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A sparse signal reconstruction perspective for source localization with sensor arrays

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Abstract

In this paper, the authors present a source localization method based on sparse representation of sensor measurements. In particular, they use SVD of the data matrix obtained from the sensors to summarize the multiple measurements. The SVD summarized data is then sparsely represented in order to detect the sources. The authors also proposed grid refinement in order to mitigate the effects of limiting estimates to a grid of spatial locations. They demonstrate the superior resolution ability with limited time samples of their method over the existing methods via various experiments.

Introduction and Background

- Source localization methods deal with finding the closely spaced sources in presence of considerable noise.
- Many advanced techniques for the localization of sources achieve super-resolution by exploiting the presence of a small number of sources. For example, the key component of the MUSIC method is the assumption of a low-dimensional signal subspace.
- Estimating the spatial locations (or directions) is a well-known problem in array signal processing.
- Three major source estimation techniques are 1. Classical methods (beamformer, MVDR) 2. Subspace methods (MUSIC, ESPRIT) 3. ML-based methods (deterministic and stochastic).
- Beamforming is simple but its resolution is limited. Subspace methods achieve super resolution, provided SNR is moderately high and sources are not strongly correlated and the number of snapshots (measurement vectors) are sufficient. ML techniques are superior than the subspace methods but require accurate initialization for global convergence.
- By turning to the sparse signal representation framework, the authors are able to achieve super-resolution without the need for a good initialization, without a large number of time samples, and with lower sensitivity to SNR and to correlation of the sources.
- The authors have developed the method for narrowband case and discussed in brief how it can be used for wideband source localization.
- Prior research has established sparse signal representation as a valuable tool for signal processing, but its application to source localization has been developed only for very limited scenarios. For example, [1, 2] is concerned with source localization in the beam-space domain, under the assumption that the sources are uncorrelated, and that a large number of time samples is available.

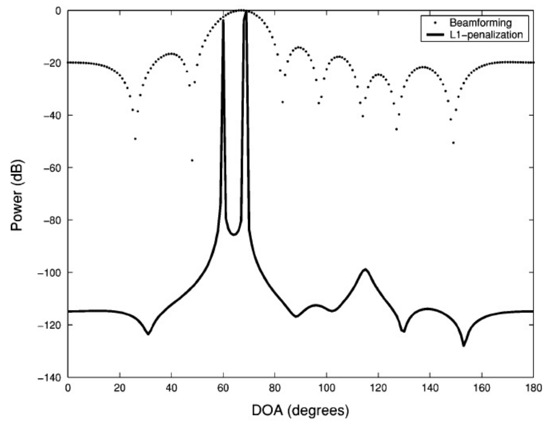


Fig. 1. Single sample source localization with ℓ_1. Spatial spectra of two sources with DOAs of 60° and 70° (SNR = 20 dB).

- In its most basic form, the problem of sparse signal representation in overcomplete bases asks to find the sparsest signal x to satisfy $y = Ax$, where $A \in C^{M \times N}$ is an overcomplete basis, i.e., $M < N$.Without the sparsity prior on x, the problem $y = Ax$ is ill-posed and has infinitely many solutions. Additional information that x should be sufficiently sparse allows one to get rid of the ill-posedness.

Source localization framework

- The goal of the source localization is to find locations of sources of wavefields that impinge on an array of sensors that are seperated by a distance less than or equal to $\lambda/2$
- Consider K narrowband signals $u_k(t), k \in \{1, 2, \cdots, K\}$, arriving at an array of M sensors, after being corrupted by additive noise $n_m(t)$, resulting in sensor outputs $y_m(t), m \in \{1, 2, \cdots, M\}$. After demodulation, the vector form of the received signal is

$$\boldsymbol{y}(t) = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{u}(t) + \boldsymbol{n}(t), \quad t \in \{t_1, \cdots, t_T\} \tag{1}$$

- $\boldsymbol{A}(\boldsymbol{\theta})$ is array manifold matrix. The $(m, k)^{th}$ element $\boldsymbol{A}$ contains the delay and gain information from the kth source (at location θ_k) to the mth sensor. The column, $a(\theta_k)$, of $\boldsymbol{A}$ are called steering vectors and is given by $a(\theta_k) = \left[e^{j\frac{2\pi}{\lambda} 1 \sin\theta_k}, e^{j\frac{2\pi}{\lambda} 2 \sin\theta_k}, \cdots, e^{j\frac{2\pi}{\lambda} M \sin\theta_k} \right]^T$
- Any source localization method aims to find the unknown locations of the sources $\theta_k, \forall k$, given $\boldsymbol{y}(t)$ and $\boldsymbol{A}$.
- We note that finding $\boldsymbol{\theta}$ is a non-linear estimation problem.

