Game-Theoretic Techniques for the Energy Efficiency of Wireless Communications and Sensor Networks

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MARIE CURIE ACTIONS

Outline

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Motivation

What is game theory? (1/2)

Wiki:

«Game theory is a study of strategic decision making. More formally, it is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers." (R. Myerson, Nobel prize in Economic Sciences in 2007)»

and

""Game theory is a sort of umbrella or 'unified field' theory for the rational side of social science, where 'social' is interpreted broadly, to include human as well as non-human players (computers, animals, plants) (R. Aumann, Nobel prize in Economic Sciences in 2005)"

What is game theory? (2/2)

It was raised to the dignity of "**theory**" by the seminal book *Theory of Games and Economic Behavior* published in 1944 by John von Neumann and Oskar Morgenstern

Game theory gives surprising clues and insight whenever **rational entities** interact and make **appropriate choices** so as to find (at the end of the game…) their own maximum utility:

- Competitors in a market
- Opponents in gambling & recreational games (rock-paper-scissors)
- Strategies of political agents
- Interaction of social groups (humans and animals)
- Distribution of resources
- Design of distributed algorithms
- **Nodes and terminals in the Internet & wireless networks**

Is game theory important for wireless applications?

What Google says:

- "game theory": 6,700,000 results
- "wireless networks": 10,700,000 results
- "game theory for wireless networks": 64,600 results

What IEEExplore says:

Ingredients of a game

- *A set of players*

- *A set of strategies*

- *A payoff function*

Motivating example (1/2)

3G mobile phones (UMTS/cdma2000) transmit on the same frequency band and in the same time slots, with non-orthogonal CDMA technology

The risk is that the (weak) signal of the FAR terminal is overwhelmed by the (strong) signal of the NEAR one

This is the NEAR-FAR issue, and the network has to implement some form of power control strategy

Motivating example (2/2)

- *Can we devise a strategy that satisfies both players?*
- *And what about the satisfaction of the network manager?*
- *Can we merge all these needs ?*

Game theory provides analytical tools that help to answer such typical questions in the context of resource allocation

Basics of noncooperative game theory

Ingredients of a game

- A set of *players*

- A set of *strategies*

- A *payoff function*

Definition and examples

A **game** can be defined as "a description of strategic interaction that includes the constraints on the actions that the players take and on the players' interests" (Osborne and Rubinstein)

Examples of a game:

- wireless terminals contending for the medium access in a network
- sport shoe factories competing in the Asian market
- a group of primitive hunters seeking for food
- peahens and peacocks competing for reproduction
- …

Taxonomy

A game can be:

- cooperative
- noncooperative
- static
- dynamic
- perfect-information
- imperfect-information
- complete-information
- incomplete-information

and many more types…

Static games

In a (noncooperative) **static** game,

(1) the players simultaneously choose their actions; and then

(2) the players receive their own payoffs that depend on the combination of actions just chosen by **all** players

The near-far effect game [14]

The two terminals, the near and the far ones, can either **transmit** at a power level *p* (incurring in a transmission cost *c*), or **wait**

- when only one terminal is silent, the other one achieves a throughput *t*, with *t*>*c*;
- when both terminals are simultaneously active, only the near terminal can reach the AP, achieving again a throughput *t*;
- when both terminals are silent, no data exchange takes place.

how can we properly describe this situation **mathematically**?

Strategic-form representation: the general case…

In its mathematical formulation, a static game can be described by its **strategic-form representation** $\mathcal{G} = [\mathcal{K}, \{\mathcal{A}_k\}, \{u_k(a)\}]$, where:

(1) $\mathcal{K} = [1, \ldots, K]$ is the finite set of players

(2) $\mathcal{A}_k$ is the set of strategies (actions) available to player k

(3) $u_k(a)$ is the utility (payoff) for player k

… and the near-far effect game (1/2)

The **strategic-form representation** of the near-far effect game is

$\mathcal{G} = [\mathcal{K}, \{\mathcal{A}_k\}, \{u_k(a)\}]$, where:

- K=2, $\mathcal{K}$ = {far player (#1), near player (#2)}
- $\mathcal{A}_k$ = {wait, transmit at power p}
- $u_k(a)$ = throughput achieved – tx cost

- this is a **finite** game, because the $\mathcal{A}_k$'s are countable
- this is a game of **complete information**, because all game ingredients (players, actions, and utilities) are common knowledge among the players

… and the near-far effect game (2/2)

The utility function of each terminal can be summarized in the **payoff matrix**:

		player 2 (NEAR)	
		wait	*transmit*
player 1 (FAR)	*wait*	0, 0	0, $t - c$
	transmit	$t - c$, 0	$-c$, $t - c$

$u_1(a_1,a_2)$, $u_2(a_1,a_2)$

*Once the game is in its strategic form, we can try to **solve** it!*

Iterated dominance (1/2)

Assume that the players are **rational**: the near far effect game can be solved by **iterated elimination of dominated strategies**

$u_2(0,0) = 0 < u_2(0,p) = t - c$

$u_2(p,p) = 0 < u_2(p,p) = t - c$

$$u_2(\bullet,0) < u_2(\bullet,p)$$

player 2 (NEAR)

player 1 (FAR)	0	*p*
0	0, 0	0, $t - c$
p	$t - c$, 0	$-c$, $t - c$

$u_1(a_1,a_2)$, $u_2(a_1,a_2)$

Player 2 is always better off when transmitting, **no matter what** player 1 does

Rationality implies that player 2 (the near terminal) will **always** play *p*

Iterated dominance (2/2)

The near-far effect game reduces to

player 2 (NEAR)

player 1 (FAR)

	p
0	0, $t - c$
p	$-c$, $t - c$

$u_1(a_1,a_2)$, $u_2(a_1,a_2)$

$$u_1(0,p) = 0 > u_1(p,p) = -c$$

*The **outcome** of the game is (wait, transmit)*

Nash equilibrium (1/4)

*Unfortunately, only **a few games** can be solved by iterated dominance*

Let's consider the **multiple-access game** [19], in which two terminals are at a comparabile distance from the AP. As before, each terminal can either **transmit** at a power *p* (incurring in a transmission cost *c*), or **wait**

(1) when only one terminal is silent, the other one achieves a throughput *t*;

(2) when both terminals are simultaneously active, they **collide** at the AP (none achieves any throughput);

(3) when both terminals are silent, no data exchange takes place.

