CS Journal Club, Apr. 18, 2013

Aliasing-Free Wideband Beamforming Using Sparse Signal Representation

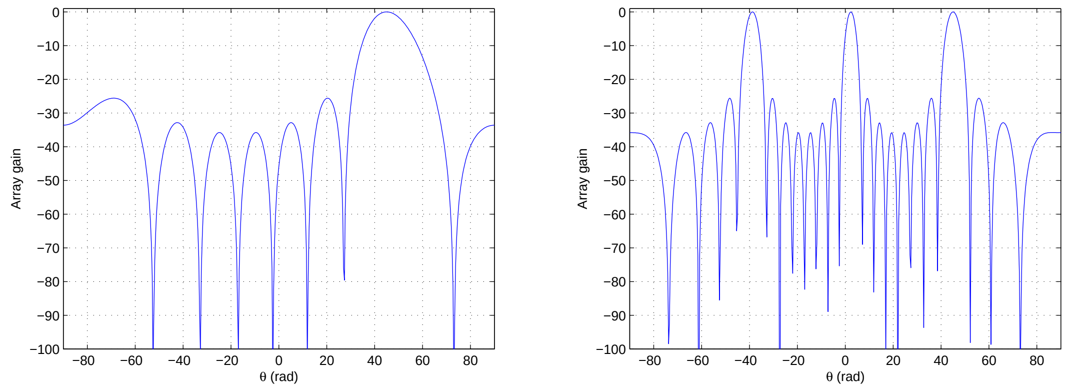
Authors: Zijian Tang, Gerrit Balcquiere, and Geert Leus
Journal: IEEE Trans. on. Sign. Proc. July, 2011.
Presenter: J. Oliver

Abstract

This paper considers the use of sparse signal representation for the wideband direction of arrival (DOA) or angle of arrival estimation problem. In particular, this paper discusses about the two ambiguities, namely, spatial and algebraic aliasing that arise in wideband-DOA. The authors of the paper suggest procedures to avoid the aliasing using multiple measurement vector and multiple dictionaries.

Introduction and Background

- A beamformer is a processor used in conjunction with an array of sensors to provide spatial filtering. The sensor array collects spatial samples of propagating wave fields, which can be processed by the beamformer.
- The objective of a beamformer is to estimate the signal arriving from a desired direction in the presence of noise and interfering signals. A beamformer thus performs spatial filtering to separate signals that have overlapping frequency content but originate from different spatial locations.
- Estimating the spatial locations (or directions) is a well-known problem in array signal processing.
- Three major DOA estimation techniques are 1. Classical methods (Delay-sum beamformer, MVDR) 2. Subspace methods (MUSIC, ESPRIT) 3. ML-based methods
- This paper discusses about beamforming and in particular wide-band beamforming.
- DOA estimation by beamforming can be subjected to ambiguity called spatial aliasing [1].
- Spatial aliasing occurs when the spacing, d, between the sensors is larger than half of the apparent wavelength, that is, $d > \lambda/2$ (See Fig. 1)
- We note from the figures, the resolution increases as d increases, but spatial aliasing also increases.
- This paper discusses how to avoid spatial aliasing (if there is any) in a wideband setting.
- Various authors [2-5] have studied sparse representation (SSR) for narrowband DOA estimation in various contexts. In [2], CS is applied to reduce the ADC sampling rate, in [3,4] it is used to improve angle resolution, in [5] it is used to reduce hardware complexity. All these works assume spatial aliasing is not present.
- However, in SSR based methods aliasing (or ambiguity) comes not only from spatial aliasing, but also from the over-completeness of the dictionary (algebraic aliasing).
- This paper discusses, how to avoid both spatial and algebraic aliasing.



(a) Array tuned to 45 deg with $d = 0.5\lambda$ (b) Array tuned to 45 deg with $d = 1.5\lambda$

Figure 1: Illustration of spatial aliasing

- In summary, the spatial aliasing can be avoided by using multiple dictionaries and the robustness to algebraic alaising can be obtained by using multiple measurement vectors.

