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| Distributed Channel Coding for Underwater Acoustic Cooperative Networks |

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| Publication: | IEEE Trans. Communications, Mar. 2014 |
| Speaker:Date: | Zafar IqbalSep. 22, 2014. |

**Short summary**: Multiuser cooperative schemes usually rely on relay selection or channel selection to avoid deep fading and achieve diversity while maintaining acceptable spectral efficiency. In some applications such as underwater acoustic communications, the low speed of the acoustic wave results in a very long delay between the channel state information (CSI) measurement time and the relay assignment time, which leads to a severely outdated CSI.

To remedy this, a distributed coding schemes that aim at achieving good diversity-multiplexing trade-off (DMT) for multiuser scenarios, where CSI is not available for resource allocation has been proposed. A network with multiple source nodes, multiple relay nodes, and a single destination is considered. First, a distributed linear block coding scheme, including Reed-Solomon codes, where each relay implements a column of the generator matrix of the code, and soft decision decoding is employed to retrieve the information at the destination side, is introduced. The end-to-end error performance of this scheme has been derived and shown that the achievable diversity equals the minimum Hamming distance of the underlying code, while its DMT outperforms that of existing schemes. Then the proposed scheme has been extended to distributed convolutional codes, and shown that achieving higher diversity orders is also possible.

# Introduction

The underwater acoustic (UWA) channel is a complex communication environment characterized by multipath propagation, low speed of sound, and frequency-dependent path loss [1]. For multipath propagation, Rayleigh and Rician are well accepted fading distributions for UWA channel [1]–[5]. In order to combat fading and at the same time maintain an acceptable spectral efficiency, different cooperative schemes have been proposed for multi-source multi-relay networks [6]–[9]. These schemes usually use the channel state information (CSI) at the transmitter side to do channel selection or relay selection to avoid deeply faded channels. However, in UWA communication systems, obtaining accurate CSI is difficult. This is due to the low speed of the acoustic wave in the water. As a result, the total traveling time of the wave between each node and the resource allocator becomes very long. During this period, the sub-channels most likely change and the reported CSI becomes outdated.

To remedy this problem, this paper proposes a distributed coding strategy, which combines cooperative communications [10] and network coding [11]. As opposed to conventional coding schemes, in distributed coding, different parts of the codeword are transmitted by different nodes through independent channels. Different strategies for distributed coding in cooperative networks have been proposed during the past decade. Laneman *et al.* [6] proposed distributed spacetime block codes (DSTBCs) based on orthogonal STBCs. Since the design of orthogonal STBCs is not possible for every number of antennas and to overcome the difficulties imposed by large numbers of relays, Yiu *et al.* [12] proposed a DSTBC which selects a subset of nodes for transmission. This idea was also extended to distributed space-time trellis codes (DSTTCs) [13] where a larger coding gain, compared to STBCs, is expected. In DSTTC, convolutional codes are usually used as constituent codes both at the source and at the relay nodes. Other proposed schemes include distributed convolutional codes [14], distributed low density parity check

(DLDPC) codes [15], and distributed turbo codes [16], [17].

The above mentioned distributed coding schemes have been developed for single-source cooperative networks. In this paper, the scenario has been extended to multi-source multi-relay networks where CSI is not available for relay assignment. For this setting, adaptive network coded cooperation (ANCC) has been proposed in [18], where each relay node decodes the received message from all source nodes and then, among the successfully decoded messages, *randomly* selects a small number of messages and generates a parity check message using them. This parity check message is transmitted to the destination alongside with a bit-map field to inform the destination about the constituents of these messages. In this way, a graph code is formed at the destination and a belief propagation decoding algorithm can then be used for decoding. In [19], a network consisting of multiple source nodes and a single destination is considered where each source relays the information of the other sources. For this network, it is shown that a diversity equal to the Hamming distance of the code is achievable. The authors show that the scheme proposed in [19] cannot be applied to distributed convolutional codes. As a result, its diversity is limited to the singleton bound. They propose a new scheme that allows for using distributed convolutional codes and achieving high diversity orders.

