorithm 3** Greedy Algorithm for PFSCRN-ATP

---

- 1: Let  $p(r)$  be the scheduled power determined by Algorithm 2
  - 2: Let  $T$  be the set of CR RSs that have been scheduled.
  - 3: Let  $U$  be the candidate CR RSs to be scheduled.
  - 4: Let  $M$  be the candidate CR MSs to be scheduled.
  - 5:  $M \leftarrow \phi$
  - 6: **for all**  $c \in \mathcal{C}$  **do**
  - 7:    $T \leftarrow \phi$
  - 8:   **for all**  $r \in \mathcal{R}$  such that  $v_{(r,c)} = 1$  and  $r \notin T$  **do**
  - 9:      $U \leftarrow \phi$
  - 10:     **for all**  $i \in \mathcal{R}$  **do**
  - 11:       **if**  $e_{(r,i)}(p(r)) = 1$  and  $v_{(i,c)} = 1$  **then**
  - 12:          $T \leftarrow i, \quad U \leftarrow i$
  - 13:       **end if**
  - 14:     **end for**
  - 15:     **for all**  $u \in U$  **do**
  - 16:        $m_{(u,c)} = \arg \max_{m \in u's \ children} \frac{R_{1_m^p}(c, p(u))}{\rho_m}$
  - 17:        $M \leftarrow M \cup \{m_{(u,c)}\}$
  - 18:     **end for**
  - 19:   **end for**
  - 20: **end for**
  - 21: Denote by  $n_{(l,c)}$  the available time-slots on link  $l$  over sub-channel  $c$
  - 22: **for all**  $m_{(u,c)} \in M$  **do**
  - 23:    $D_m = R_{m_{(u,c)}}(c, p(u))n_{(l_{m_{(u,c)}}^p, c)}$
  - 24:    $SchedData \leftarrow 0$
  - 25:   **while**  $SchedData < D_m$  and CR BS has available sub-channels **do**
  - 26:      $c' \leftarrow \arg \max_{c \in \mathcal{C} \text{ and } v_{(BS,c)}=1} R_{1_u^p}(c)$
  - 27:     Allocate available time-slots of sub-channel  $c'$  on link  $l_u^p$  to CR RS  $u$
  - 28:      $SchedData \leftarrow SchedData + R_{1_u^p c'}()$
  - 29:   **end while**
  - 30:   Allocate time-slots of sub-channel  $c$  on link  $l_{m_{(u,c)}}^p$  to  $m_{(u,c)}$
  - 31: **end for**
-

## V. PERFORMANCE EVALUATION

The simulation environment of an IEEE 802.16j relay network has been constructed to evaluate the proportional fairness of proposed algorithms, and to compare the system throughput of each algorithm.

A. *Simulation Setup*: two scenarios for evaluation of performance of proposed algorithms.

Total CR BS =1 (located in center of the cell), Total CR RS = 4, Total CR MS = 20 or 40  
 in low system load or high system load case, Total 40 random topologies are generated,  
 Total Frames = 2000, No of sub-channels = 64

Scenario 1 {purpose} Signal enhancement	Scenario 2 {purpose} Range extension
CR RSs within 1200m of CR BS	CR RSs within 1500m of CR BS
CR MSs uniformly distributed in cell, with in 1800m of CR BS	CR MSs uniformly distributed with in 300m of randomly selected CR RS

