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Compressive Sensing for Spread Spectrum Receivers

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Short summary: Compressive sensing enables the receiver to sample below the Shannon-Nyquist sampling rate, which may lead to a decrease power efficiency and production cost. This paper investigates the use of CS in a general Code Division Multiple Access (CDMA) receiver. Furthermore, they numerically evaluate the proposed receiver in terms of bit error rate under different signal to noise ratio conditions and compare it with other receiver structures.

I. INTRODUCTION

- As wireless communication devices are becoming more and more widespread and ubiquitous, the need for power efficiency and low production cost becomes paramount.
- Recently, a new concept termed CS has been attracting more and more attention in the signal processing community. If a signal is sparse in some arbitrary basis, it may be sampled at a rate lower than the Nyquist frequency.
- In the spread spectrum area, some researchers have studied the general use of CS for spread spectrum communication systems.
- In their work they apply CS to a general CDMA system. And they show that a random demodulation implementation may be used to subsample the CDMA signal, but they also develop a simplified version of the RD which performs equally well for CDMA signals but is simpler and cheaper to implement.

II. SIGNAL MODEL

Each slot contains an independent CDMA signal and the slots decoded sequentially and independently of each other.

For one slot, define a discrete QPSK baseband signal, $\mathbf{x} \in \mathbb{C}^{N\times 1}$ as:

$$\mathbf{x} = \mathbf{\Psi}\boldsymbol{\alpha} \tag{1}$$

where $\mathbf{\Psi} \in S_{\Psi} \subset \{\pm 1\}^{N\times N}$ is an orthogonal or near orthogonal dictionary, containing spreading waveforms for transmission, S_{Ψ} is the subset of $\{\pm 1\}^{N\times N}$ that contains orthogonal or near-orthogonal dictionaries and $\boldsymbol{\alpha} \in \{\pm 1 \pm j, 0\}^{N\times 1}$ is a sparse vector, that selects which spreading waveform(s) and what QPSK constellation point(s) to send.

Each node has a unique CDMA sequence assigned, which it uses to transfer information and each node does not know which neighbors it has, but it knows all possible CDMA sequences. Note that in this signal model $\boldsymbol{\alpha}$ is defined so that all users have identical amplitude.

In cases where the number of active nodes or users in a network is smaller than the total number of possible users, the vector $\boldsymbol{\alpha}$ may be assumed sparse, which is the enabling factor for CS.

At the receiver the following signal is observed:

$$\mathbf{y} = \mathbf{\Theta}(\mathbf{x} + \mathbf{w}) = \mathbf{\Theta}\mathbf{\Psi}\boldsymbol{\alpha} + \mathbf{\Theta}\mathbf{w} \tag{2}$$

Where $\mathbf{\Theta}$ is a measurement matrix, which we shall treat later, and $\mathbf{w} \in \mathbb{C}^{N\times 1}$ is Additive White Gaussian Noise (AWGN). Notice here that we take into account noise folding as the noise is folded down into the compressed domain together with the signal. This makes the noise has an impact on the demodulation performance, because each time the sampling rate is reduced by one half, the Signal to Noise Ratio (SNR) is decreased by $3dB$.

A. Spread Spectrum Dictionary of Gold Sequences

In spread spectrum signals, a possible dictionary $\mathbf{\Psi}$ is a set of Gold sequences, as used in e.g. GPS technology. A set of Gold sequences is a special dictionary of binary sequences with very low auto and cross-correlation properties.

When using such a CDMA dictionary, the received signal must be sampled at a rate corresponding to the chip rate, where a chip is one entry in the received Gold sequences. If $\boldsymbol{\alpha}$ is sparse the information rate of the signal is much lower and it may be possible to decrease the sampling rate by using CS.

III. Compressive Sensing

CS is novel sampling scheme, developed to lower the number of samples required to obtain some desired signal.

Denote by $\mathbf{\Theta}_{\kappa} \in \mathbb{R}^{M \times N}$ a CS measurement matrix, where $\kappa \in \mathbb{N}$ is the subsampling ratio when compared to the Nyquist rate and $M = N / \kappa$. This measurement matrix is then responsible for mapping the N-dimensional signal $\mathbf{x}$ to a M-dimensional signal $\mathbf{y}$. Normally this would make it impossible to recover the original signal, but under the assumption that $\mathbf{x}$ is sparse in some basis, it is possible to reconstruct the original signal from the sampled, M-dimensional signal $\mathbf{y}$.