Normally if the aforementioned schemes are used together with decode-and-forward (DF) relaying, then detection errors at the relays have to be taken into account in the code design and the performance analysis, however most papers, such as [18], usually assume that the relays can decode correctly. In this paper, the possibility of unsuccessful decoding at the relays has been considered in the performance analysis, making the proposed scheme more realistic.

*A. Contributions of This Paper*

The contributions of this paper can be summarized as follows:

**1)** We propose distributed coding schemes that aim at achieving good diversity-multiplexing tradeoff (DMT) without the need to have the CSI available for relay assignment. We consider both block codes and convolutional codes.

**2)** The DMT of the proposed scheme outperforms that of existing schemes. This property is related to employing soft-decision decoding at the destination which can achieve a diversity equal to the Hamming distance of the underlying code. As opposed to [18] where the code generator matrix is sparse and each node *randomly* picks only a few symbols to thin the code graph, the proposed scheme uses a predetermined plan for relaying. As a result of this nonrandom nature, we do not need a high number of transmissions to ensure specific target diversity, hence we expect a better DMT.

**3)** Since the proposed scheme is not adaptive, the relays do not need to send an extra bit-map field to inform the destination about the corresponding messages included in the packet. In this way, fixed relays are assigned to each source node and thus, the management of the network will be much easier. In [19], when a node cannot decode the message of other nodes to contribute to cooperation, it just retransmits its own information. As a result, each node needs to inform the destination about the contents of its message, which requires a considerable overhead.

**4)** To the best of our knowledge, the proposed scheme offers the best diversity among competitive schemes except for distributed space-time schemes. Those space-time schemes require very careful inter-user synchronization at the bit/baud level, but the proposed scheme is free of this limitation.

**5)** In the literature, to obtain diversity using channel codes, different coded symbols should undergo different channel fades. Therefore, the use of an interleaver to scramble the coded symbols before transmission is necessary. The proposed scheme, however, is free of this limitation, because different coded symbols are transmitted over completely separate channels. Hence, the system will not suffer from the corresponding delay and it is suitable for quasi-static channels.

# Proposed Scheme

## System Model

Consider the network shown in Fig. 1, which consists of *k* source nodes, *n* relay nodes, and a single destination. All relays operate in a half-duplex mode. The fading in all source-relay (*S*-*R*) and relay-destination (*R*-*D*) channels is assumed to be independent but not identically distributed

(i.n.d.) according to the Rayleigh distribution. Since the underlying system model is an underwater acoustic channel, we can assume that the nodes are relatively far apart from each other (tens or hundreds of meters). This justifies the independence among the sub-channels. We assume there is no direct link between the sources and the destination. A two-phase relay mode is employed. In the first phase, the source nodes broadcast their messages using *k* orthogonal channels and the relays receive. These orthogonal channels could be in time, frequency or code. In the second phase, the relay nodes transmit using orthogonal channels and the destination receives.



Fig 1. An example of the proposed distributed linear block code relaying. Four relays only decode and forward the received signal while the other three relays transmit the XOR of the received signal from the three sources.

## Distributed Block Code Relaying

Given the underlying network, an (*n, k*) linear block code is implemented in a distributed manner. Let us denote the generator matrix of the mentioned code by **G**. As an example, Fig. 1 shows how the relays form the standard (7*,* 4) Hamming code in a distributed manner. Each relay corresponds to one column of the generator matrix **Gk***×***n**. When *Gi,j* is 1, relay *j* has to decode the message of source *i*. Since there are multiple 1s in *n − k* columns of **G**, the corresponding relays are responsible for multiple sources. These relays should perform an exclusive-OR operation on the decoded bits from the corresponding sources and then retransmit the result to the destination. Each relay corresponding to one of the *k* remaining columns of **G**, only decodes and forwards the information from one source. If the channels between any relay node and its corresponding source nodes are not sufficiently good to allow successful decoding, this relay node remains silent.

