

# Simplified Relay Selection and Power Allocation in Cooperative Cognitive Radio Systems

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Publication: IEEE Trans. Wireless Comm., Jan. 2011

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**Short summary:** In this paper authors propose solution of a combined problem; relay selection and power allocation to secondary users under the constraint of limited interference to primary users in cognitive radio (CR) system. Objective of the joint problem was to maximize system throughput. A high complexity optimal solution and a low complexity suboptimal solution are proposed. The presented solutions show over 50% improvement in system throughput.

## I. INTRODUCTION

Cooperative technique for spectrum sensing and sharing in CR networks has been investigated in the literature. It can obtain spatial diversity and combat detrimental effects of wireless channels however it has some limitations associated. For example, while doing relay selection and resource allocation one must also consider spectrum efficiency and interference limitation as well. Authors in this paper consider these combined issues i.e. relay selection and power allocation with interference limitation.

## II. SYSTEM DESCRIPTION

In order to have effective cooperation following decisions must be made prior to cooperation:

- When to cooperate
- To whom cooperate with
- What resources to share and how to share?

These decisions are basis of relay selection and power allocation problem.

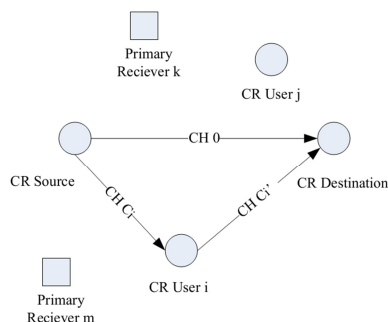


Figure 1: The structure of a cooperative CR network

A simple three-node relay system where each CR user can only help one CR transceiver pair is shown in the figure. The source node transmits data to the destination and the relay simultaneously using orthogonal channels, Channel (CH) 0 and CH  $C_i$ , respectively. The relay node forwards scaled version of the received signal from CH  $C_i$  to the destination node using CH  $C'_i$ .

**Power modeling:**

Existing relay selection schemes does not consider interference issue. In order to prevent primary users from interference the transmission powers on channels (CH 0), CH  $C_i$  and CH  $C'_i$  must satisfy:

$$\begin{aligned} P_{1,i} |h_{s,p,d}|^2 &\leq I_1 \\ P_{2,i} |h_{s,p,i}|^2 &\leq I_2 \\ P_{3,i} |h_{i,p}|^2 &\leq I_3 \end{aligned} \quad (1)$$

Where  $h_{s,p,i}$  and  $h_{s,p,d}$  are channel gain between CR source and primary users of CH  $C_i$  and (CH 0) respectively while  $h_{i,p}$  is channel gain between CR source and primary user of channel CH  $C'_i$ .  $I_1$ ,  $I_2$  and  $I_3$  are acceptable interference powers of primary users over channel CH 0, CH  $C_i$  and CH  $C'_i$  respectively. The overall transmission power of CR source and relay nodes are limited as:

$$P_{1,i} + P_{2,i} \leq P_{total} \text{ and } P_{3,i} \leq P_3$$

The channels under consideration i.e. CH 0, CH  $C_i$  and CH  $C'_i$  are  $\mathcal{N}(0, \sigma^2)$  with known channel gains at CR source and CR relay nodes. If  $i^{\text{th}}$  CR user is relay node then signal and noise powers at destination from relay is:

$$\begin{aligned} P_{s,i} &= \frac{P_{3,i} P_{2,i} |h_{s,i}|^2 |h_{i,d}|^2}{P_{2,i} |h_{s,i}|^2 + \sigma^2} \\ P_{n,i} &= \left( \frac{P_{3,i} |h_{i,d}|^2}{P_{2,i} |h_{s,i}|^2 + \sigma^2} + 1 \right) \sigma^2 \\ SNR_i &= \frac{P_{s,i}}{P_{n,i}} \end{aligned}$$

Where  $h_{s,i}$  and  $h_{i,d}$  are channel gains between source and relay and relay and destination nodes respectively and  $SNR_i$  is SNR value at destination from  $i^{\text{th}}$  relay channel.

**Throughput Calculation:**

The system throughput for  $i^{\text{th}}$  relay is:

$$T_i(P_{1,i}, P_{2,i}, P_{3,i}) = (1 - \alpha) \log_2 \left( 1 + \frac{P_{1,i} |h_{s,d}|^2}{\sigma^2} \right) + (1 - \alpha) \log_2 \left( 1 + \frac{P_{3,i} |h_{i,d}|^2 P_{2,i} |h_{s,i}|^2}{(P_{3,i} |h_{i,d}|^2 + P_{2,i} |h_{s,i}|^2 + \sigma^2) \sigma^2} \right) \quad (2)$$

Where  $\alpha$  is mis-detection probability of spectrum sensing.