Nash equilibrium (2/4)

		player 2	
		wait	transmit
player 1	wait	0, 0	0, $t-c$
	transmit	$t-c$, 0	$-c$, $-c$

$u_1(a_1,a_2)$, $u_2(a_1,a_2)$

We resort to the concept of **best response**: the strategy chosen by a self-optimizing player is the best response to the action taken by the others

Nash equilibrium (3/4)

John F. Nash, Jr., 1928-

player 1 \ player 2	wait	transmit
wait	0, 0	0, $t - c$
transmit	$t - c$, 0	$-c, -c$

$u_1(a_1,a_2)$, $u_2(a_1,a_2)$

(transmit, wait) and (wait, transmit) are the two (pure-strategy) Nash equilibria of the game

Nash equilibrium (4/4)

More in general, the **Nash equilibrium** represents a stable outcome of the noncooperative game in which multiple agents (the players) with conflicting interests:

- compete through **self-optimization**, and
- reach a point where no player has no incentive to **unilaterally** deviate from

Exercise: Verify that (wait, transmit) is the only Nash equilibrium of the near-far effect game

Mixed strategies (1/8)

We try to apply what we've just learned to the **jamming game** [19]:

In an FDMA network with **two** channels, a **malicious** transmitter (say player 1) tries to prevent a **licensed** user (say player 2) from a successful transmission by transmitting on the same channel:

(1) if the jammer picks the **same** channel chosen by the licensed user, then **player 1 wins** and player 2 loses;

(2) if the two channels **do not match**, then **player 2 wins** and player 1 loses.

This game (a **zeros-sum** game) captures a number of different situations:

- soccer player (p2) vs goalkeeper (p1) in a penalty kick: left or right?
- card players in a poker game: bluffing (p2) or good hand (p1)?
- pitcher (p2) vs batter (p1) in a baseball game: fastball or curve?

Mixed strategies (2/8)

The strategic-form representation of the jamming game is:

player 2 (licensed)
(wins on no match)

player 1 (jammer) (wins on match)	chan. A	chan. B
chan. A	+1, −1	−1, +1
chan. B	−1, +1	+1, −1

$u_1(a_1,a_2)$, $u_2(a_1,a_2)$

No Nash equilibrium appears in this game: then what?

Mixed strategies (3/8)

Instead of adopting a deterministic strategy (in game theory parlance, using a **pure strategy**), we define a probability distribution over the strategy set (in game theory parlance, we use a **mixed strategy**)

For instance, let

$$q_k = \Pr\{\text{ Player } k \text{ chooses channel A }\}, \qquad 0 \le q_k \le 1$$

$$1 - q_k = \Pr\{\text{ Player } k \text{ chooses channel B }\}$$

For player 1, a mixed strategy is the **probability distribution** $\sigma_1=(q_1, 1 - q_1)$, whereas for player 2 it is the **probability distribution** $\sigma_2=(q_2, 1 - q_2)$

Note: Pure strategies are degenerated forms of mixed strategies. Player 1's pure strategy channel A is simply the mixed strategy $\sigma_1= (q_1=1 , 1 - q_1=0) = (1,0)$.

Mixed strategies (4/8)

Each user wants to **maximize** his/her own payoff

		player 2 (no match)	
		ch. A	ch. B
player 1 (match)	ch. A	+1, –1	–1, +1
	ch. B	–1, +1	+1, –1

$u_1(a_1,a_2)$, $u_2(a_1,a_2)$

Let's identify player 2's best response to player 1's mixed strategy $\sigma_1=(q_1, 1-q_1)$

$$u_2(\sigma_1, \sigma_2) = (-1)\, q_1 q_2 + (+1)\, q_1 (1-q_2) + (+1)(1-q_1)\, q_2 + (-1)(1-q_1)(1-q_2) = q_2(2-4q_1) + (2q_1-1)$$

if $2-4q_1 > 0$, i.e., if $q_1 < 1/2$, then $q_2(q_1) = 1$

if $2-4q_1 < 0$, i.e., if $q_1 > 1/2$, then $q_2(q_1) = 0$

if $2-4q_1 = 0$, i.e., if $q_1 = 1/2$, then $q_2(q_1) = [0,1]$

Mixed strategies (5/8)

if $2 - 4\,q_1 > 0$, i.e., if $q_1 < 1/2$, then $q_2(q_1) = 1$

if $2 - 4\,q_1 < 0$, i.e., if $q_1 > 1/2$, then $q_2(q_1) = 0$

if $2 - 4\,q_1 = 0$, i.e., if $q_1 = 1/2$, then $q_2(q_1) = [0,1]$

Mixed strategies (6/8)

Let's now identify player 1's best response to player 2's mixed strategy $\sigma_2=(q_2, 1-q_2)$

$$u_1(\sigma_1, \sigma_2) = (+1)\, q_1 q_2 + (-1)\, q_1 (1-q_2) + (-1)(1-q_1)\, q_2 + (+1)(1-q_1)(1-q_2) = q_1(4q_2-2) + (1-2q_2)$$

player 1 (match) \ player 2 (no match)	ch. A	ch. B
ch. A	+1, –1	–1, +1
ch. B	–1, +1	+1, –1

$u_1(a_1,a_2)$, $u_2(a_1,a_2)$

if $4q_2 - 2 > 0$, i.e., if $q_2 > 1/2$, then $q_1(q_2) = 1$

if $4q_2 - 2 < 0$, i.e., if $q_2 < 1/2$, then $q_1(q_2) = 0$

if $4q_2 - 2 = 0$, i.e., if $q_2 = 1/2$, then $q_1(q_2) = [0,1]$

Mixed strategies (7/8)

if $4\,q_2 - 2 > 0$, i.e., if $q_2 > 1/2$, then $q_1(q_2) = 1$

if $4\,q_2 - 2 < 0$, i.e., if $q_2 < 1/2$, then $q_1(q_2) = 0$

if $4\,q_2 - 2 = 0$, i.e., if $q_2 = 1/2$, then $q_1(q_2) = [0,1]$

Mixed strategies (8/8)

Mixing the two **best-response correspondences** together yields:

The **mixed-strategy Nash equilibrium** of the jamming game is $q_1=q_2=1/2$: i.e., the best solution in terms of game theory, **random choice** between chan. A and chan. B for both players, corresponds to the trivial solution

Nash equilibrium: A summary

Theorem [Nash, 1950]: In the strategic-form game $\mathcal{G} = [\mathcal{K}, \{\mathcal{A}_k\}, \{u_k(a)\}]$, if K is finite and $\mathcal{A}_k$ is finite for every k (i.e., in a finite game), then there exists at least one Nash equilibrium, possibily involving mixed strategies

Some implications:

- it establishes the existence of (at least) one steady state solution for every finite game (i.e., we're not taking the risk of studying the empty set);
- a finite game may have no pure-strategy equilibria (e.g., jamming game), one pure-strategy equilibrium (e.g., near-far effect game), or multiple pure-strategy equilibria (e.g., multiple-access game)

Exercise: Prove that (wait, transmit) in the only Nash equilibrium in the near-far effect game (both in pure and mixed strategies)

Exercise: Find the third Nash equilibrium of the multiple-access game in mixed strategies (solution: q_k = Pr { Player k waits } = c/t for k=1,2)

Pareto and social optimality (1/5)

Let's consider the **forwarder's dilemma** [19]:

- to communicate with its receiver (and thus achieve a throughput *t*), each sender should use the **other player as a relay** that forwards the packet
- when dropping the packet to be forwarded, each player can **save energy** (getting a positive reward *s*, $0<s<t$)

Hence the dilemma: to forward or not to forward?