## Distributed Reed-Solomon Code Relaying

RS codes are non-binary cyclic codes where the input symbols and the generator matrix elements are selected from *GF*(*q*) [20]. The RS code is optimal in the sense that its minimum distance is the highest for a linear code of the same size; this is known as the Singleton bound. Furthermore, efficient algorithms are proposed for soft decision decoding of RS codes, such as [21], where a polynomial-time soft decision algebraic list-decoding is presented. These two properties make RS codes a promising candidate for distributed code relaying. As an example, let us consider the RS code with the following generator matrix (which is also used in simulations):

**** (1)

The structure of the scheme is similar to Fig. 1. The only difference is that instead of exclusive-OR operation at the relays, we need to perform summation in the Galois field *GF*(*q*). It will be shown that by using these codes, a more bandwidth efficient code compared to distributed Hamming codes is achieved.

# Diversity Order Analysis

In order to express the end-to-end (E2E) diversity analysis of the proposed scheme, first the following events are defined.

*Am*: *m* relays cannot successfully decode the required messages;

*B*: decoding error happens at the destination.

The E2E average error probability can be expressed in terms of the conditional probabilities

 (2)

Here, in the first scenario, all of the required information symbols are correctly decoded at the relays,, but a wrong codeword is decoded at the destination,. In the second scenario, detection error happens at one or more relays. In this section, the diversity order offered by each term of (2) is analyzed, and then the resulting E2E diversity is expressed.

: Using the model of [22] for DF, if the mutual information between the source *Si* and the relay *Rj* is greater than the spectral efficiency , the relay is able to successfully decode the transmitted signal from this source. Assuming Rayleigh fading channels, the distribution of the corresponding SNR value is exponential and the probability of unsuccessful decoding over *Si*-*Rj* link becomes [22, Eq. (5)]

 (3)

where is the average SNR of the *Si*-*Rj* link. Now, let us consider the set of all *Si*-*Rj* links that contribute to form the distributed codeword. These links correspond to the nonzero elements of **G**. We denote this set and its cardinality byand, respectively. Then, we have

 . (4)

Obviously, as the SNR approaches infinity (), the value of  approaches one and the diversity offered by  is zero.

: Analysis of this scenario is a part of the present literature [23, Section 2.4] [24], [25] where it is proved that diversity *dmin* is achieved at the destination.

: This event means that *m* relays could not decode their required information. To analyze this case, we assume that  is partitioned into two sets, namely  and , where  is the set of the links corresponding to unsuccessful decoding at the relays and  is the set of the links corresponding to successful decoding at the relays. In this case, we have

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As it was mentioned before, does not introduce any diversity to the system, therefore the only contributing terms are with  and for good average SNR values, we obtain

. (5)

To analyze the performance for high average SNR values, we can calculate the Taylor expansion in terms of  around zero, i.e., we replace  with . The result clearly shows that *Am* achieves diversity *m*.

: For this scenario, we express the corresponding diversity as a theorem:

*Theorem 1:* If *m* relays (*m ≤ dmin*) remain silent due to unsuccessful decoding, the diversity order at the destination drops to *dmin − m*.

*Proof:* The proof is presented in Appendix A.

Now, the E2E diversity of the proposed scheme can be analyzed. Here, the E2E expression for *PE*2*E* has been rewritten where the contribution of each term to the diversity order is already calculated and is written below it.

*.* (6)

Since the slope of the multiplication of two curves is the sum of their slopes and the minimum diversity in (6) is dominant, the E2E diversity turns out to be *m* + *dmin − m* = *dmin*.

# Extension to Distributed Convolutional Code Relaying

In this section, we introduce a distributed convolutional code based relaying scheme. We should mention that simple extension of the last model (Fig. 1) to convolutional codes fails to achieve a good performance. To explain this fact, let us consider a rate *k/n* convolutional code with free distance *dfree* and the following generator matrix.

*****.* (7)

Since each element of the generator matrix is a polynomial, its implementation requires delayed (buffered) copies of the input signal. The constraint length *ν* specifies the amount of this delay.