B. *Proportional Fairness*:

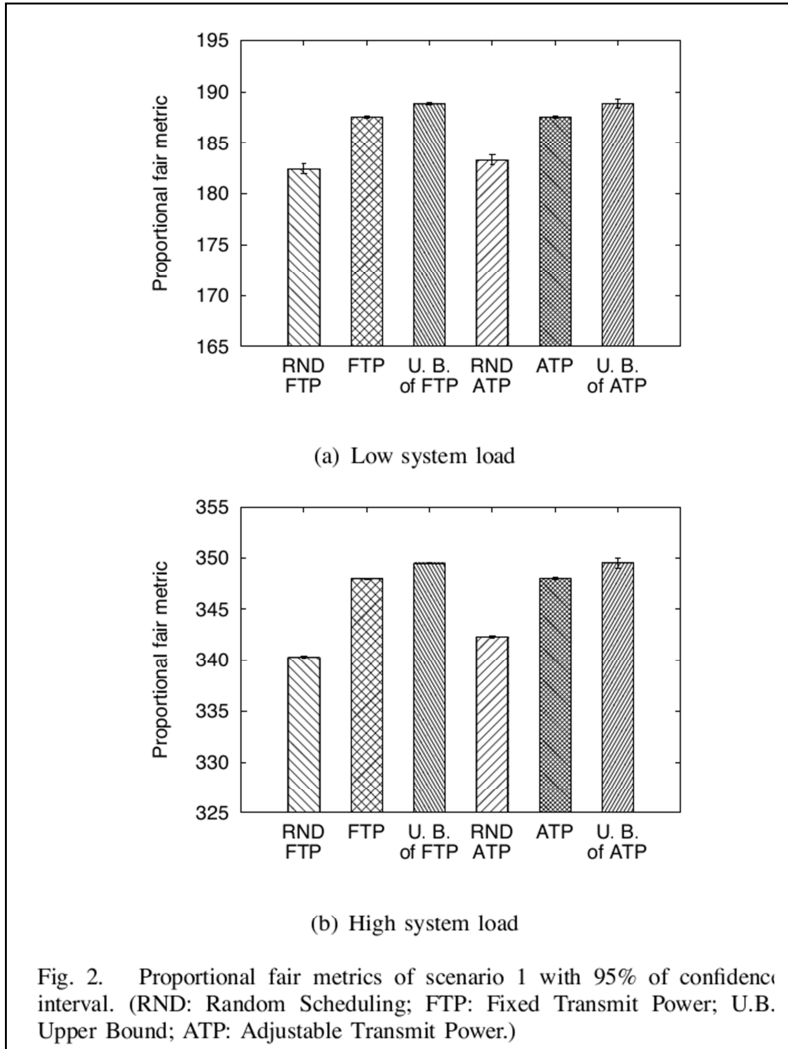


Fig. 2. Proportional fair metrics of scenario 1 with 95% of confidence interval. (RND: Random Scheduling; FTP: Fixed Transmit Power; U.B.: Upper Bound; ATP: Adjustable Transmit Power.)

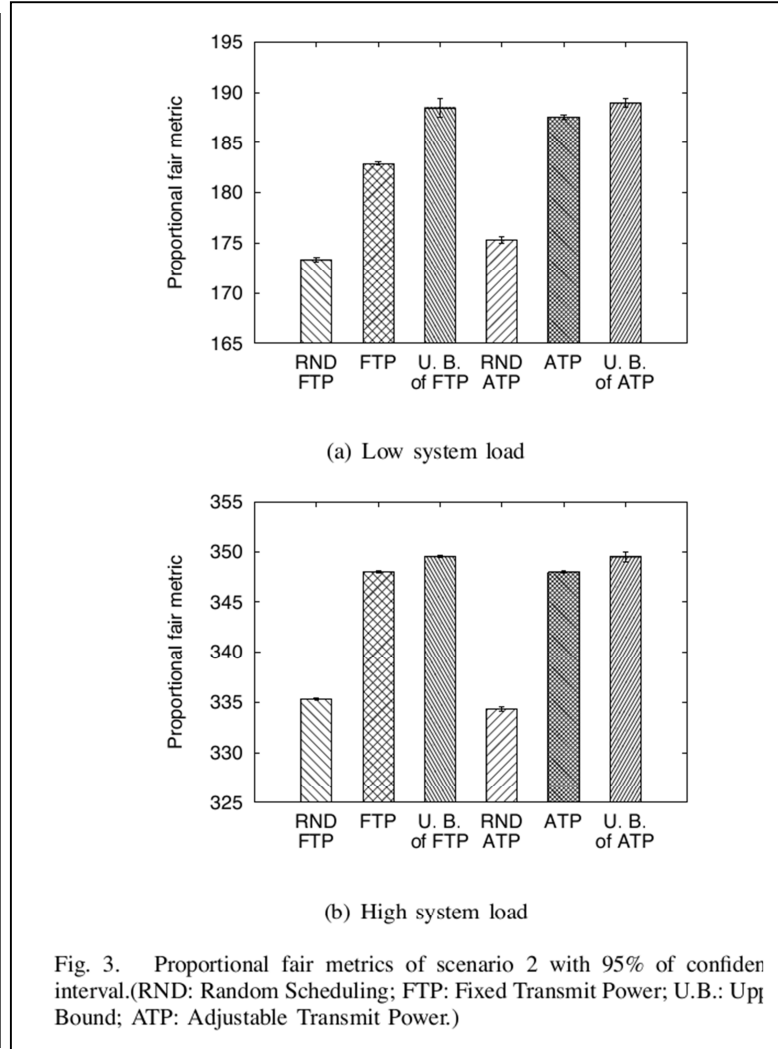
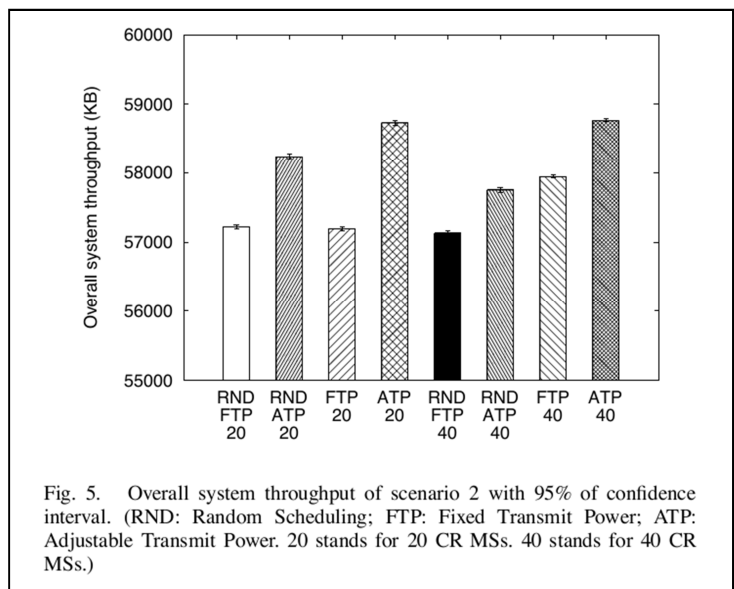
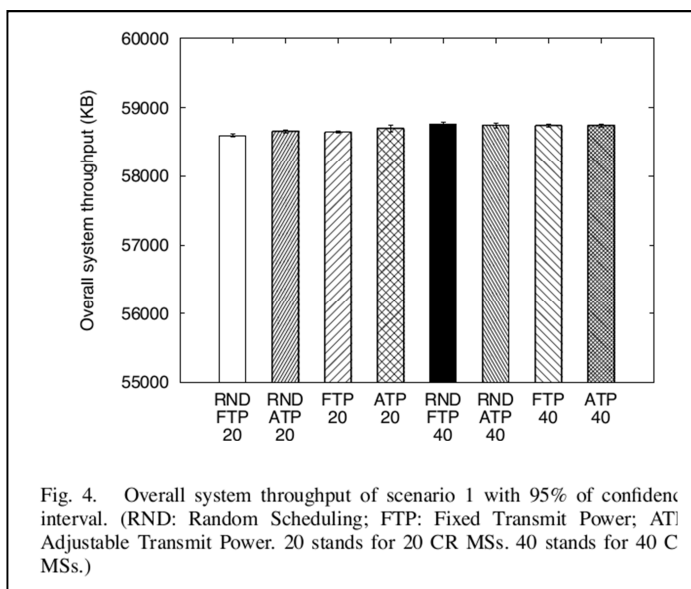