Data model

- A uniform linear array (ULA) comprised of N channels indexed by which are equally spaced on a line with spacing d. It receives signals radiated from Q point sources.

- The signal at each channel after time-sampling is partitioned into P segments, where for each segment, K frequency subbands are computed by e.g., a filter bank or the discrete Fourier transform (DFT).

- Let $S_{q,k}(p)$ denote the kth subband (frequency) coefficient computed for the pth segment of the signal that is radiated from the Qth target; similarly, let $y_{n,k}(p)$ denote the kth subband (frequency) coefficient for the pth segment of the signal received at the nth channel.

- With narrow-band assumption, the received signal at the nth sensor at the kth DFT bin is given [1] by

$$y_{n,k}(p) = \sum_{q=0}^{Q-1} e^{j2\pi f_k \frac{d}{c} n \sin\theta_{m_q}} \; S_{q,k}(p) \tag{1}$$

- The aim of this paper is to estimate the target DOAs $\{\theta_0, \theta_1, \cdots, \theta_{Q-1}\}$

- The matrix-vector form of Eqn. (1) is

$$\boldsymbol{y}_{k,p} = \sum_{q=0}^{Q-1} \boldsymbol{a}_{k,m_q} \; S_{q,k}(p) = A_k \boldsymbol{S}_{k,p} \tag{2}$$

where $\boldsymbol{a}_{k,m_q} = \left[1, e^{j2\pi f_k \frac{d}{c} 1 \sin\theta_{m_q}}, \cdots, e^{j2\pi f_k \frac{d}{c}(N-1)\sin\theta_{m_q}}\right]^T$ is called array response vector and A_k steering matrix.

Assumption 1: The array response vectors corresponding to different targets are mutually independent.

Classical beamforming

- Classical beamforming (in this paper, delay-sum beamformer) sets the beamformer coefficients corresponding to a single target angle.
- For example, if the beamformer wants to listen to angle $\theta = 30\,\text{deg}$, then it sets its coefficient vector as $\left[1, e^{j2\pi f_k \frac{d}{c} 1 \sin\frac{\pi}{6}}, \cdots, e^{j2\pi f_k \frac{d}{c}(N-1)\sin\frac{\pi}{6}}\right]^T$ and forms the product $\boldsymbol{a}_{k,m}^H \boldsymbol{y}_{k,p}$
- The angle domain is divided into M points $\Theta = \{\theta_0, \cdots, \theta_{M-1}\}$
- In many applications, such as sonar, a range (time)-bearing(angle) image is desired which can be made by repeating the above procedure for all the subbands; the outputs are then combined and transformed back into the time domain by means of e.g., an inverse Fourier transform.
- In the end, the signal for the pth segment at the mth angle in the range-bearing image $I(p, m)$ can be computed as

$$I(p, m) = \left| \frac{1}{N} \sum_{k=0}^{K-1} \boldsymbol{a}_{k,m}^H \boldsymbol{y}_{q,k}(p) e^{j2\pi f_k p} \right|^2 \tag{3}$$

- $I(p, m)$ can be interpreted as the power of the output of a spatial-temporal filter steered to the direction θ_m. The DOAs are estimated by seeking those θ_m whose corresponding values in $\sum_{p=0}^{P-1} I(p, m)$ are the largest.
- The delay-sum beamformer is subject to spatial aliasing. That is, when the spacing d is larger than apparent wavelength, it is possible to find another $\theta_{m'} \neq \theta_m$ such that for an arbitrary integer j

$$f_k \frac{d}{c} \sin\theta_m = f_k \frac{d}{c} \sin\theta_{m'} + j \tag{4}$$

holds and thus $\boldsymbol{a}_{k,m} = \boldsymbol{a}_{k,m'}$, which gives multiple peaks in the range-bearing image $I(p, m)$.