For this model, the general analysis in (6) is still valid, but the diversity offered by each term is different. This is because the diversity achieved at the destination is determined by the reduced free distance of the code, and the free distance of the code is proportional to the number of unsuccessfully decoded messages at the relays. It is well known that using soft decision decoding and interleaving, convolutional codes can achieve a diversity equal to their free distance, i.e. Pr{*B|A*0}= *dfree* [23, Section 2.4.2] [24], [25]. The diversity offered by the term Pr{*Am*}is the same as before. However, The diversity corresponding to the term Pr{*B|Am*}is different and is expressed by the following lemma.

*Lemma 1:* If the received signal at the relays achieves diversity *ν*, then the diversity offered by Pr{*B|Am*}is equal to *dfree − mν*.

*Proof:* If *m* relays cannot successfully decode the message of the source nodes at time *t*, then the output of those relays could be affected within the time interval [*t, t*+*ν −*1], because each decoded bit stays in the shift registers of the relays for *ν* time intervals and can affect the construction of up to *ν* coded bits. This clearly means that the distance of the codeword will be decreased by *m×ν* which is valid as far as *mν ≤ dfree*.

Using this result, for the E2E diversity we have

. (8)

The minimum diversity among the above additive terms is dominant. As an example, for the **G** in (7), we have *dfree* = 4, *ν* = 3 and therefore for *m* = 1 we have (*m* + *dfree −ν*) = 2 which is much less than the potential diversity of the underlying code.

To solve this problem, we need to improve the diversity of Pr{*Am*}to compensate for the diversity loss in Pr{*B|Am*}. For this purpose, we propose to use the model shown in Fig. 2 where separate convolutional codes are used to increase the diversity of Pr{*Am*}to *mν*. The free distance of these convolutional codes should be equal to or more than *ν* (which is the constraint length of the distributed convolutional code). For this model, the E2E performance of the scheme is:

*,* (9)



Fig. 2. An example of the proposed distributed convolutional code-based relaying scheme.

where by increasing the diversity at the relays to *ν*, the E2E diversity is increased to *dfree*.

The choice between convolutional codes and linear block codes is rather a choice of trade-off between performance and complexity. By increasing the trace-back length in Viterbi decoding, the complexity of convolutional codes increases exponentially, whereas for linear block codes, it increases linearly. On the other hand, RS codes are rather suitable for applications where errors occur in bursts; this is mainly because if several bits in a symbol are corrupted, they only count as a single error. Thus, for an RS code operating on 8-bit symbols, which is very common because of the prevalence of 8-bit processors, 16 erroneous bits count as two erroneous symbols. However, in our case this is not an issue, because in distributed coding scheme, different parts of the codeword are transmitted through different channels.

# Diversity Multiplexing Analysis

There exists an important tradeoff between spatial multiplexing and transmit diversity gains [26]. To calculate the DMT function, the source transmission rate  (bits/s/Hz) is first parameterized in terms of the transmission SNR as

**, (10)

where *r* is called the multiplexing gain and is asymptotically equivalent to the gradient of the rate or capacity curves. For our proposed distributed linear block codes and RS codes, since there are *k* + *n* orthogonal channels used to transmit *k* information symbols, the average transmission rate is . On the other hand, the diversity gain relationship can be reformulated as

**. (11)

Inserting (10) into (11) and solving w.r.t. *d* yields the DMT expression. The DMT shows the relationship between the reliability of the transmitted data and the spectral efficiency. For our proposed distributed linear block codes and RS codes, we use (2) to express the error probability in terms of and SNR. For this purpose, we need to express Pr{*Am*}and Pr{*B|Am*}in terms of and SNR. Since we already derived Pr{*Am*}in (5), we only need to express Pr{*B|Am*}in terms of and SNR. Note that in (11), we can use the average error probability or outage probability interchangeably, because for good average SNR values in the logarithmic scale, both curves have the same slope (diversity). Thus, the only difference between them is a constant, which becomes zero in (11), i.e.

