**Outage Probability of Cognitive Relay Networks with Interference Constraints**

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**Summary**

The authors evaluate the outage probability of cognitive relay networks with cooperation between secondary users based on the underlay approach, while adhering to the interference constraint on the primary user. A relay selection criterion, suitable for cognitive relay networks, is provided, and using it, they derive the outage probability.

It is shown that the outage probability of cognitive relay networks is higher than that of conventional relay networks due to the interference constraint. In addition, the outage probability is affected by the distance ratio of the interference link (between the secondary transmitter and the primary receiver) to the relaying link (between the secondary transmitter and the secondary receiver). They also prove that cognitive relay networks achieve the same full selection diversity order as conventional relay networks, and that the decrease in outage probability achieved by increasing the selection diversity (the number of relays) is not less than that in conventional relay networks.

**I. Introduction**

In cognitive radio networks, unlicensed users (secondary users) are permitted to use the licensed band so long as they protect the data transmission of the licensed user (primary user) using spectrum underlay, overlay and interweave approaches [1].

* *Underlay* approach: The secondary user is allowed to use the spectrum of the primary user only when the interference from the secondary user is less than the interference level which the primary user can tolerate. Hence, the transmission power of the secondary user is constrained not to exceed the interference level.
* *Overlay* approach: The secondary user uses the same spectrum concurrently with the primary user while maintaining or improving the transmission of the primary user by applying sophisticated signal processing and coding.
* *Interweave* approach: The secondary user utilizes the spectrum not currently being used by the primary user, known as a *spectrum hole*, after performing detection on the spectrum.

Relay networks have been proposed as a way to enhance the total throughput and coverage of wireless networks [2]. The advantage of relay networks lies in reducing the overall path loss achieved by using a relay between a source and a destination. Inspired by cognitive radio and relay networks, *cognitive relay networks (CRN)* have recently been investigated as a potential way to improve secondary user throughput using one of two approaches: cooperation between primary and secondary users [3], and cooperation between secondary users [4]-[7].

For cooperation between secondary users, approximate [4] and exact [5] outage probabilities of cognitive relay networks have been presented considering the impact of the spectrum sensing accuracy in overlay coexistence. Also, it is shown that full diversity cannot be achieved under imperfect spectrum sensing [4]. Decentralized schemes for transmit power allocation for secondary relays have been proposed to minimize the overall transmit power or to maximize the received signal to noise-interference ratio (SINR) in [6]. In addition, the performance of cognitive relay networks in licensed and unlicensed bands is compared to that of conventional relay networks [7]. Although the location of the relay and appropriate relay selection are important [8], no prior work has considered *an appropriate relay selection procedure* for cognitive relay networks.

The goal of this paper is to evaluate the performance of cognitive relay networks using cooperation between secondary users based on the *underlay* approach while adhering to the interference constraint on the primary user. It first provides a relay selection criterion suitable for cognitive relay networks and then derives the outage probability of cognitive relay networks.

The main contributions are to show that:

1. The outage performance of cognitive relay networks is affected by the distance ratio of the interference link (from the secondary transmitter to the primary receiver) to the relaying link (between the secondary transmitter and the secondary receiver);
2. The outage probability of cognitive relay networks consists of (or can be divided into) the conventional relay network outage probability and the increase in outage probability resulting from the interference constraint;
3. Cognitive relay networks achieve the same relay selection diversity order as conventional relay networks, but the decrease in outage probability of cognitive relay networks achieved by increasing the selection diversity is equal to or greater than that in conventional relay networks.