DOA estimation via SSR

Problem formulation

- Divide the whole angle search range into a fine grid $\Theta = \{\theta_0, \theta_1, \cdots, \theta_{M-1}\}$.
- Each θ_m corresponds to a certain array response vector $\boldsymbol{a}_{k,m}$, which depends on f_k.
- Construct $N \times M$ steering matrix $A_k = [\boldsymbol{a}_{k,0}, \cdots, \boldsymbol{a}_{k,M-1}]$ (dictionary)
- Assumption 2: The DOAs of the targets $\{\theta_{m_0}, \theta_{m_1}, \cdots, \theta_{m_{Q-1}}\} \in \Theta \quad \Omega = \{m_0, m_1, \cdots, m_{Q-1}\}$
- Data model : $\quad \boldsymbol{y}_{k,p} = A_k \boldsymbol{x}_{k,p} \quad \boldsymbol{x}_{k,p}$ is a Q-sparse signal ($Q < N$)
- We have P such snapshots (measurement vector), then we can form $Y_k = A_k X_k \quad Y_k$ is $N \times P$, A_k is $N \times M$ and X_k is $M \times P$
- Assume that the DOAs during the span of P snapshots remain unchanged, then the columns of X_k share a common sparsity.
- Let $\mathcal{R}(A)$ denote an operation that collects the indexes of all the nonzero rows of a matrix A. $\mathcal{R}(X_k) = \Omega$ and $|\mathcal{R}(X_k)| = Q$.
- With these notations, we can formulate the sparse recovery problem as

$$\min_{\widehat{X}_k} \ |\mathcal{R}(\widehat{X}_k)| \quad \textit{subject to} \quad Y_k = A_k X_k \tag{5}$$

Aliasing Suppression

- As mentioned earlier, spatial aliasing occurs if d is larger than half of the apparent wavelength, which leads to similar columns in the steering matrix
- In classical beamforming, the DOAs are sought by steering a beamformer to different potential angles.
- However, the SSR-based method recovers X_k first and then estimates the DOAs by locating the rows of X_k that contain dominant entries.
- The over-completeness of the SSR dictionary gives rise to non-unique solutions and thus ambiguity in DOA estimation, which is termed as algebraic aliasing.
- Algebraic aliasing is essentially related to the "goodness" of the sensing matrices (steering matrix) for the DOA recovery.

Proposition 1: Under Assumption 1, if the number of targets Q and channels N satisfy

$$N > 2Q - rank(Y_k) \tag{6}$$

then the SSR-based method will not suffer from algebraic aliasing.

Proof: Algebraic aliasing will not exist if we can find unique solution $\widehat{X}_k$ satisfying $Y_k = A_k\widehat{X}_k$. This is only possible if the Kruskal-rank of A_k is larger than $2Q - rank(Y_k)$ [6, Theorem 2.4]. Since A_k is a Vandermonde matrix, whose Kruskal-rank is equal to its rank, N.

Kruskal-rank (or k-rank) of a matrix A is defined as the largest integer r for which every set of r columns of A is linearly independent.

Remarks:

- If $rank(Y_k) = 1$ ($P = 1$), then $Q < N/2$, that is we can discriminate at most $N/2$ targets.
- On the other side, $rank(Y_k) \le rank(X_k) \le Q$, suggests that $Q < N$.
- Thus, using multiple measurement vectors the authors argue that it is possible to counter the algebraic aliasing.

So far, we have concentrated on the data model for a single frequency f_m. We can obtain different measurements and different dictionaries $A_k \neq A_l$ if we use different frequency $f_k \neq f_l$. We will next show that using multiple dictionaries enables us to eliminate spatial aliasing.

- Let Γ_k denote the support of all possible DOA solutions for the k-th dictionary $\Gamma_k = \{\mathcal{R}(\widehat{X}_k^{(0)}), \mathcal{R}(\widehat{X}_k^{(1)}), \cdots\}$
- Spatial aliasing is frequency-dependent, which means that for different center frequencies, the resulting ambiguity will not (completely) overlap. Therefore, we can imagine that if we solve Eqn. (5) for several frequencies: $f_0, f_1, \cdots, f_{K-1}$ and combine the solutions in a judicious way, the ambiguity due to spatial aliasing will at least be reduced.