This equation tells us that data will be lost if interference happens.

### III. ALGORITHM DEVELOPMENT

Optimal and suboptimal algorithms are developed for power allocation and relay selection problem.

#### A. Optimal Approach

Optimization problem is formulated as:

$$(P_{1,i}^*, P_{2,i}^*, P_{3,i}^*) = \arg \max_{P_{1,i}, P_{2,i}, P_{3,i}} T_i(P_{1,i}, P_{2,i}, P_{3,i}) \quad (3a)$$

$$i = \arg \max_i T_i(P_{1,i}^*, P_{2,i}^*, P_{3,i}^*) \quad (3b)$$

subject to

$$P_{1,i} + P_{2,i} \leq P_{total} \quad (3c)$$

$$P_{3,i} \leq P_3 \quad (3d)$$

$$0 \leq P_{1,i} |h_{s,p,d}|^2 \leq I_1 \quad (3e)$$

$$0 \leq P_{2,i} |h_{s,p,i}|^2 \leq I_2 \quad (3f)$$

$$0 \leq P_{3,i} |h_{i,p}|^2 \leq I_3 \quad (3g)$$

Lagrange multiplier is then used to divide the problem into subproblems and then obtain solution for subproblems:

$$\begin{aligned} L(P_{1,i}, P_{2,i}, \lambda_1, \lambda_2, \lambda_3) = & -(1 - \alpha) \log_2 \left( 1 + \frac{P_{1,i} |h_{s,d}|^2}{\sigma^2} \right) - (1 - \alpha) \log_2 \left( 1 + \frac{P_{3,i} |h_{i,d}|^2 P_{2,i} |h_{s,i}|^2}{(P_{3,i} |h_{i,d}|^2 + P_{2,i} |h_{s,i}|^2 + \sigma^2) \sigma^2} \right) \\ & + \lambda_1 (P_{1,i} + P_{2,i} - P_{total}) + \lambda_2 (P_{1,i} |h_{s,p,d}|^2 - I_1) + \lambda_3 (P_{2,i} |h_{s,p,i}|^2 - I_2) \end{aligned} \quad (4)$$

According to Karush-Kuhn-Tucker conditions:

$$0 \leq P_{1,i}, 0 \leq P_{2,i} \text{ and } \lambda_i \geq 0 \quad \forall i$$

$$\lambda_1(P_{1,i} + P_{2,i} - P_{total}) = 0,$$

$$\lambda_2(P_{1,i} |h_{s,p,d}|^2 - I_1) = 0,$$

$$\lambda_3(P_{2,i} |h_{s,p,i}|^2 - I_2) = 0$$

and

$$\frac{\partial L}{\partial P_{1,i}} = 0 \text{ and } \frac{\partial L}{\partial P_{2,i}} = 0$$

Solving them by using dual-domain and sub-gradient method we get solution for Lagrangian dual variables as:

$$\lambda_1^{(n+1)} = \left[ \lambda_1^n + \mu^n \left( P_{1,i}^n(\lambda_1^n, \lambda_2^n, \lambda_3^n) + P_{2,i}^n(\lambda_1^n, \lambda_2^n, \lambda_3^n) - P_{total} \right) \right]^+,$$

$$\lambda_2^{(n+1)} = \left[ \lambda_2^n + \mu^n \left( P_{1,i}^n(\lambda_1^n, \lambda_2^n, \lambda_3^n) |h_{s,p,d}|^2 - I_1 \right) \right]^+,$$

$$\lambda_3^{(n+1)} = \left[ \lambda_3^n + \mu^n \left( P_{2,i}^n(\lambda_1^n, \lambda_2^n, \lambda_3^n) |h_{s,p,i}|^2 - I_2 \right) \right]^+$$

Here ‘ $\mu^n$ ’ is sequence of scalar step-sizes. Once we get  $\lambda_i, \forall i$  we can calculate  $P_{1,i}$  and  $P_{2,i}$  as follows;

$$P_{1,i} = \left[ \frac{1-\alpha}{(\lambda_1 + \lambda_2) |h_{s,p,d}|^2 \ln 2} - \frac{\sigma^2}{|h_{s,d}|^2} \right]^+,$$

$$P_{2,i} = \left[ \frac{\sqrt{P_{3,i}^2 |h_{i,d}|^4 + 4P_{3,i} |h_{i,d}|^2 |h_{s,i}|^2 K} - P_{3,i} |h_{i,d}|^2 + 2\sigma^2}{2|h_{s,i}|^2} \right]^+$$