Pareto and social optimality (2/5)

Let's solve the game using the best-response technique:

player 2

player 1	*drop*	*forward*
drop	s, s	$t+s, 0$
forward	$0, t+s$	t, t

$u_1(a_1,a_2),\ u_2(a_1,a_2)$

Vilfredo Pareto, 1848-1923

Neither player 1 nor player 2 has any incentive to deviate from (drop, drop) **unilaterally**

But, if both players could **jointly** change their strategies, they would opt to play (forward, forward), as $t>s$

Pareto and social optimality (3/5)

To evaluate the **efficiency** of a Nash equilibrium, we often use the concept of Pareto optimality:

> In general, a strategy profile is **Pareto-optimal** if we cannot increase the payoff of one player without *decreasing that of at least one other player*

- the profile (d,d) is a Nash equilibrium, **but not** Pareto-optimal;
- the profiles (d,f), (f,d), and (f,f) are Pareto-optimal, **but not** Nash equilibria;
- any Pareto-optimal strategy Pareto-dominates the Nash equilibrium, which is said to be **inefficient**;

player 2

player 1	d	f
d	s, s	$t+s, 0$
f	$0, t+s$	t, t

$u_1(a_1,a_2),\ u_2(a_1,a_2)$

Pareto and social optimality (4/5)

Beyond Pareto optimality, the performance (in terms of **social efficiency**) of a Nash equilibrium can be measured by comparing it to the **socially optimal profile** $\mathbf{a}_{\text{so}} = \arg\max_{\mathbf{a}\in\boldsymbol{\mathcal{A}}} \sum_{k\in\mathcal{K}} u_k(\mathbf{a})$

In the forwarder's dilemma, a_{so}=(f, f), that gives a **maximum social welfare** 2*t*

The **price of anarchy** η_a and the **price of stability** η_s correspond to the ratio between the maximum social welfare and the "**worst**" and the "**best**" Nash equilibria (in a social sense), respectively:

$$\eta_a = \frac{\max_{\mathbf{a}\in\boldsymbol{\mathcal{A}}} \sum_k u_k(\mathbf{a})}{\min_{\mathbf{a}\in\boldsymbol{\varepsilon}} \sum_k u_k(\mathbf{a})} \qquad \eta_s = \frac{\max_{\mathbf{a}\in\boldsymbol{\mathcal{A}}} \sum_k u_k(\mathbf{a})}{\max_{\mathbf{a}\in\boldsymbol{\varepsilon}} \sum_k u_k(\mathbf{a})}$$

where ε is the set collecting all Nash equilibria

Pareto and social optimality (5/5)

- $1 \leq \eta_s \leq \eta_a$
- if $|\varepsilon|=1$ (i.e., the Nash equiibrium is unique), $\eta_s = \eta_a$
- if $\eta_a=1$, the Nash equilibrium is **socially efficient**

In the forwarder's dilemma, $\eta_s = \eta_a = t/s > 1$

The **price of anarchy** η_a and the **price of stability** η_s correspond to the ratio between the maximum social welfare and the "**worst**" and the "**best**" Nash equilibria (in a social sense), respectively:

$$\eta_a = \frac{\max_{\mathbf{a}\in\mathcal{A}} \sum_k u_k(\mathbf{a})}{\min_{\mathbf{a}\in\varepsilon} \sum_k u_k(\mathbf{a})} \qquad \eta_s = \frac{\max_{\mathbf{a}\in\mathcal{A}} \sum_k u_k(\mathbf{a})}{\max_{\mathbf{a}\in\varepsilon} \sum_k u_k(\mathbf{a})}$$

where ε is the set collecting all Nash equilibria

An example of an infinite game [21, 22] (1/4)

Again a **near-far power control** game: this time the K=2 terminals can select their powers a_k from a **continuous** set:

$$\mathcal{A}_k = \{p \in \mathbb{R} : 0 \leq p \leq \overline{p}\}$$

The utility is measured in terms of **energy efficiency**, i.e., number of correct bits delivered per unit of energy consumed [bit/Joule]:

$$u_k(\mathbf{a}) = \frac{t_k}{a_k} = R \cdot \frac{f(\gamma_k)}{a_k}$$

bit-rate

packet success rate

where

$$\gamma_k = \frac{M \cdot h_k a_k}{\sigma^2 + h_{\backslash k} a_{\backslash k}}$$

is the **signal-to-interference-plus-noise ratio** (SINR), M is the CDMA spreading factor, σ^2 is the AWGN power, and $\backslash k$=2 if k=1, and $\backslash k$=1 if k=2

An example of an infinite game (2/4)

To compute the Nash equilibrium of the game, we can focus on the **best response** $b_k(a_{\backslash k})$:

$$b_k(a_{\backslash k}) = \arg \max_{a_k \in \mathcal{A}_k} u_k([a_k, a_{\backslash k}])$$

To better visualize the problem, let's plot the utility for a **fixed** interfering power $a_{\backslash k}$

An example of an infinite game (3/4)

Using standard techniques, $b_k(a_{\backslash k}) = \min\left(\overline{p}, \gamma^\star(\sigma^2 + h_{\backslash k} a_{\backslash k})/(M h_k)\right)$

where $\gamma^\star$ is such that $df(\gamma_k)/d\gamma_k|_{\gamma_k=\gamma^\star} = f(\gamma^\star)/\gamma^\star$

The **unique** Nash equilibrium of this game is $(a_1^\star, a_2^\star)$

An example of an infinite game (4/4)

In an infinite game, **sufficient conditions** (that apply in this case) for the **existence** of a (pure-strategy) Nash equilibrium are:

- non-emptiness, compactness and convexity of the strategy space $\mathcal{A}_k$
- quasi-convexity of the utility function $u_k(a)$ for all k