Theorem 1: With Proposition 1 met, if there exist at least two dictionaries, whose corresponding frequencies, say f_k and f_l , satisfy

$$0 < |f_k - f_l| < \frac{c}{2d} \tag{7}$$

then the intersection of the solution support related to different dictionaries will contain exclusively the target DOAs, i.e.,

$$\bigcap_k \Gamma_k = \Omega \tag{8}$$

Proof: With proposition 1 satisfied, we can exclude the ambiguity due to algebraic aliasing and need to focus only on spatial aliasing. Let us proceed with a counter-example. Suppose θ_m is one of the target angles and $\theta_m \neq \theta_{m'}$ is spatial aliasing contained in both dictionaries corresponding to f_k and f_l , which implies that $\{\theta_m, \theta_{m'}\}$ belongs to both Γ_k and Γ_l. In accordance with Eqn. (4) we then have

$$f_k \frac{d}{c} \sin\theta_m - f_k \frac{d}{c} \sin\theta_{m'} = j_1$$

$$f_l \frac{d}{c} \sin\theta_m - f_l \frac{d}{c} \sin\theta_{m'} = j_2$$

$$f_k \frac{d}{c} \sin\theta_m - f_k \frac{d}{c} \sin\theta_{m'} - f_l \frac{d}{c} \sin\theta_m + f_l \frac{d}{c} \sin\theta_{m'} = j_1 - j_2 = j_3$$

where j_1, j_2, j_3 are integers and j_1, j_2 are not equal to 0. Using trignometric identities, the above equations can be written as

$$-2 f_k \frac{d}{c} \sin\frac{\theta_m - \theta_{m'}}{2} \cos\frac{\theta_m + \theta_{m'}}{2} = j_1 \tag{9}$$

$$-2 f_l \frac{d}{c} \sin\frac{\theta_m - \theta_{m'}}{2} \cos\frac{\theta_m + \theta_{m'}}{2} = j_2 \tag{10}$$

$$-2(f_k - f_l) \frac{d}{c} \sin\frac{\theta_m - \theta_{m'}}{2} \cos\frac{\theta_m + \theta_{m'}}{2} = j3 \tag{11}$$

Since $0 < |f_k - f_l| < \frac{c}{2d}$, it is only possible for Eqn. (11) to hold if the integer $j_3 = j_1 - j_2 = 0$. On the other hand, from the above equations we know that j_1 and j_2 are not zero and they cannot be equal. Therefore, the angle $\theta_{m'}$ cannot be contained simultaneously in Γ_k and Γ_l, which concludes the proof. A judicious choice of frequencies can not only prevent spatial aliasing, but also enhance the performance in a noisy environment.

Aliasing-Free SSR Recovery

Based on the analysis in the previous section, the authors formulate the following multi-dictionary (MD) joint optimization problem with the joint-sparsity constraint:

$$\min_{\widehat{X}_k} \; |\mathcal{R}(\widehat{X}_k)| \quad for\ k = 0, 1, \cdots, K-1$$

$$subject\ to \quad Y_k = A_k X_k, \quad and\ \ \mathcal{R}(\widehat{X}_k) = \mathcal{R}(\widehat{X}_l)\ for\ k \neq l \tag{12}$$

whose solution will be free from any ambiguity under Theorem 1.

The authors have not proposed any new algorithm. They have used OMP in their simulations.

Numerical Examples

1. The authors demonstrate their approach using synthetic and real data.

2. For both cases, they considered ULA with $N = 16$ hydrophones, with a spacing of $d = 0.06\ m$. The speed of the signal wave is assumed to be $c = 1500\ m/s$

3. In the synthetic data they consider two sinusoids $Q = 2$ with frequencies $f_0 = 25$ kHz and $f_1 = 35$ kHz. The DOA are $\{35°, 39°\}$. The search gird is defined as $\Theta = \{-90°, -89.75°, \cdots, 90°\}$. With P=100 snapshots, each dictionary has a dimension of 16×720