Sparse representation for a single time sample, that is, $T = 1$

- To cast a sparse representation problem, the authors introduce an overcomplete representation of $\boldsymbol{A}$ in terms of all possible source locations.
- Let $\{\tilde{\theta}_1, \tilde{\theta}_2, \cdots, \tilde{\theta}_N\}$ be a sampling grid of all source locations of interest.
- The number of potential sources N will typically be much greater than the number of actual sources K and the number of sensors M.

- A matrix composed of steering vectors corresponding to each potential source location as its columns constitute an over-complete dictionary, that is, $\boldsymbol{A} = \left[a(\tilde{\theta}_1), a(\tilde{\theta}_2), \cdots, a(\tilde{\theta}_N)\right]$. We note that $\boldsymbol{A}$ is known and does not depend on the actual source locations.

- The signal vector is $\boldsymbol{s}(t)$ with the nth element $s_n(t) = u_k(t)$ if the source k comes from θ_n for some k and zero otherwise. For $T = 1$, then the source localization problem reduces to

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{n} \tag{2}$$

- In effect, this overcomplete representation allows us to exchange the problem of parameter estimation of $\boldsymbol{\theta}$ for the problem of sparse spectrum estimation of $\boldsymbol{s}$.

- With the key assumption that the source numbers are less, the underlying spatial spectrum is sparse (i.e., has only a few nonzero elements), and hence we can solve this inverse problem via l_1 methodology, $min\|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{s}\|_2^2 + \lambda\|\boldsymbol{s}\|_1$

- The data for the model is complex-valued; hence, neither linear nor quadratic programming can be used for numerical optimization. Instead, the authors adopt an SOC programming framework and find $\boldsymbol{s}$. Once $\boldsymbol{s}$ is found, the estimates of the source locations correspond to the locations of the peaks in $\boldsymbol{s}$.

Source location with multiple time samples and $l_1 - SVD$

- Source localization with multiple snapshots from potentially correlated sources is of greater practical importance.

- When we bring time into the picture, the overcomplete representation is easily extended and it has the following form:

$$\boldsymbol{y}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t), \qquad t \in \{t_1, t_2, \cdots, t_T\} \tag{3}$$

Single and Joint inverse problem

- The first thought that comes to mind when we switch from one time sample to several time samples is to solve each problem indexed by separately. In that case, we would have a set of solutions $\hat{\boldsymbol{s}}(t)$.

- If the sources are moving fast, then the evolution of the sources is of interest, and the approach is suitable for displaying it.

- When the sources are stationary over several time samples, then it is preferable to combine the independent estimates to get one representative estimate of source locations from them, for example, by averaging or by clustering.

- Now, we consider a simple approach that uses different time samples together. Let $\boldsymbol{Y} = [\boldsymbol{y}(t_1), \boldsymbol{y}(t_2), \cdots, \boldsymbol{y}(t_T)]$, and define $\boldsymbol{S}$ and $\boldsymbol{N}$ similarly. Then, we have

$$\boldsymbol{Y} = \boldsymbol{A}\boldsymbol{S} + \boldsymbol{N} \tag{4}$$

- We note that the matrix $\boldsymbol{S}$ is parametrized temporally and spatially, but sparsity only has to be enforced in time not in space.

- To accommodate this issue in the optimization problem, the authors first compute the l_2 norm of all time-samples of a particular space index of s, that is, $\boldsymbol{s}_i^{l_2} = \|[s_i(t_1), s_i(t_2), \cdots, s_i(t_T)]\|_2$.