In this case, the **uniqueness** of the Nash equilibrium is ensured by the **standard properties** (positivity, monotonicity, and scalability) of the best response $b_k(a_{\backslash k})$ and apply to a generic number of terminals K

Similarly to the forwarder's dilemma, the equilibrium point is **socially inefficient**

Special classes of static games

Among **noncooperative static games**, there are special classes of games which are particularly **relevant** to address problems in wireless and communication networks:

- Supermodular games
- Potential games
- Generalized Nash games

Supermodular games

A **supermodular** game is a static noncooperative game, which has:

(1) a set of players $\mathcal{K}$ = [1, …, *K*]

(2) $\mathcal{A}_k$ is the set of strategies (actions) available to player *k*

(3) $u_k(a)$ is the utility (payoff) for player *k*

where, for any $a'_k \geq a_k$ and for any $\mathbf{a}'_{\backslash k} \geq \mathbf{a}_{\backslash k}$ (component-wise),

$$u_k([a'_k, \mathbf{a}'_{\backslash k}]) - u_k([a_k, \mathbf{a}'_{\backslash k}]) \geq u_k([a'_k, \mathbf{a}_{\backslash k}]) - u_k([a_k, \mathbf{a}_{\backslash k}])$$

The utility function has **increasing differences**: if a player takes a higher action, then the other players are better off when taking higher actions as well

Supermodular games: An example [24] (1/2)

Supermodular games are attractive, as they possess **pure-strategy Nash equilibria**, that can be reached through **best-response** iterative algorithms

Let's consider again the near-far power control game with continuous power sets $\mathcal{A}_k$=[0, $\underline{p}$], but using a **modified utility**:

$$\tilde{u}_k(\mathbf{a}) = u_k(\mathbf{a}) - \alpha \cdot a_k = R \cdot \frac{f(\gamma_k)}{a_k} - \alpha \cdot a_k$$

pricing factor

Pricing introduces some form of **externality**, that charges players proportionally to the powers they radiate (that causes **multiple-access interference**)

To prove that $\tilde{u}_k(\mathbf{a})$ has increasing differences, it is sufficient to check that

$$\frac{\partial^2 \tilde{u}_k(\mathbf{a})}{\partial a_k \partial a_j} \geq 0 \quad \forall k, j \in \mathcal{K}, k \neq j$$

Supermodular games: An example (2/2)

This modified formulation improves the **efficiency** of the Nash equilibrium (NE):

$a^\star$: NE of the original game

$\tilde{a}^\star$: NE of the pricing game

Intuitively, this form of externality encourages players to use the resources **more efficiently**

Potential games (1/2)

A **potential** game is a static noncooperative game, which has:

(1) a set of players $\mathcal{K}$ = [1, …, *K*]

(2) $\mathcal{A}_k$ is the set of strategies *a* (actions) available to player *k*

(3) $u_k(a)$ is the utility (payoff) for player *k*

where, for all players and all other players' profile $a_{\backslash k}$,

$$u_k([a_k, \mathbf{a}_{\backslash k}]) - u_k([a'_k, \mathbf{a}_{\backslash k}]) = \Phi([a_k, \mathbf{a}_{\backslash k}]) - \Phi([a'_k, \mathbf{a}_{\backslash k}]) \quad \forall a_k, a'_k \in \mathcal{A}_k$$

potential function

no dependence on user index!

(**exact potential game**)

Potential games (2/2)

A **potential** game is a static noncooperative game, which has:

(1) a set of players $\mathcal{K}$ = [1, …, *K*]

(2) $\mathcal{A}_k$ is the set of strategies (actions) available to player *k*

(3) $u_k(a)$ is the utility (payoff) for player *k*

where, for all players and all other players' profile $a_{\backslash k}$,

$$\mathrm{sgn}[u_k([a_k, \mathbf{a}_{\backslash k}]) - u_k([a'_k, \mathbf{a}_{\backslash k}])] = \mathrm{sgn}[\Phi([a_k, \mathbf{a}_{\backslash k}]) - \Phi([a'_k, \mathbf{a}_{\backslash k}])] \quad \forall a_k, a'_k \in \mathcal{A}_k$$

(**ordinal potential game**)

Potential games: An example [26] (1/4)

Uplink power allocation for an OFDMA cellular network, with *N* subcarriers and *K* terminals

Each terminal *k*'s objective: allocate the power so as to maximize its own achievable rate given a constraint on the maximum total radiated power $\bar{p}_k$ to exploit the channel frequency diversity

Potential games: An example (2/4)

The strategic-form representation of the game is the following:

(1) **players:** $\mathcal{K}$ = [1, …, *K*] terminals in the network

(2) **strategies:** $\mathcal{A}_k = \left\{ \mathbf{p}_k \in \mathbb{R}^N : p_k(n) \geq 0 \, \forall n, \sum_{n=1}^{N} p_k(n) \leq \overline{p}_k \right\}$

(3) **utilities:** $u_k([\mathbf{p}_k, \mathbf{p}_{\setminus k}]) = \sum_{n=1}^{N} \log(1 + \gamma_k(n))$ *Shannon capacity*

where

$$\gamma_k(n) = \frac{h_k(n) p_k(n)}{\sigma^2 + \sum_{j \neq k} h_j(n) p_j(n)}$$

and $h_k(n)$ are terminal *k*'s SINR and channel power gain over subcarrier *n*, resp.

Potential games: An example (3/4)

This is an **exact** potential game, with potential function

$$\Phi([\mathbf{p}_k, \mathbf{p}_{\backslash k}]) = \sum_{n=1}^{N} \log \left(\sigma^2 + \sum_{j=1}^{K} h_j(n) p_j(n) \right)$$

Exercise: Check that $u_k([\mathbf{p}_k, \mathbf{p}_{\backslash k}]) - u_k([\mathbf{p}'_k, \mathbf{p}_{\backslash k}]) = \Phi([\mathbf{p}_k, \mathbf{p}_{\backslash k}]) - \Phi([\mathbf{p}'_k, \mathbf{p}_{\backslash k}])$

In an infinite potential game (as occurs in this case), a pure-strategy Nash equilibrium **exists** if:

- the strategy sets $\mathcal{A}_k$ are compact
- the potential function Φ is upper semi-continuous on $\mathcal{A}=\mathcal{A}_1\times\ldots\times\mathcal{A}_K$

The interest in potential games stems from the **guarantee** of existence of pure-strategy Nash equilibria

Potential games: An example (4/4)