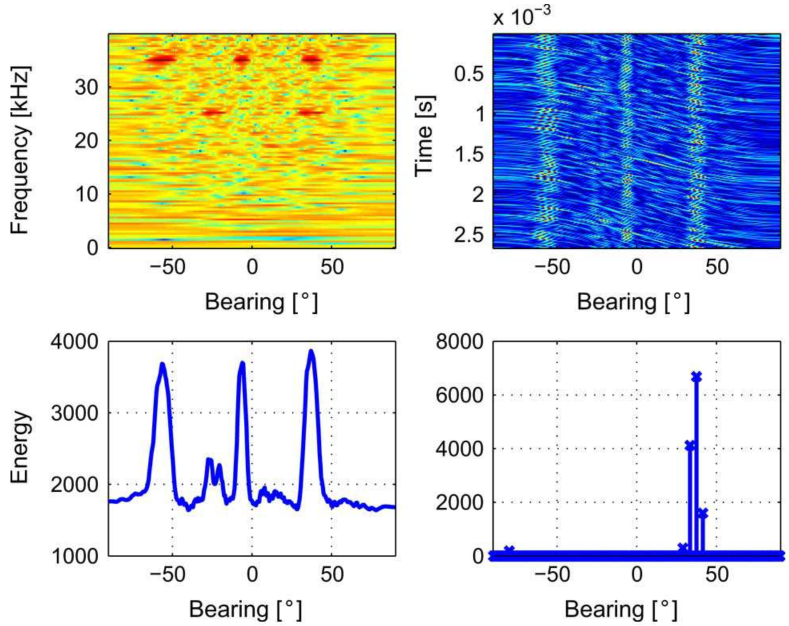


Figure 2: Comparison of classical beamforming with the proposed method. Upper-left subplot: the frequency-bearing image after beamforming; upper-right subplot: the time-bearing image after beamforming; lower-left subplot: the integrated energy of the time-bearing image; lower-right subplot: the result yielded by the proposed method.

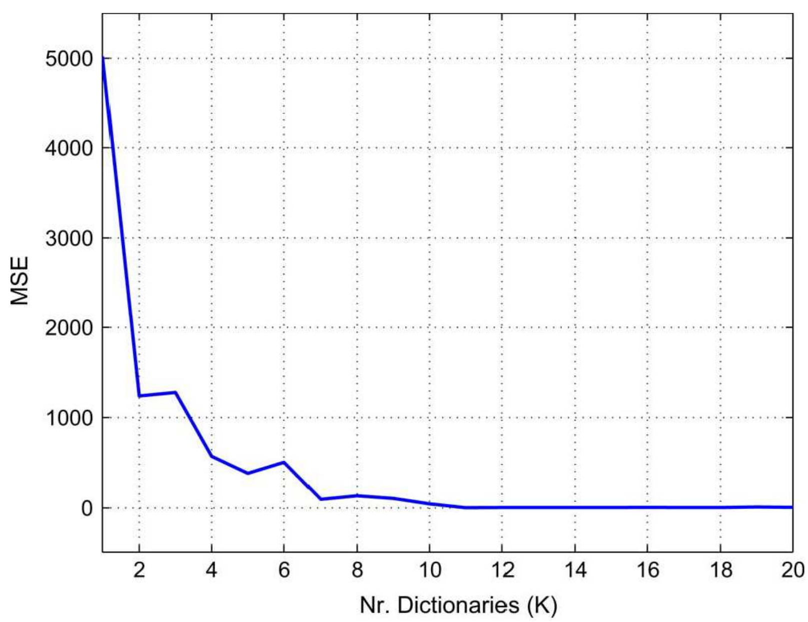


Figure 3: MSE performance against the number of utilized frequencies

4. They halt OMP after 5 iterations. MSE $= \frac{1}{N_s}\sum_{q=0}^{Q-1}(\widehat{\theta}_m - \theta_m)$

1. In the real data experiment, the direction of the divers has to be estimated based on their exhaling sound. Two divers who are 150 m away from the hydrophone are considered. The received signals are from 52° and 60°, respectively.

2. We can see that the frequencies lower than 10 kHz are completely useless for DOA estimation: the diver signal is subdued by the ambient noise dominated by the ship traffic in the harbor.