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Here, *κ* is a constant. Thus, following the same lines as in Appendix A, we have

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For Rayleigh fading channels, *|hi|*2 follows the exponential distribution and the probability density function (PDF) of the sum of independent RVs is the convolution of their RVs. By employing the moment-generating function (MGF) based approach and using [27, Eqs. (5A.72) and (5A.73)] we arrive at the MGF of the intended probability. Then we can use the initial value theorem to calculate the first terms in the Taylor expansion of the result. After some manipulations, we obtain

*,* (12)

where  denotes the exponential equality and 1*/λj* is the average SNR of the *Rj* -*D* link. By plugging (5) and (12) into (2), we obtain

*.* (13)

Note that here we have assumed that all of the *λj*s are distinct. Using [27, Eqs. (5A.72) and (5A.73)], the same result can be achieved for the case when some of the channels have the same average SNR. This result clearly shows that if the average SNR of all the channels are scaled by Λ, then the *PE*2*E* (*,* SNR) is scaled by Λ*−dmin*.

For our proposed distributed linear block codes and RS codes, *dmin* is calculated as 3 (Hamming codes) and *n−k*+1, respectively. By inserting (13) into (11) and solving w.r.t. *d*, we arrive at the general DMT expression

**. (14)

For the proposed distributed convolutional coding scheme, let us denote the rate of the parallel convolutional codes that are employed in the *S*-*R* channels by *k*1*/n*1. In order to transmit one bit from each source node, the scheme needs *n*+*kn*1*/k*1 orthogonal channels. Thus, similar to (14), the DMT turns out to be

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The results are depicted in Figure 3, which also shows the DMT of some other cooperative relaying protocols. All protocols are considered in a cooperative configuration where there is no direct channel between the source and the destination. The first one is a simple cooperative system consisting of a single source, a single relay and a single destination. This protocol clearly achieves no diversity (diversity one) and multiplexing gain 0*.*5. The second scheme is the repetition-based relaying where one source node uses [*n/k*] relays to transmit its information to the destination [28, p. 19]. This scheme clearly yields diversity [*n/k*] while the multiplexing gain is 1*/* (1 + [*n/k*]) (because there is one *S*-*R* channel and [*n/k*] *R*-*D* channels and the same data is repeated [*n/k*] times [28]). The third scheme is the one proposed in [19], where a diversity equal to Hamming distance of the RS codes is achieved with a multiplexing gain equal to 1*/n*. Using the model proposed in [19], we can achieve better multiplexing gains by decreasing diversity [19, Eq. (14)]. In this figure, we did not consider the extra overhead of the scheme proposed in [19]. The DMT is not depicted for the proposed distributed convolutional codes and for the ANCC scheme described in Section I, because there is no explicit formula to express their diversity. As for the ANCC scheme, since there is no coordination among the relays, each information symbol is relayed a random number of times and we need a larger number of relays to achieve a target diversity for all of the information symbols. Thus we expect a better DMT for our proposed scheme. Finally, we have depicted the DMT of distributed STBC as a benchmark. However, as we mentioned before, such codes are nearly impractical to implement in underwater acoustic environments for synchronization reasons.



Fig. 3. Diversity-multiplexing analysis of the proposed distributed RS relaying compared to repetition code relaying and simple cooperation where we see the superiority of the proposed scheme.

# Simulation and Discussion

In this section, we illustrate the Monte-Carlo simulation of the proposed schemes and compare that to other existing scenarios. Except for the ANCC scheme [18] and repetition based relaying, the only scheme relevant to our proposed scheme is sequential relaying [29]. In this scheme, the *r*th best *R*-*D* channel is scheduled to help the user with the *r*th poorest *S*-*D* channel. In order to have a relatively fair comparison, we assume that the CSI used for relay assignment in this scheme is “outdated” for the *R*-*D* channels, because we want to compare the results with our scheme which does not use CSI for relay assignment at all. Note that if we assume that the CSI

of the *S*-*R* channels is not available for relay assignment in sequential relaying, the performance of this scenario reduces to that of partial relay selection [30]. We denote by  the circularly symmetric complex Gaussian channel gain between source *Si* and relay *Rj* and we denote by the partially known channel corresponding to  at the time of relay assignment. The outdated CSI for the channel is modeled as  [31], [32], where *w* is a circularly symmetric complex Gaussian random variable (RV) having the same variance as and *ρ* is a fixed parameter specifying the correlation coefficient between and .