The **convergence** to one of the Nash equilibria of the game can be achieved using an iterative algorithm based on the **best-response** criterion:

$$\mathbf{p}_k^{(t+1)} = \arg \max_{\mathbf{p}_k \in \mathbb{R}^N} \Phi\left(\left[\mathbf{p}_k, \mathbf{p}_{\backslash k}^{(t)}\right]\right)$$

$$\text{s.t.} \sum_{n=1}^{N} p_k(n) \leq \overline{p}_k \text{ and } p_k(n) \geq 0, \forall n$$

The solution is given by the well-known **iterative water-filling** algorithm:

$$p_k^{(t+1)}(n) = \max\left(0, \frac{1}{\lambda_k} - \frac{p_k^{(t)}(n)}{\gamma_k^{(t)}(n)}\right)$$

fed back by the base station

where λ_k is such that

$$\sum_{n=1}^{N} \max\left(0, 1/\lambda_k - p_k^{(t)}(n)/\gamma_k^{(t)}(n)\right) = \overline{p}_k$$

Generalized Nash games

A **generalized Nash game** is a static noncooperative game, with:

(1) a set of players $\mathcal{K} = [1, \ldots, K]$

(2) player k's set of strategies $\mathcal{A}_k$ depends on **all others' actions** $a_{\backslash k}$

(3) $u_k(a)$ is the utility (payoff) for player k

The **interplay** between strategy sets typically occurs when placing **constraints** on the game formulation

Generalized Nash games: An example [28] (1/2)

Let's focus again on the uplink of an OFDMA cellular network, this time during the (contention-based) **network association** phase, in which the subcarriers are shared by the terminals in a **multicarrier CDMA** fashion

Each terminal *k*'s objective: allocate the power so as to **minimize the energy** spent to get a successful association given quality of service (QoS) constraints in terms of **false code locks**

Generalized Nash games: An example (2/2)

This situation can be modeled by the following strategic-form representation:

(1) **players:** $\mathcal{K}$ = [1, …, K] terminals in the network

(2) **strategies:** power strategy sets $\mathcal{A}_k$=[0, $\bar{p}_k$]

(3) **QoS constraints:** max. probabilty of false alarm

max. mean square error on timing estimation

(4) **utilities:** $u_k(\mathbf{a}) = \frac{\Pi_k(\mathbf{a})}{T \cdot a_k}$

probability of correct network association

energy spent per correct

The solution of generalized Nash games is the **generalized Nash equilibrium**: in this case, it exists and is unique if and only if the number of terminals K is below a **certain threshold**

Beyond Nash equilibrium: other equilibrium concepts

In addition to the concept of mixed-strategy and pure-strategy Nash equilibria in noncooperative static games, it is worth mentioning:

- the **correlated equilibrium**: a generalization of the Nash equilibrium, where an arbitrator (not necessarily an intelligent entity) helps the players to correlate their actions, so as to favor a decision process **in between** non-cooperation and cooperation

  example: it may select one of the two pure-strategy Nash equilibria in the multiple-access game

- the **Wardrop equilibrium**: a limiting case of the Nash equilibrium when the population of users becomes **infinite**

  example: it can be used as a decision concept to select routing strategies in ad-hoc networks

Taxonomy revisited

classes: zero-sum, potential, etc.
tools: iterated dominance, mixed strategies, etc.
solution: Nash e., correlated e., etc.

Dynamic games

There could be situations in which the players are allowed to have a **sequential** interaction, meaning that the move of one player is conditioned by the previous moves in the game: This is the class of **dynamic games**

Consider a **sequential multiple-access game**, where the packet transmission is successful when there is no collision, in which:

(1) player 1 can either transmit (with a transmission cost c) or wait;

(2) then, **after observing player 1's action**, player 2 chooses whether to transmit or not

Extensive-form representation (1/2)

A convenient way to describe a sequential game such as the sequential multiple-access game is the **extensive-form representation**, which consists of:

(1) a set of *players*;

(2a) the *order* of moves – i.e., who moves when;

(2b) what the players' *choices* are when they move;

(2c) the *information* each player has when he/she makes his/her choices;

(3) the *payoff* received by each player for each combination of strategies that could be chosen by the players.

Extensive-form representation (2/2)

The extensive-form representation of the sequential multiple-access game [29] is

- this is a game with perfect information, because the player with the move knows the full history of the play of the game thus far;
- this is a game with complete information, because the $u_k(a)$'s are common knowledge;
- this game is a finite-horizon game, because the number of stages is finite.

Nash equilibrium

Any game (both static and dynamic) can **indifferently** be described by its strategic-form representation and its extensive-form representation. The options (the **strategy**) we show for player 2 are his/her response conditioned on what player 1 does (wait/transmit), i.e., a plan for **every** history of the game

Player 2 (follower)

Player 1 (leader)	*W W*	*W T*	*T W*	*T T*
W	0, 0	0, 0	0, $t-c$	0, $t-c$
T	$t-c$, 0	$-c$, $-c$	$t-c$, 0	$-c$, $-c$

There are three **Nash equilibria**: (*T*, (*W*,*W*)), (*T*,(*T*,*W*)), and (*W*,(*T*,*T*))

Backward induction & subgame-perfect Nash equilibrium

Let's go back to the extensive-form representation:

Are the three Nash equilibria, (*T*, (*W*,W)), (*T*, (*T*,*W*)), and (*W*, (*T*,*T*)), all **credible**?

To answer this question, let's resort to the **backward induction** of the game

The strategy (*T*, (*T*,*W*)) is the only **subgame-perfect Nash equilibrium**

Dynamic games with imperfect information

Imperfect information occurs when, at some stage(s) of the game, some players do **not** know the full history of the play thus far

Example: a situation in which the two terminals play the sequential multiple-access game first, and then the (simultaneous) multiple-access game in case of a collision during the first stage, is a game with imperfect information

Static games are a special case of dynamic games with imperfect information

Complete information	≠	Perfect information
(knowledge of the structure of the game: player, strategies, utilities, etc.)		(knowledge of the full history of the game)

Subgame-perfect Nash equilibrium: A summary

A **subgame-perfect Nash equilibrium** is a strategy profile which prescribes actions that are optimal at each game stage for **every history** of the game, i.e., for any **possible** unfolding of the game

Otherwise stated, a subgame-perfect Nash equilibrium is a Nash equilibrium of **any proper subgame** of the original game (i.e., it is a **credible** Nash equilibrium that has **survived** backward induction

Some implications:

- any finite game with complete information has a subgame-perfect Nash equilibrium, perhaps in mixed strategies;
- any finite game with complete and perfect information has (at least) one pure-strategy subgame-perfect Nash equilibrium;

Stackelberg games: An example [32]

Games such as the sequential multiple-access game are a particular class of dynamic games: **Stackelberg games**, that have a **leader** and a **follower**