- Then the authors minimize the l_1 norm of $\boldsymbol{s}^{l_2} = \left[s_1^{l_2}, s_2^{l_2}, \cdots, s_N^{l_2}\right]$. Now the problem becomes

$$min\|\boldsymbol{Y} - \boldsymbol{AS}\|_f^2 + \lambda\|\boldsymbol{s}^{l_2}\|_1 \tag{5}$$

- Note in Eqn. (5), the optimization is performed over the matrix $\boldsymbol{S}$ and once the estimate of $\boldsymbol{S}$ is computed the peaks of $\boldsymbol{S}$ provide the source locations.
- The main drawback of this technique is its computational cost. The size of the inverse problem increases linearly with T, and the computational effort required to solve it increases superlinearly with T. In order to alleviate this, the authors propose a SVD based solution.

l_1- SVD

- To reduce both the computational complexity and the sensitivity to noise, the authors propose to use the SVD of the $M \times T$data matrix $\boldsymbol{Y}$.
- The idea is to decompose the data matrix into the signal and noise subspaces.
- With the signal subspace, mold the problem as multiple-vector sparse spectrum estimation problem similar to Eqn. (4).
- Without noise on the sensors, the set of vectors of $\boldsymbol{Y}$ would lie in a K-dimensional subspace.
- If we can relate the basis of this K-dimensional subspace (set of K vectors) to the source matrix $\boldsymbol{S}$, then we can just keep K vectors (instead of T) for the estimation problem.
- Take the SVD $\boldsymbol{Y} = \boldsymbol{ULV'}$ and form a $M \times K$ dimensional matrix $\boldsymbol{Y}_{sv}$ as $\boldsymbol{Y}_{sv} = \boldsymbol{YVD}_k$, where $\boldsymbol{D}_k$ is an $T \times K$ matrix given as $\boldsymbol{D}_k = [\boldsymbol{I}_K\boldsymbol{0}']$
- Now $\boldsymbol{Y}_{sv}$ can be written as

$$\begin{aligned} \boldsymbol{Y}_{sv} &= \boldsymbol{YVD}_k \\ &= (\boldsymbol{AS} + \boldsymbol{N})\boldsymbol{VD}_k \\ &= \boldsymbol{ASVD}_k + \boldsymbol{NVD}_k \\ &= \boldsymbol{AS}_{sv} + \boldsymbol{N}_{sv} \end{aligned} \tag{6}$$

- We note that the sparsity structure of $\boldsymbol{S}$ is retained in $\boldsymbol{S}_{sv}$.
- Considering the k-th column of Eqn. (6) we have

$$\boldsymbol{y}^{sv}(k) = \boldsymbol{As}^{sv}(k) + \boldsymbol{n}^{sv}(k), \qquad k = 1, 2, \cdots, K \tag{7}$$

  This is exactly the same form as multiple-vector model in Eqn. (3), expect that indexing is by singular vector, k.
- By bringing SVD, the problem size is reduced from T to K. This reduction is substantial, because in typical situations $K \ll T$.
- Now in the matrix $\boldsymbol{S}_{sv}$, the sparsity is along the spatial domain and not in the singular vector domain.
- To accommodate the true sparsity in the minimization problem, the authors define $\tilde{s}_i^{l_2} = \|[s_i^{sv}(1), s_i^{sv}(2), \cdots, s_i^{sv}(K)\|_2$. The sparsity of the $N \times 1$ vector $\tilde{s}_i^{l_2}$ is the sparsity of the spatial spectrum, which can be found by minimizing

$$\|\boldsymbol{Y}_{sv} - \boldsymbol{AS}_{sv}\|_f^2 + \lambda\|\tilde{\boldsymbol{s}}^{l_2}\|_1 \tag{8}$$

- In this paper, the authors have solved the above problem using SOC programming (see paper for details)

Multi-resolution grid refinement

- Thus far, in this paper, the estimates of the source locations are confined to a grid.
- We cannot make the grid very fine uniformly since this would increase the computational complexity and also the columns of $\boldsymbol{A}$ becomes more linearly dependent.
- Hence, the authors explore the idea of adaptively refining the grid in order to achieve better precision
- Instead of having a universally fine grid, we make the grid fine only around the regions where sources are present.
- This requires an approximate knowledge of the locations of the sources, which can be obtained by using a coarse grid first.