A. Compressive Spread Spectrum Measurement Matrix

In most CS literature a choice of measurement matrix or structure must be made. The Random Demodulator (RD) sampling structure is one of the most well-known measurement matrix structures developed, which is well suited for practical implementation. In the RD a Pseudo-Random Noise (PRN) sequence is mixed with the received signal. Because a spread spectrum transmitter has already spread the signal before transmission, we show that the RD structure can be improved so that the mixing with a PRN sequence at the receiver may be skipped.

The proposed measurement matrix may therefore be defined similarly to the definition of the RD matrix in “Beyond Nyquist : Efficient Sampling of Sparse Bandlimited Signals”.

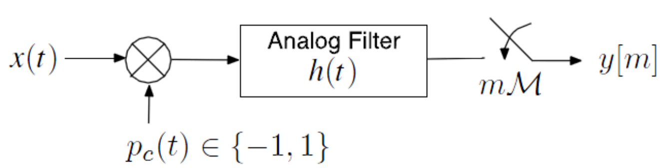

Figure 1 Pseudo-random demodulation scheme

In their work, the measurement matrix is based on two matrices, $\mathbf{D}$ and $\mathbf{H}$. First, let $\varepsilon_0, \varepsilon_1, \cdots, \varepsilon_N \in \{\pm 1\}$ be the chipping sequence used in the RD for a signal of length N. The mapping $\mathbf{x} \rightarrow \mathbf{Dx}$ signifies the demodulation mapping with the chipping sequence, where $\mathbf{D}$ is the diagonal matrix:

$$\mathbf{D} = \begin{bmatrix} \varepsilon_0 & & & \\ & \varepsilon_1 & & \\ & & \ddots & \\ & & & \varepsilon_N \end{bmatrix}$$

Second, the $\mathbf{H}$ matrix denotes the accumulate-and-dump action performed after mixing. Let M denote the number of samples taken. Then each sample is the sum of N/M consecutive entries of the demodulated signal. An example with $M = 3$ and $N = 6$ is :

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & & & & \\ & & 1 & 1 & & \\ & & & & 1 & 1 \end{bmatrix}$$

The reason for applying a chipping sequence is to spread the signal across the frequency spectrum, so that information is aliased down into the lower frequency area, which is left untouched by the low-pass filtering.

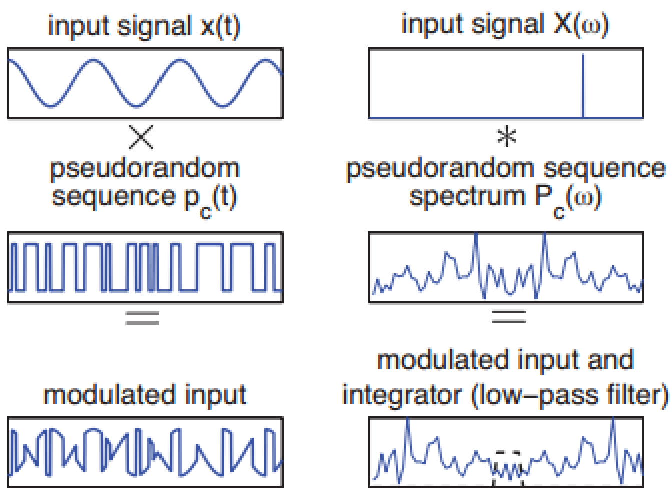

Figure 2 Action of the demodulator on a pure tone. The demodulation process multiplies the continuous-time input signal by a random sequence wave. The action of the system on a single tone is illustrated in the time domain(left) and the frequency domain (right). The dashed line indicates the frequency response of the lowpass filter.

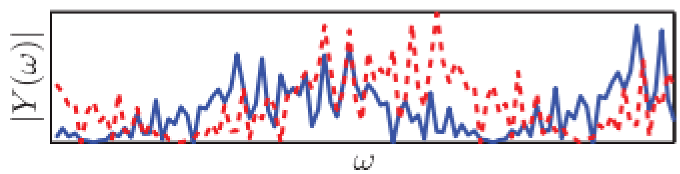

Figure 3 Signatures of two different tones. The random demodulator furnishes each frequency with a unique signature. This image enlarges the filter's passband region of the demodulator's output for two input tones (solid and dashed). The two signatures are nearly orthogonal.