3. In the midfrequency range (between 10 and 12.5kHz), where the hydrophone array is not subject to aliasing, there is a strong interference signal at a direction around −40°, which possibly comes from a

departing ship blowing the horn.

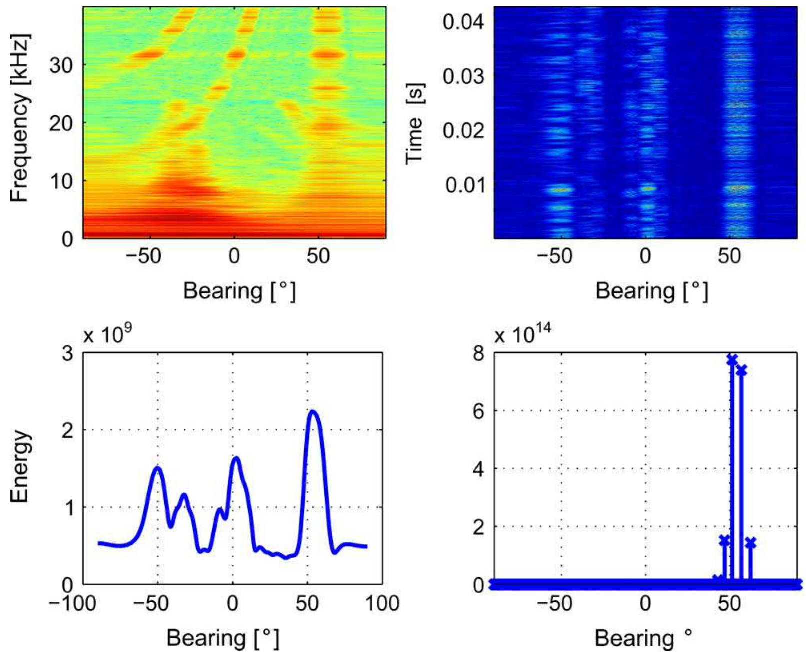


Figure 4: Comparison of classical beamforming with the proposed method for the diver signal. Upper-left subplot: the frequency-bearing image after beamforming; upper-right subplot: the time-bearing image after beamforming (only signals above 25 kHz are taken); lower-left subplot: the integrated energy of the time-bearing image; lower-right subplot: the result yielded by the proposed method.

In Summary, the authors have applied sparse signal reconstruction for DOA estimation (for ULA). They formed an MD optimization problem with joint sparsity constraints. They show how to avoid ambiguities (spatial and algebraic) by using multiple dictionaries and multiple measurement vectors, respectively. They have demonstrated their findings through synthetic and real-life examples.

References

[1] H. L. van Trees, Optimum Array Processing (Detection, Estimation and Modulation Theory, Part IV). New York: Wiley, 2002.

[2] V. Cevher, A. C. Grbz, J. H. McClellan, and R. Chellappa, Compressive wireless arrays for bearing estimation of sparse sources in angle domain, in Proc. Int. Conf. Acoust., Speech, Signal Process. (ICASSP), Mar. 2008, pp. 24972500.

[3] J. J. Fuchs, On the application of the global matched filter to DOA estimation with uniform circular arrays, IEEE Trans. Signal Process., vol. 49, no. 4, pp. 702709, Apr. 2001.

[4] D. Malioutov, M. Cetin, and A. S. Willsky, A sparse signal reconstruction perspective for source localization with sensor arrays, IEEE Trans. Signal Process., vol. 53, no. 8, pp. 30103022, Aug. 2005.

[5] Y. Wang, G. Leus, and A. Pandharipande, Direction estimation using compressive sampling array processing, presented at the IEEE Workshop on Statistical Signal Processing, Cardiff, U.K., 2009.

[6] J. Chen and X. Huo, Theoretical results on sparse representations of multiple-measurement vectors, IEEE Trans. Signal Process., vol. 54, no. 12, pp. 46344643, Dec. 2006.

[7] D. Baron, M. F. Duarte, M. B. Wakin, S. Sarvotham, and R. G. Baraniuk, Distributed compressive sensing, 2009. Available: http://arxiv.org/abs/0901.3403