First, consider the network configuration of Fig. 1 for the (7*,* 4) Hamming code. We assume that the channels in each hop (*S*-*R* and *R*-*D*) are i.i.d. Rayleigh fading channels. We compare the performance of the proposed scheme with sequential relaying when relay assignment is based on the “outdated” CSI of *R*-*D* channels. Fig. 4 shows the Monte-Carlo simulation of *PE* versus *Eb/N*0 (the energy per bit to noise power spectral density ratio). As it is seen in this figure, our proposed scheme achieves diversity *dmin* = 3. This figure obviously shows that the outdated CSI significantly affects the performance of sequential relaying [33]. This degradation in the performance is both in the diversity order and the coding gain, i.e., by selecting any value for *ρ* other than one, the achieved diversity order drops to one. For low average SNR values, our proposed scheme has a lower performance compared to sequential relaying, which is a result of employing the channel code in our scheme.



Fig. 4. Monte-Carlo simulation of the average error probability of the proposed distributed linear block code relaying versus the normalized average SNR of the *S*-*R* and *R*-*D* links

In the next simulation, we check the sensitivity of the proposed scheme (same network as in the last simulation) against different average SNR values at the *S*-*R* and the *R-D* links. Fig. 5 shows the results. Since Rayleigh flat fading is assumed for all of the underlying links, the SNR follows

exponential distribution, i.e., *f*Γ(*γ*) = *λ* exp(*−λγ*). Let us denote the average SNR value for the first and the second hop by 1*/λ*1 and 1*/λ*2, respectively. This simulation shows that the different average SNR values in the first and the second hop results in a degradation of the performance; however, the proposed scheme retains the same diversity order. Also we can see that the coding gain for *λ*1 = 0*.*5*λ*2 is better than the coding gain for *λ*2 = 0*.*5*λ*1 and *λ*2 = *λ*1. This fact shows that the sensitivity of this scheme to the SNR values in the first hop is more than that of the second hop. This result can be explained based on the discussion in Section III.



Fig. 5. Monte-Carlo simulation of the average error probability when the*S*-*R* and the *R*-*D* channels have different average SNR values

Next, we consider the performance of (7*,* 3) distributed RS code relaying. Matrix **G** for this code can be obtained by writing down all polynomials of degree 2 with coefficients over *GF*(8), then evaluating them at the nonzero elements in the field. This matrix is presented in (1). We compare the performance with the outdated CSI based sequential relaying and repetition-based relaying. For both scenarios, the transmitted symbols are selected from *GF*(8). Fig. 6 shows the Monte-Carlo simulation of the symbol-error-rate (SER) versus *Es/N*0 (the energy per symbol to noise power spectral density ratio). It is seen that our proposed scheme achieves diversity 5 which is far better compared to all other schemes. Another interesting observation is the superiority of the proposed scheme, even for low average SNR values. This superiority is due to the inherent performance superiority of RS codes.



Fig. 6. Monte-Carlo simulation of the average error probability of the distributed RS codes compared to sequential relaying with outdated CSI of the second hop

In the last figure, we simulate the performance of the proposed distributed convolutional code relaying corresponding to **G**in (7) and also we check its sensitivity against different average SNR values for the *S*-*R* and the *R*-*D* links. Fig. 7 shows the results, where we can see that increasing the imbalance of the average SNR values in the *S*-*R* and *R*-*D* channels results in a degradation of the performance; however, the proposed scheme retains the same diversity order. Also we can see that the coding gain for *λ*1 = 0*.*5*λ*2 is better than the coding gain for *λ*2 = 0*.*5*λ*1. Again, this fact shows that the sensitivity of this scheme to the *S*-*R* SNR values is more than that to the *R*-*D* SNR values.