In the proposed receiver this mixing is unnecessary because the signal has already been spread at the transmitter. The proposed receiver may therefore be simplified to:

$$\mathbf{y} = \mathbf{H}\mathbf{x} \tag{3}$$

This is significantly simpler to implement in hardware than the RD. The use of a CDMA dictionary introduces a random-like dictionary matrix, which spreads the signal out so that each sample contains a little bit of the original information signal. Therefore, the sampling process may be rewritten as:

$$\mathbf{y} = \mathbf{H}\mathbf{x} = \mathbf{H}\boldsymbol{\Psi}\boldsymbol{\alpha} = \boldsymbol{\Theta}\boldsymbol{\alpha} \tag{4}$$

Here, the measurement matrix becomes $\boldsymbol{\Theta} = \mathbf{H}\boldsymbol{\Psi}$

B. *Subspace Pursuit*

To reconstruct the signal a reconstruction algorithm must be chosen. Many different approaches have been developed, but two main classes of reconstruction algorithms are in widespread use: l_1 minimization and greedy algorithms. Often, l_1 minimization provides the best solution, but if the matrices $\mathbf{\Psi}$ and $\mathbf{\Theta}$ are very large, it is much more efficient to use the simpler greedy algorithms. Therefore, we choose to use greedy algorithms in this work.

Recall that $\mathbf{\Theta}_\kappa$ is a measurement matrix with N columns and N/κ rows and define $\mathbf{A} = \mathbf{\Theta}_\kappa \mathbf{\Psi}$. Then we define the Subspace Pursuit algorithm as in Algorithm 1. In each algorithm iteration, the pseudo-inverse is calculated as the least-squares solution as this is less computationally demanding.

Algorithm 1 Subspace Pursuit Algorithm [3]
Input:
Sparsity S, measurement and dictionary matrices combined $\mathbf{A}$ and received, sampled signal $\mathbf{y}$
Initialization:
$T^0 = \{$indices of the S largest absolute magnitude entries in the vector $\mathbf{A}^T\mathbf{y}\}$
$\mathbf{y}_r^0 = \mathbf{y} - \mathbf{A}_{T^0}\mathbf{A}_{T^0}^T\mathbf{y}$
Repeat
$l \leftarrow l+1$
$T^l \leftarrow T^{l-1} \cup \{$indices of the S largest absolute magnitude entries in the vector $\mathbf{A}^T\mathbf{y}_r^{l-1}\}$
$T^l \leftarrow \{$indices of the S largest absolute magnitude entries in the vector $\mathbf{A}_{T^l}^\dagger\mathbf{y}\}$
$\mathbf{y}_r^l \leftarrow \mathbf{y} - \mathbf{A}_{T^l}\mathbf{A}_{T^l}^\dagger\mathbf{y}$
Until $\|\mathbf{y}_r^l\|_2 > \|\mathbf{y}_r^{l-1}\|_2$, $l \geq S$

To demonstrate the performance of the Subspace Pursuit algorithm with the Gold dictionary, they have performed numerical experiments to find the phase transition in the noise-less case for various choices of measurement matrices.

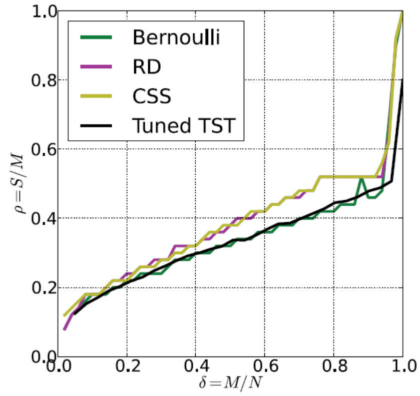

Figure 4 Phase Transition Diagrams for the three different measurement matrices (Rademacher, RD and CSS measurement matrix). The black line is the phase transition line for the Tuned Two Stage Thresholding(TST) algorithm from "Optimally tuned iterative reconstruction algorithms for compressed sensing"

IV. Conclusion

In this work they apply CS to a general CDMA system and they show that it is possible to use a very simple measurement scheme at the receiver side to enable subsampling of the CDMA signal.

V. Discussion

After meeting, please write discussion in the meeting and update your presentation file.

Appendix

Reference

[1] W. Dai and O. Milenkovic, "Subspace pursuit for compressive sensing signal reconstruction," IEEE Trans.Inf. Theory, vol. 55, no. 5, pp. 2230–2249, May 2009.
[2] A. Maleki and D. L. Donoho, "Optimally tuned iterative reconstruction algorithms for compressed sensing," IEEE J. Sel. Topics Signal Process.,vol. 4, no. 2, pp. 330–341, Apr. 2010.
[3] W. Dai and O. Milenkovic, "Subspace pursuit for compressive sensing signal reconstruction," IEEE Trans. Inf. Theory, vol. 55, no. 5, pp. 2230–2249, May 2009.