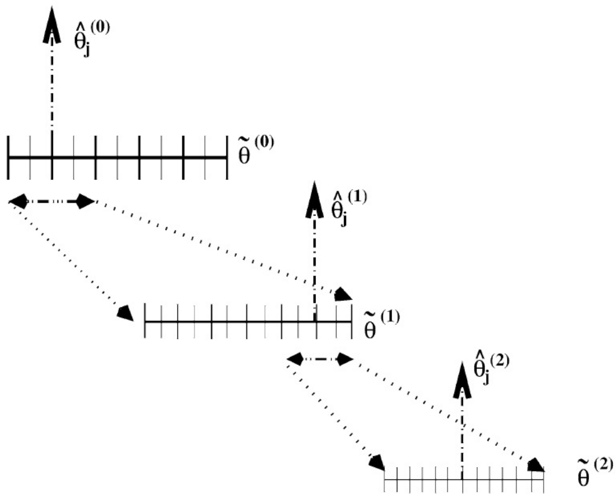


Fig. 3. Illustration of grid refinement.

- The grid refinement algorithm goes like this

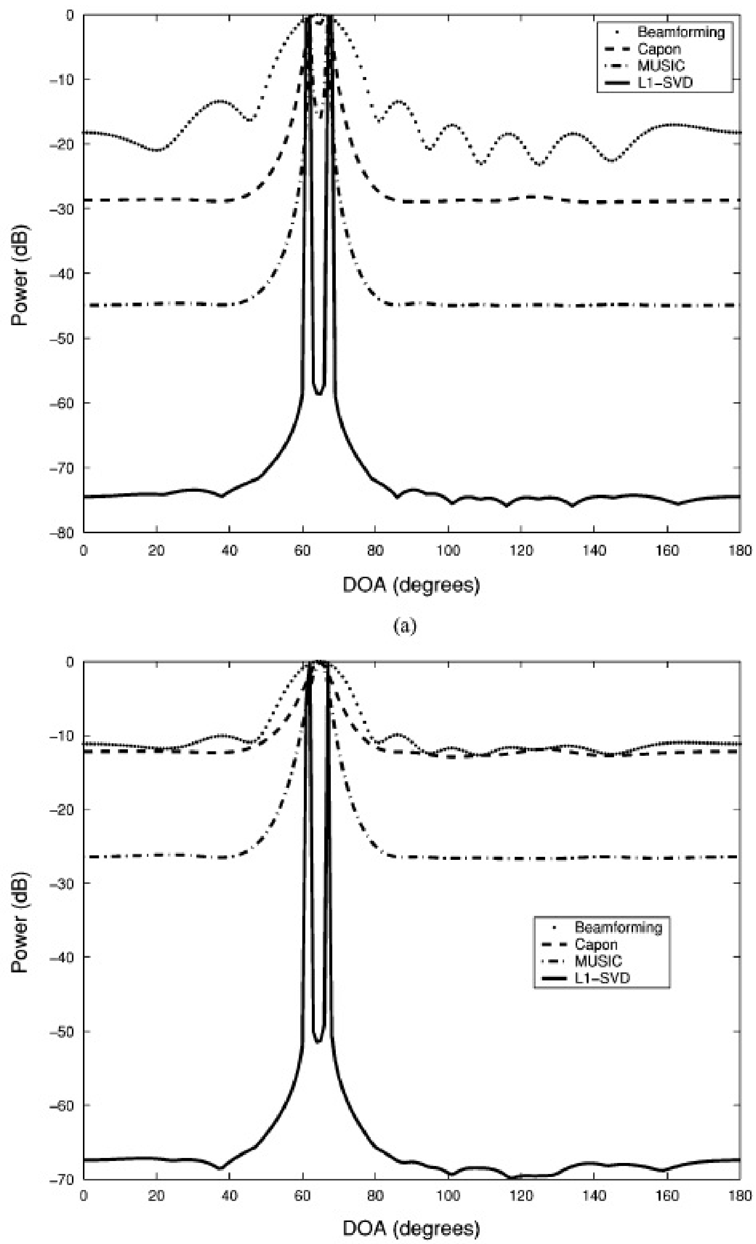
1. Create a rough grid of potential source locations $\tilde{\theta}^{(0)}$, for $i = 1, 2, \cdots, N$. Set $r = 0$.
2. Form $\boldsymbol{A}_r = \boldsymbol{A}(\tilde{\boldsymbol{\theta}}^{(r)})$, where $\tilde{\boldsymbol{\theta}}^{(r)} = \left[\tilde{\theta}_1^{(r)}, \tilde{\theta}_2^{(r)}, \cdots, \tilde{\theta}_N^{(r)}\right]$. Use the SOC minimization to find the estimates of the source locations and set $r = r + 1$.
3. Get a refined grid $\tilde{\boldsymbol{\theta}}^{(r)}$ around the locations of the peak, $\hat{\theta}_j^{(r-1)}$ (explained below).
4. Return to step 2, until the grid is fine enough.