**II. System Model**

A primary user coexists with secondary users as shown in Fig. 1. In the figure, *PURX*, *SUS*, and *SUD* represent a primary receiver, a secondary source, and a secondary destination, respectively. In addition,

there are *K* potential secondary user relays which are denoted by *SUk* (*k* = 1*, … ,K*). The relay mode is regenerative mode, so a relay decodes the received data and then forwards it to a secondary destination. Let **M** = *{S, D,* 1*,* 2*, … , K}* be the set of secondary user indexes. We assume that a primary receiver occupies multiple primary spectra (frequency channels) spaced with greater frequency separation than coherence bandwidth, and each relay uses different frequency channel among the primary spectra. On the *i*th frequency channel used by *SUi*, the instantaneous channel of the link between *SUj* and *SUi*, the relaying link in Fig. 1, is represented by *hij*. The channels of the links from *SUS* and *SUi* to *PURX*, the interference links in Fig. 1, are denoted by *hSiP* and *hiP*, respectively. The channels are assumed to consist of path loss and an independent fading effect as where αand *dij* denote the pathloss exponent and the distance between two users, respectively. The fading coefficient, 𝜒, is a complex Gaussian random variable with unit variance. Hence, the channel gain is an exponential distributed random variable with the mean value 1*/*𝜆*𝑖𝑗* , and the average channel power is defined as where *E*[.] denotes expectation. Since the link distance of *hSiP* is *dSP* for all *i* ∈ **M**, 𝜆*SiP* is the same as 𝜆*SP* . The perfect channel information such as at the secondary users is also assumed.



Fig. 1. System model of cognitive relay networks

In the underlay approach of this paper, the secondary user can utilize the primary user’s spectrum so long as the interference it generates on the primary receiver remains below the interference threshold (), which is the maximum tolerable interference level at which the primary user can still maintain reliable communication [1]. For this reason, the secondary user’s transmission power is constrained as , where *Pk* is the transmission power of *SUk*; this constraint is called *the interference constraint*. In addition, there is also *the maximum transmission power constraint*, where is the maximum transmission power [6]. By those two constraints, the transmission power constraint of *SUk* becomes as follows [6], [9]:

 (1)

If the secondary source transmits data with the help of the *k*th potential relay in dual-hop, known to be the most efficient multi-hop transmission with respect to system capacity [8], the capacity of the secondary user based on a unit bandwidth becomes as follows [8]:

 (2)

In (2), 1/2 is from the dual-hop transmission in two time slots and *N*0is the noise variance. In addition, any interference from the primary transmitter is assumed to be negligible.

**III. Outage Analysis of Cognitive Relay Networks**

This section presents an appropriate relay selection criterion for cognitive relay networks. It also analyzes the relay transmission performance in terms of the outage probability, and compares it to that of conventional relay networks.

1. **Cognitive Relay Networks**

A variety of criteria for relay selection have been proposed as a way of maximizing capacity or minimizing outage probability in conventional relay networks [15]-[17]. The *max-min* criterion, which maximizes the minimum of signal-to-noise ratios (SNRs) of the source-relay link and relay-destination link, has proven optimal for this purpose [16]. Hence, a relay selection criterion in conventional relay networks is defined as follows [16], [17]:

 (3)

Where *l*\* is the selected relay index in conventional relay networks. Thus, the relay is chosen based on the channel

gains of the relaying links. Hence, under the assumption that all users use the maximum transmission power, the outage probability based on the *k*th relay is defined as where and *CT*

is the target bandwidth efficiency [16]. Hence, the outage probability in conventional relay networks when the relay

selection is applied becomes the following:

 (4)

From (4), we can see that is solely a function of the distances of relaying links. Moreover, minimizing the outage probability is equal to minimizing , and the outage probability has a symmetric form per and . Since the selection criterion in (3) is to minimize the outage probability, we can deduce from (4) that the relay located at exactly the midpoint between the source and destination would be the one most likely to be selected in average.

In contrast to conventional relay networks, an additional transmission power constraint, *the interference constraint*,

exists in cognitive relay networks, making the conventional relay selection criterion inappropriate. For instance, a relay which has good channel conditions on its source-relay and relay-destination links is likely to be selected in conventional relay networks, but may be an inappropriate relay in cognitive relay networks if the relay generates a lot of interference at the primary receiver. Hence, as a *max-min* criterion for maximizing relay transmission performance, the relay selection criterion must be redefined while considering both the interference constraint and the maximum power constraint as follows:

 (5)

where *k\** is the relay index selected for cognitive relay networks.