Fig. 7. Monte-Carlo simulation of the average error probability for distributed convolutional code relaying when the *S*-*R* and the *R*-*D* channels have different average SNR values

# Conclusion

In this paper, novel schemes have been proposed to achieve diversity and spatial multiplexing in a relay network consisting of *k* sources, *n* relays and a single destination. The proposed schemes are based on distributed implementation of linear block codes or convolutional codes. The proposed schemes achieve diversity without using any CSI for relay assignment, and do not need strict inter-user synchronization unlike some distributed space-time schemes. These properties alongside with their higher DMT compared to rival scenarios, make them suitable for many applications such as underwater acoustic communications. Using the proposed schemes, all source nodes achieve the same diversity. We proposed distributed linear block codes where each relay implements a column of the generator matrix of the code and soft decision decoding is employed at the destination. We expressed the E2E performance of this scheme in terms of the number of unsuccessful decoded messages at relays. This analysis provided an insight to the performance of the proposed scheme and helped to extend the proposed scheme to distributed convolutional codes. We proposed a new model for distributed convolutional codes where parallel convolutional codes were employed at each source to provide the required diversity at the relays and achieve the desired E2E performance.

# discussion

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Appendix A

PROOF OF THEOREM 1

We assume that there are *m* silent relays among *n* relays. We assume that coded bits are transmitted equivalently as *−*1 (for 0) and as 1 (for 1). This assumption normalizes the energy per coded bit to unity. Without loss of generality, we assume that the codeword without error was **c**0 = [*−*1*,−*1*, . . . ,−*1*, cdmin*+1*, . . . , cn*], and the decoded signal at the relays is

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where the first *m* bits are not correctly decoded. Then we have [23], [25]. In general, *PE* is very difficult to compute; therefore using the union bound, we can upper bound this expression by a simpler expression , where is the pairwise error probability (PEP) of receiving a codeword “closer” to the incorrect codeword **c**1 when **c**0 is transmitted. The received codeword at the destination is

*****.*

Here, **h** = [*h*1*, h*2*, . . . , hn*] and **w** = [*w*1*, w*2*, . . ., wn*] denote the complex gains of the *R*-*D* channels and the corresponding noise in each channel, respectively. The ML-decoder at the

destination minimizes the Euclidean distance between the received signal **r** and the vector **c***m* = [*h*1*u*1*, h*2*u*2*, . . . , hnun*] over all the codewords **u**. Let us assume that the codeword

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is a valid codeword. Then the error is described by *Distance* (**r***,* **c**0 *◦* **h**) *≥ Distance* (**r***,* **c**1 *◦* **h**), where *◦* denotes the elementwise multiplication. It means that the Euclidean distance between the received signal **r** and **c**0 is larger than the distance between **r** and **c**1. This implies that

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where *Eα*(*a, b*) denotes norm-α Euclidean distance of *a* and *b* and  denotes the real part. Simplifying the above expression is difficult for general values of *α*. Assuming *α* = 2 and simplifying the above inequality yields

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where the expectation is over the fading coefficients. Let us denote by *ρcb* the average SNR per coded bit. By plugging the PDF of the instantaneous SNR we have

*.* (15)

In order to proceed, we use [27, Eq. (9.11)] for Rayleigh fading channels

*,* (16)

where (*s*) is the MGF of the SNR distribution and *g* is a fixed number specifying the modulation type. For Rayleigh fading, we have. Plugging this equation into (16) and using the same approach as in [27, Eqs. (5A.76) and (5A.77)], after some manipulations we obtain

*.* (17)

The diversity order offered by (17) is obviously *dmin − m*, because it is proportional to the inverse of every where *j* = 1*, . . . , dmin − m*.

References

As in the original paper.