Fig. 3. Proportional fair metrics of scenario 2 with 95% of confidence interval. (RND: Random Scheduling; FTP: Fixed Transmit Power; U.B.: Upper Bound; ATP: Adjustable Transmit Power.)



The upper bounds of FTP case in both figures are derived as that presented in III-A. The upper bounds of ATP in both figures are derived by using the power levels determined by Algorithm 2. The random scheduling is lower than proposed algorithms because in scenario 1, the CR RSs are placed closer to the CR BS. The channel condition of each CR MS would be less variant. On the contrary, in scenario 2, the CR RSs are placed far apart from each other as well as from the CR BS, and the channel condition of each CR MS varies a lot. However proposed algorithms are insensitive to the changes of scenario and system load.

**C. System Throughput:** Overall system throughput is the sum of the services that each CR MS receives during the whole simulation period.



In scenario 1, the overall system throughput of 20 and 40 CR-MSs does not differ much. The overall system throughput of FTP and ATP does not differ much, either. The reason is that CR RSs are placed closer to the CR BS. As a result, CR RSs will easily interfere with each other even with the lowest transmit power level. In such circumstance where cell coverage is small and CR MSs are evenly distributed, transmit power control will not help much. Also, in scenario 1, the channel condition of each CR MS does not differ much. Hence, the throughput does not differ much in all settings.

In scenario 2 the CR RSs are placed farther from the CR BS. Also, CR MSs tend to form hot-spot near the CR RSs. In systems where CR RSs are apart far enough and CR MSs are clustered in a certain extent, the interference among CR RSs will be highly controllable through adjustment of transmit power levels. Thus, as shown in Fig. 5, the performance of ATP is better than that of FTP.

**D. Algorithm Running Time:** The implemented algorithms are executed with 4 CR RSs, 40 CR MSs, 64 sub-channels, 48 time-slots per frame, and 2000 frames. The proposed FTP and ATP requires 0.38 ms and 0.75ms, respectively. however random FTP and random ATP require 0.14 ms and 0.23ms, respectively. Although they are slightly less than proposed algorithms, yet proposed algorithms outperform random scheduling algorithms as shown in Figs. 2–5.