***Lemma 1:***

Using the outage probability of conventional relay networks, , the outage probability for cognitive relay networks after applying the relay selection described in (5) can be defined as follows:

 (6)

where  is the increase in outage probability due to the interference constraint. The outage probability in cognitive relay networks is defined as , and presented as,

 (7)

From Lemma 1, we can see that the outage probability when the *k*th relay is used in cognitive relay networks consists of the outage probability in conventional relay networks, and an increase due to the interference constraint, . becomes zero as goes to zero. Moreover, if goes to infinity then becomes

zero as goes to infinity, and one as goes to zero. Hence, has a range of 0 *≤*  *≤* 1. Therefore, it can be said that the outage probability in cognitive relay networks is always equal to or greater than that in conventional relay networks.

Moreover, the outage probability depends not on the distances of the interference links, but on the distance ratio of the interference link to the relaying link. This means that the relative distance of the interference link based on the relaying link distance has more of an impact on the outage probability than the absolute distance of the interference link.

1. **Comparison of Relay Selection Diversity**

This subsection compares the relay selection diversity in cognitive relay networks to that in conventional relay networks.

For simplicity, it is assumed that each link has an equal mean of channel gain, ,, and for all *k* [17], [18], since this assumption provides a simple form for the analysis without changing the diversity order. With this assumption, and become equal for all *k* as and . Thus, the outage probabilities in conventional relay networks and cognitive relay networks become and . Using the Binomial theorem,  can be redefined as follows,

 (8)

where

 (9)

Hence, we can see that the outage probability of cognitive relay networks with relay selection is divided into the outage probability in conventional relay networks, , and the increased outage probability arising from the interference constraint, .

***Lemma 2:***

Cognitive relay networks achieve the full relay selection diversity order of *K*.

Note that this is the same as the selection diversity order in conventional relay networks [18]. Hence, with full selection diversity order, the outage probabilities for both cognitive and conventional relay networks decrease as the number of relays increases. However, the reduction in the amount of outage probability achieved by adding a relay in cognitive relay networks is different from what occurs in conventional relay networks. We designate this *the diversity gain*. In the generally considered range of the outage probability (i.e., ), the following lemma on the diversity gain can be obtained.

***Lemma 3:***

The diversity gain in cognitive relay networks is always equal to or greater than that in conventional relay networks when .

From Lemma 2 and Lemma 3, it can be seen that cognitive relay networks have the same relay selection diversity

order as conventional relay networks, but that the decrease in outage probability achieved by increasing the selection

diversity is larger. Hence, *the outage probability of cognitive relay networks, generally larger than that of conventional relay networks, approaches that of conventional relay networks as the number of relays increases.*

**IV. Numerical Results**

In this section, the performance of cognitive relay networks is examined based on the outage probability. Simulations were conducted to verify the derived outage probabilities in (7) and (8), and the results closely match the analysis, as shown in Figs. 3-5.

Figure 2 shows the distribution of probabilities that a relay in a specific location will be selected in cognitive relay and conventional relay networks. To obtain more precise distributions, 400 potential relays are located equidistantly (marked with ‘x’ in Fig. 2). The numbers on the contour lines represent the probability that a specific relay will be selected, and the relay that is most likely to be selected is indicated by a square. In conventional relay networks, the relay located exactly at the midpoint between source and destination (near the coordinates (0*.*5*,* 0*.*5)) is the one most likely to be selected. However, in cognitive relay networks, the relay located farthest from the primary receiver (near the coordinates (0*.*35*,* 0*.*35)), is the one most apt to be selected. Accordingly, the conventional relay selection

criterion is not suitable in cognitive relay networks.