Stackelberg games are particularly suitable to analyze wireless networks: e.g., they can investigate power allocation schemes for **femtocell networks**

leaders: macrocell base stations (BSs)

followers: femtocell access points (FAPs)

each player's objective: allocate the power so as to maximize the achievable rate according to the Shannon capacity formula

This multi-leader/multi-follower formulation leads to **multiple** Stackelberg equilibria

Repeated games (1/2)

Repeated games are a subclass of dynamic games, in which the players face the **same** single-stage (static) game every period

Suppose the forwarder's dilemma is played ***n* times** [33], with $n < \infty$

Both players move **simultaneously** after knowing all previous actions

Using backward induction, both players will end up dropping the packets at every stage, and **no cooperation** is enforced: since the stage game has a unique Nash equilibrium, it is played at every stage

Repeated games (2/2)

What if the stage game is repeated an infinite number of times? In this case (infinite-horizon game), a free rider behavior may be punished in subsequent rounds

Cooperation can be automatically enforced by threatening future punishments (grim-trigger strategy):

- Keep playing the social-optimal solution unless the other player has defected: in this case, switch to the noncooperative (selfish) solution

In the repeated forwarder's dilemma, the subgame-perfect equilibrium strategy is for each player to play 'forward' unless the other player has dropped his/her message in the previous stage

Repeated games: An example [34]

We can also consider the utility maximization over time (**long-sighted games**), by introducing the concept of **discount**, that measures the **patience** of the players:

discount factor

$$u_k^\delta(\mathbf{a}) = (1-\delta)\sum_{n=0}^{\infty}\delta^n u_k(\mathbf{a}(n))$$

Let's suppose to play the continuous-power near-far power control game an **infinite** number of times:

There exists a critical $\underline{\delta}$ such that, for all $\delta > \underline{\delta}$ (i.e., for **delay-tolerant** users), the grim-trigger strategy achieves the maximum social welfare

Games with incomplete information

In many situations,

- a player may not be *fully* informed about his opponents; or
- a player may not know *how well* opponents are informed.

This kind of interactions can be modeled as noncooperative games with **incomplete information**

A strategic game with incomplete information is called a **Bayesian game**

Bayesian games: An example [36] (1/5)

Suppose a packet forwarding problem in a network, in which the two nodes have **asymmetic information**:

- the sender knows that the receiver is a **regular** node
- the receiver knows that the sender can be a **malicious** node, with probability β, or a **regular** node, with probability $1 - \beta$

receiver (player 1)

$\mathcal{T}_1$={regular}

sender (player 2)

$\mathcal{T}_2$={malicious, regular}

- A malicious sender can either **attack** the receiver or **forward** the packet
- A regular sender can just **forward** the packet

- The receiver can either **check** the packet to prevent the attack (with a **monitoring cost** c), or just **accept** the received packet
- A **successful** attack yields a payoff +*g* to the malicious sender, and –*g* to the receiver, and viceversa is the attack is **prevented**, with *g*>c

Bayesian games: An example (2/5)

We can investigate this situation using a **strategic-form representation**:

player 2 (malicious sender)

player 1 (receiver)	forward	attack
accept	0, 0	–g, +g
check	–c, 0	+g–c, –g

t_2 = malicious

player 2 (regular sender)

player 1 (receiver)	forward
accept	0, 0
check	–c, 0

t_2 = regular

Let's compute player 2's **best response**:

- Of course, the sender (player 2) knows its own nature t_2 (in game theory parlance, its **type**)
- The receiver (player 1) knows that $\Pr\{t_2 = \text{malicious}\} = 1 - \Pr\{t_2 = \text{regular}\} = \beta$ (i.e., it has a **belief** on player 2's type)

Bayesian games: An example (3/5)

Now let's focus on player 1's **rationality**: since probabilities are involved, we can obtain player 1's best-response strategy by computing its **expected payoffs** for all possible combinations of player 2's types and actions:

player 1 (receiver)	(f, f)	(a, f)
accept	0	$-\beta g$
check	$-c$	$\beta g - c$

(f, f) means: player 2 plays 'forward' when t_2=malicious, and 'forward' when t_2=regular

(a, f) means: player 2 plays 'attack' when t_2=malicious, and 'forward' when t_2=regular

$$u_1(\text{accept}, (f,f)) = \beta \cdot 0 + (1-\beta) \cdot 0 = 0$$

$$u_1(\text{check}, (f,f)) = \beta \cdot (-c) + (1-\beta) \cdot (-c) = -c$$

$$u_1(\text{accept}, (a,f)) = \beta \cdot (-g) + (1-\beta) \cdot 0 = -\beta g$$

$$u_1(\text{check}, (a,f)) = \beta \cdot (g-c) + (1-\beta) \cdot (-c) = \beta g - c$$

Bayesian games: An example (4/5)

Now let's focus on player 1's **rationality**: since probabilities are involved, we can obtain player 1's best-response strategy by computing its **expected payoffs** for all possible combinations of player 2's types and actions:

player 1 (receiver)	(f, f)	(a, f)
accept	0	$-\beta g$
check	$-c$	$\beta g - c$

(f, f) means: player 2 plays 'forward' when t_2=malicious, and 'forward' when t_2=regular

(a, f) means: player 2 plays 'attack' when t_2=malicious, and 'forward' when t_2=regular

Let's compute player 1's **best response**: $-\beta g > \beta g - c \iff \beta < g/(2c)$

player 2 (malicious sender)

player 1 (receiver)	forward	attack
accept	0, 0	$-g$, $+g$
check	$-c$, 0	$+g-c$, $-g$

player 2 (regular sender)

player 1 (receiver)	forward
accept	0, 0
check	$-c$, 0

Bayesian games: An example (5/5)

To sum up, the strategy (accept, (attack, forward)) (i.e., player 1 playing 'accept', no matter player 2's type is, and player 2 attacking when is malicious, and forwarding when is regular) is a **Bayesian Nash equilibrium**

Suppose now $\beta < g/(2c)$:

player 1 (receiver)		
accept	0	$-\beta g$
check	$-c$	$\beta g - c$

(f, f) means: player 2 plays 'forward' when t_2=malicious, and 'forward' when t_2=regular

(a, f) means: player 2 plays 'attack' when t_2=malicious, and 'forward' when t_2=regular

player 1 (receiver)	player 2 (malicious sender): forward	player 2 (malicious sender): attack
accept	0, 0	$-g, +g$
check	$-c, 0$	$+g-c, -g$

player 1 (receiver)	player 2 (regular sender): forward
accept	0, 0
check	$-c, 0$

In this case, **no** Bayesian Nash equilibria exist in this game

Bayesian games: A summary

A **static** game with incomplete information is composed by:

(1) a set of players $\mathcal{K} = [1, \ldots, K]$

(2) each player k's possible **types** $\mathcal{T}_k$

(3) each player k's set of strategies $\mathcal{A}_k(t_k)$

(4) each player k's **beliefs** about the other players $\Pr_k\{t_{\setminus k} \mid t_k\}$

(5) each player k's utility function $u_k(a;t)$

At the **Bayesian Nash equilibrium**, no player wants to change his/her own strategy, even if the change involves only one action by one type

Games with incomplete information are also useful to model **auctions**

Games with strict incomplete information

In many practical problems, the hypothesis of **correct** beliefs available to all players in the game is not **realistic**

Example: a CDMA wireless network, with a mixed population of **out-of-sync** users (during their network association phase) and **in-sync** users (already exchanging data with the base station) [37]

Uplink power control based on **SINR feedback** from the BS

Using continuous powers, an equilibrium point (the **ex-post Nash equilibrium**) exists and can be reached using an iterative best-response algorithm, even if no **a-priori information** is available locally at each terminal (including the number of players)

Taxonomy revisited

Basics of cooperative game theory

Definition

Cooperative game theory studies the interactions among rational players when they are willing to **cooperate**

Cooperative games can be classified into two main categories:

- Bargaining problems
- Coalitional games

Bargaining game theory (1/3)

A **bargaining process** takes place when a set of players need to **agree** on the terms of cooperation

The need for an agreement is a consequence of **conflicting interests** among players: however, the players can **mutually benefit** from reaching an agreement, and hence **cooperate**

Some examples:

- a buyer and a seller trading the purchase of a painting
- investors trading how to split the company revenues
- communication nodes trading how to share a network resource

Bargaining game theory (2/3)

This kind of interactions can be studied using the following model:

(1) a set of players $\mathcal{K} = [1, \ldots, K]$

(2) each player k's set of strategies $\mathcal{A}_k$

(3) each player k's utility function $u_k(a)$

(4) a **disagreement outcome** $d=[d_1, \ldots, d_K]$

The disagreement outcome d_k represents the payoff each player k receives should all players **fail** to reach an agreement

Bargaining game theory (3/3)

Two reasonable choices for the disagreement outcome are:

- d_k=0
- d_k=u_k($a^\star$), where $a^\star$ is the Nash equilibrium of the noncooperative version of the game

A way to solve the bargaining problem is to compute

$$\check{\mathbf{a}} = \arg\max_{\mathbf{a}\in\mathcal{A}} \prod_{k=1}^{K} (u_k(\mathbf{a}) - d_k) \quad \text{s.t. } u_k(\mathbf{a}) \geq d_k \, \forall k$$

- The (unique) strategy ǎ is called the Nash bargaining solution (NBS)
- When d_k=0, the NBS coincides with the proportional fairness allocation, widely used in resource allocation for wireless communications

Bargaining problems: An example [40] (1/3)

Nash bargaining for downlink beamforming in a multiple-input single-output (MISO) Gaussian interference channel

$$y_1 = \mathbf{h}_{11}^T \mathbf{w}_1 s_1 + \mathbf{h}_{21}^T \mathbf{w}_2 s_2 + n_1$$
$$y_2 = \mathbf{h}_{22}^T \mathbf{w}_2 s_2 + \mathbf{h}_{12}^T \mathbf{w}_1 s_1 + n_2$$
$$n_1, n_2 \sim \mathcal{N}(0, \sigma^2)$$

Each link *k*'s objective: select the (normalized) beamforming vector $\mathbf{w}_k$ so as to maximize its own achievable rate $R_k = \log(1+\gamma_k)$, where

$$\gamma_k = \frac{|\mathbf{w}_k^T h_{kk}|^2}{\sigma^2 + |\mathbf{w}_{\backslash k}^T h_{\backslash kk}|^2}$$

Bargaining problems: An example (2/3)

This problem can be modeled as follows:

(1) set of players: $\mathcal{K}$ = [1, 2]

(2) strategies: $\mathcal{A}_k = \left\{ \mathbf{w}_k \in \mathbb{R}^L : \|\mathbf{w}_k\|^2 \leq 1 \right\}$

(3) utilities: $u_k(\mathbf{a}) = \log(1 + \gamma_k)$

(4) disagreement outcome: $d_k = u_k(\mathbf{a}^\star) = \log(1 + \gamma_k^\star)$

where $$a_k^\star = \frac{\mathbf{h}_{kk}^\dagger}{\|\mathbf{h}_{kk}\|}$$ (Nash equilibrium)

$$\gamma_k^\star = \frac{\|\mathbf{h}_{kk}\|^2}{\sigma^2 + |\mathbf{h}_{\backslash k \backslash k}^\dagger \mathbf{h}_{\backslash kk}|^2 / \left\|\mathbf{h}_{\backslash k \backslash k}\right\|^2}$$ (maximal-ratio combining)

Bargaining problems: An example (3/3)

The NBS is the **joint** strategy ă that solves

$$\breve{\mathbf{a}} = \arg\max_{\mathbf{a}} \left(u_1(\mathbf{a}) - u_1(\mathbf{a}^\star)\right) \cdot \left(u_2(\mathbf{a}) - u_2(\mathbf{a}^\star)\right)$$

such that $u_k(\breve{\mathbf{a}}) \geq u_k(\mathbf{a}^\star)$ for *k*=1,2

The NBS lies on the Pareto-optimal frontier, but does **not** coincides with the maximum social welfare: the player who is already in a good position (here, player 2) is stronger in a **negotiation**

The two base stations must **communicate** to achieve the NBS

Beyond the Nash bargaining solution

The NBS, although a widely used concept for bargaining problems, is **not** the only possible approach

For instance, we can account for **unequal** players' bargaining powers ρ_k:

$$\check{\mathbf{a}} = \arg\max_{\mathbf{a}} \prod_{k=1}^{K} (u_k(\mathbf{a}) - d_k)^{\rho_k}$$
$$\text{s. t.} \quad u_k(\mathbf{a}) \geq d_k \quad \forall k \in \mathcal{K}$$
$$\sum_{k=1}^{K} \rho_k = 1$$

generalized Nash bargaining solution

- The NBS (also in its generalized version) requires the **convexity** of the utility space: to overcome this limitation, other solutions exist, e.g., the **Kalai-Smorodinsky solution**
- How can the players negotiate so as to reach the NBS? This issue can be addressed by **dynamic bargaining theory**

Coalitional game theory (1/2)

Coalitional game theory investigates the formation of cooperating groups of players (**coalitions**) that can improve the players' positions in a game, with:

(1) a set of players $\mathcal{K} = [1, \ldots, K]$

(2) each player k's capacity to **form agreements** through the action set $\mathcal{A}_k$

(3) a **coalition value** $v(\mathcal{S})$ for each coalition $\mathcal{S} \subseteq \mathcal{K}$

Coalitional game theory (2/2)

Coalitional games can be grouped in three main categories:

(1) canonical coalitional games

(2) coalition formation games

(3) coalition graph games

Canonical coalitional games (1/2)

In a canonical coalitional game, the coalition of all users $\mathcal{K}$, called the **grand coalition**, represents the optimal structure

Key question: how to **stabilize** the grand coalition, i.e., how to **encourage** all players to bind agreements among each other?