Where  $K = \frac{1-\alpha}{(\lambda_1 + \lambda_3) |h_{s,p,i}|^2 \ln 2}$  and  $[\cdot]^+ = \max(\cdot, 0)$

By using these values we get optimal power allocation  $P_{1,i}^*, P_{2,i}^*$ . Note that  $P_{3,i}^* = \min \left\{ \frac{I_3}{|h_{i,p}|^2}, P_3 \right\}$ .

Finally system throughput  $T_i^*$  when  $i^{\text{th}}$  CR node acts as relay is then calculated from equation ...

### B. Sub-Optimal Approach

Joint relay selection and power allocation problem provides optimal throughput yet it is quite complex algorithm. A low complexity, sub-optimal version of the problem can be defined as follows:

Transmission power constraints of CR nodes are defined as:

$$P_{1,i} = \frac{I_1}{|h_{s,p,d}|^2}$$

$$P_{2,i} = \frac{I_2}{|h_{s,p,i}|^2}$$

In order to calculate system throughput by using equation 2 we need to choose relay as:

$$i = \arg \max_i \frac{P_{3,i} |h_{s,i}|^2 |h_{i,d}|^2}{P_{3,i} |h_{i,d}|^2 |h_{s,p,i}|^2 + I_2 |h_{s,i}|^2 + \sigma^2 |h_{s,p,i}|^2}$$

The optimal power limit and other constraints in (2) are taken as total power limit.

#### IV. SIMULATION RESULTS

##### Parameters:

Interference limits:=  $I_1 = I_2 = I_3 = 0.1 mW$  , Path loss exponent= 4, channel of unit bandwidth is used. Channel fading follows Rayleigh distribution with  $\sigma = 6dB$

##### Results:

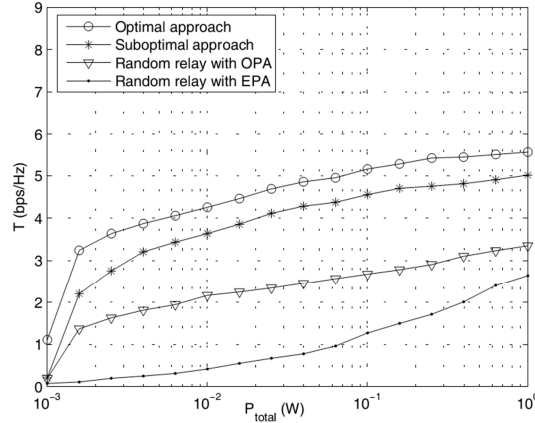


Fig. 2. System throughput versus transmission power limit of CR source

Transmission power limit of relay node is  $P_3 = 0.5W$  and no. of candidate relay nodes = 20. From figure we see proposed scheme achieves about 50% throughput achievement over optimal power allocation (OPA) and equal power allocation (EPA) schemes. Comparing sub-optimal scheme with optimal scheme we see only about 15% degradation in throughput is observed. Moreover in low  $P_{total}$  region the system throughput increase rapidly however for high  $P_{total}$  region the growth is restricted due to interference limits.

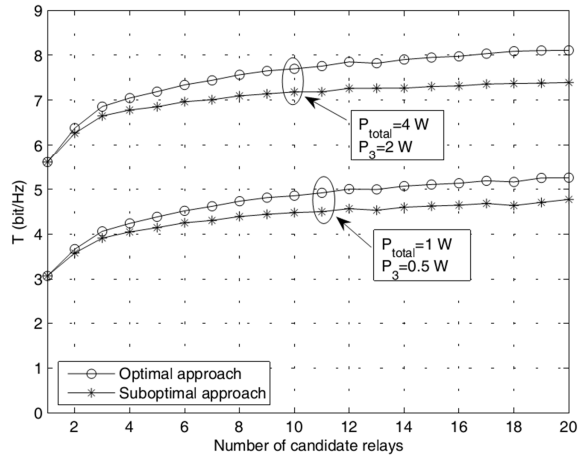


Fig. 3. System throughput versus number of candidate relays.

Figure 3 shows that gap between optimal and sub-optimal schemes is small well when the number of candidate users is small. This shows that sub-optimal approach performs well when number of relays are small.