Fig. 2. Percentages of relay selection in conventional relay networks (dotted line), and in cognitive relay networks (solid line)

Figure 3 depicts the outage performances versus. For this simulation, we vary the interference threshold while is fixed, so that the outage performance of the conventional relay network is not affected by the variation of . In Fig. 3, if we compare the results when the primary receiver’s locations are (1, 1) (dotted lines) and (1*.*5, 1*.*5) (solid lines), we see that the outage performance of cognitive relay networks improves when the primary receiver is located farther away from the secondary users. In addition, the outage probability of cognitive relay networks approaches that of conventional relay networks as the interference threshold increases. In general, we can see that the relaying transmission achieves better outage performance than direct transmission. However, direct transmission can outperform relay transmission when the primary receiver is located close to the secondary users

or the interference threshold is low, as shown in Fig. 3. This results from the fact that, in those cases, the loss from using twice the resources (e.g., 1*/*2 term in (2)) is more dominant than the improved capacity achieved via relay transmission. In addition, we note that the relay transmission is more sensitive to any variation in the interference threshold or the location of the primary receiver than direct transmission.



Fig. 3. Comparison of outage probabilities of cognitive relay networks (Cog-Relay), conventional relay networks (Conv-Relay), and direct transmission in cognitive radio networks (Cog-Direct) according to the interference threshold compared to the maximum transmission power, , and 20 relays distributed in a rectangular area.

Figures 4-6 are to show the diversity order and the gain when the primary receiver is located at the coordinates (1*.*5*,* 1*.*5), and each channel’s mean is assumed to be equal, , , and for all *k*, as described in Section III.B.

In Fig. 4, the curves obtained by are included as references to explicitly verify the diversity order. Comparing the slopes of the results confirms that cognitive relay networks indeed achieve full diversity order equivalent to conventional relay networks. However, due to *the transmission power constraint* in (1), the outage probability of cognitive relay networks converges to a constant when is high, e.g., in the area Ain Fig. 4. In this case, the transmission power is with high probability. So, the diversity order needs to be verified according to as shown in Fig. 5, which shows the outage probability when is high, i.e., . In Fig. 5, the same slopes of the results also confirm the full diversity order of cognitive relay networks. We can thus see that the

interference constraint in cognitive relay networks does not change the diversity order.



Fig. 4. Comparison of outage probabilities of cognitive relay networks (Cog-Relay) and conventional relay networks (Conv-Relay) according to the ratio of the maximum transmission power to the noise, .



Fig. 5. Outage probability of cognitive relay networks (Cog-Relay) according to the ratio of the interference threshold to the noise, .

In Fig. 6, both the outage probabilities of cognitive relay networks and conventional relay networks decrease and become similar to each other as the number of relays increases. On comparing and , we can see that the diversity gain achieved by increasing the number of relays is greater in cognitive relay networks than in conventional relay networks, as is discussed in Section III.B. For instance, if the number of relays is increased from 3 to 4, the outage probability reduction amounts are 0.05 in cognitive relay networks and 0.01 in conventional relay networks. We can therefore achieve a greater reduction in outage probability in cognitive relay networks than in conventional relay networks by providing additional relays.



Fig. 6. Comparison of outage probabilities of cognitive relay networks (Cog-Relay), conventional relay networks (Conv-Relay), and direct transmission in cognitive radio networks (Cog-Direct) according to the number of potential relays, *K*. 

**V. Conclusions**

The overall contribution of this paper is the evaluation of the outage probability of cognitive relay networks when a suitable relay selection criterion is applied. As opposed to conventional relay networks, the outage probability in cognitive relay networks is affected by the distance-ratio of the interference link to the relaying link, and not the absolute distances. Moreover, it is always higher than that of conventional relay networks due to the interference constraint, and the difference between them, depending on the interference threshold and the maximum transmission power, is quantified. The outage probability of cognitive relay networks decreases with full selection diversity order the same as conventional relay networks, but the decrease in outage probability achieved by increasing the number

of relays is greater than that of conventional relay networks. These observations give quantitative insight into the effect of the interference constraint and the number of potential relays on the outage probability of cognitive relay networks.

**References**

\*The numbers shown in this summary are the same as the original paper reference numbers. Please refer to the paper for references.