- There are many ways of refining the grid; the authors have chosen a simple equispaced grid refinement.
- Suppose at step r, we have a uniform grid with spacing δ_r. Also, we have an estimate $\hat{\theta}_j^{(r)}$

- Pick an interval around the jth detected source with two grid spacing on either side, that is, $[\hat{\theta}_j^{(r)} - 2\delta_r, \hat{\theta}_j^{(r)} + 2\delta_r]$, for $j = 1, 2, \cdots, K$.
- In the intervals around the peak, select a new grid whose spacing is a fraction of the old one $\delta_{r+1} = \delta_r/\gamma$

Simulation results

- The authors consider $M = 8$ sensors separated by half a wavelength. $K = 2 \quad (62°, 65°), T = 200, N = 180$.



(b)

Fig. 4. (a) and (b). Spatial spectra for beamforming, Capon's method, MUSIC, and the proposed method (ℓ_1-SVD) for uncorrelated sources. DOAs: 62° and 67°. Top: SNR = 10 dB. Bottom: SNR = 0 dB.

- For correlated sources, the result is as follows

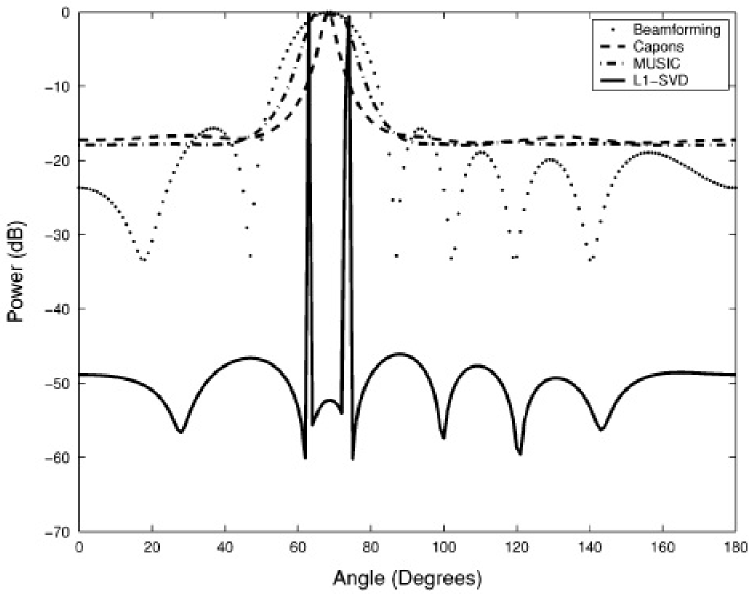


Fig. 5. Spectra for correlated sources. SNR = 20 dB. DOAs: 63° and 73°.

summary

In this paper, the authors have proposed a source location estimation based on sparse representation. The SVD of the sensor measurements summarizes the large chunk of data which is then used as a model for identifying the sources. This method is applicable for both narrow and wideband beamforming. The authors have also presented a grid refinement method in order to obtain fine estimates. The advantages of the proposed method include superior resolution ability with limited time samples for both correlated and uncorrelated sources.

References

[1] J. J. Fuchs, “Linear programming in spectral estimation. Application to array processing,” in: Proc. IEEE Int. Conf. Acoust., Speech, Signal Process., vol. 6, 1996, pp. 3161-3164.

[2] J. J. Fuchs, “On the application of the global matched filter to DOA estimation with uniform circular arrays,” IEEE Trans. Signal Process., vol. 49, no. 4, pp. 702-709, Apr. 2001.