Canonical coalitional games (2/2)

Let's focus on **transferable-utility** (TU) games, i.e., games in which, for all coalitions $\mathcal{S} \subseteq \mathcal{K}$, there exists a **payoff vector** $\mathbf{x} \in \mathbb{R}^{|\mathcal{S}|}$ s.t.

$$\sum_{k \in \mathcal{S}} x_k = v(\mathcal{S})$$

x_k is the amount of utility a player $k \in \mathcal{S}$ receives when joining the coalition $\mathcal{S}$

Key assumption: when forming a larger coalition, the players cannot do **worse** than by acting alone, i.e.,

$$v(\mathcal{S}_1 \cup \mathcal{S}_2) \geq v(\mathcal{S}_1) + v(\mathcal{S}_2) \quad \forall \mathcal{S}_1, \mathcal{S}_2 \subset \mathcal{K}, \mathcal{S}_1 \cap \mathcal{S}_2 = \emptyset$$

(superadditive game)

How to solve a canonical coalitional game

To encourage players to form a **stable** grand coalition $\mathcal{K}$, we should find a payoff allocation x (the **core** $\mathcal{C}$) that guarantees that no group of players has incentive to leave $\mathcal{K}$:

$$\mathcal{C} = \left\{ \mathbf{x} \in \mathbb{R}^K : \sum_{k \in \mathcal{K}} x_k = v(\mathcal{K}) \text{ and } \sum_{k \in \mathcal{S}} x_k \geq v(\mathcal{S}) \, \forall \mathcal{S} \subseteq \mathcal{K} \right\}$$

To improve the **fairness** of the solution, other concepts may be used:

- the **Shapley value**, that makes use of some general axioms to introduce fairness
- the **nucleolus**, that minimizes the dissatisfaction of the players

Canonical coalitional games: An example [43]

How can we **fairly** allocate the **rates** among nodes in a multiple-access Gaussian channel?

- the coalitional value $v(\mathcal{S})$ is the **maximum sum-rate** achievable by terminals $k \in \mathcal{S}$
- every terminal that does not obtain a fair allocation of the rate can threaten to act on its own, by **jamming** the transmission of all others

This situation, modeled as a canonical coalitional game, shows a **non-empty core**: all terminals are encouraged to form the grand coalition

To compute a **single** fair allocation that lies in the core, we can resort to the concept of Shapley value

Coalition formation games (1/2)

In a coalition formation game, the **network structure** that takes place depends on **gains** and **costs** induced by cooperation

Key question: how to form a proper coalitional structure (i.e., a **topology**) and how to study its properties?

Coalition formation games (2/2)

Coalition formation game theory can be used to investigate situations in which the interactions do **not** lead to the property of superadditivity

This often occurs when forming a coalition incurs in a **cost** for the players (e.g., due to the negotiation process or to the exchange of information among players)

The coalition formation process if usually a **dynamic** procedure

The merge&split algorithm

To find an optimal **partition** of the set of players $\mathcal{K}$ with *reasonable complexity*, we can use the **merge&split** algorithm:

- **merge rule:** any set of coalitions $\{\mathcal{S}_1, \ldots, \mathcal{S}_L\}$ may be merged whenever $\mathcal{S}' = \bigcup_{\ell=1}^{L} \mathcal{S}_\ell$ is preferred to $\{\mathcal{S}_1, \ldots, \mathcal{S}_L\}$ by **all** players, i.e.,

$$v(\mathcal{S}') > \sum_{\ell=1}^{L} v(\mathcal{S}_\ell)$$

- **split rule:** any coalition $\mathcal{S}'$ may be split in a set of coalitions $\{\mathcal{S}_1, \ldots, \mathcal{S}_L\}$ such that $\bigcup_{\ell=1}^{L} \mathcal{S}_\ell = \mathcal{S}'$ whenever it is preferred to $\mathcal{S}'$ by **all** players, i.e.,

$$\sum_{\ell=1}^{L} v(\mathcal{S}_\ell) > v(\mathcal{S}')$$

Irrespectively of the initial partition, using this algorithm we can converge to a **suitable network partition** $\{\mathcal{S}_1, \ldots, \mathcal{S}_M\}$ such that $\mathcal{S}_i \cap \mathcal{S}_j = \emptyset \; \forall i, j$ and $\bigcup_{m=1}^{M} \mathcal{S}_m = \mathcal{K}$

Coalition formation games: An example [44]

Cooperative spectrum sensing in a cognitive network: *K* secondary nodes collaborate by exchanging information on the spectrum owned by a primary user

each secondary node senses the spectrum using an **energy detector** and exchanges this results with the **coalition head**

Network's objective: to form proper coalitions so as to maximize the **probability of detection** using an OR rule given a target probability of false alarm

Using a **merge&split algorithm**, we can come to a stable partition of the nodes that **outperforms** an individual (noncooperative) spectrum sensing

Coalition graph games

In a coalition graph game, players' interactions are regulated by a **communication graph structure**

In this case, how the players are **interconnected** withint the coalition impacts on the coalitional value $v(\mathcal{S})$

Key question: how to form a stable grand coalition or to form a network structure considering the **communication graph**?

Coalition graph games: An example [45]

Relay station (RS) development in next-generation cellular networks:
RSs cooperate to improve the performance of the uplink transmission

the mobile stations (MSs) can reach the BS through **multihop** communication using decode-and-forward (DF) relays

Network designer's objective: to build the **tree** based on quality-of-service parameters such as delay and packet loss

Using the tools borrowed from noncooperative game theory and mapped to network graphs, we can derive a **distributed network-formation algorithm**

Taxonomy revisited

classes: static g., dynamic g.
tools: Nash bargaining
solution: Nash bargaining solution (NBS), Kalai-Smorodinsky solution, etc.

classes: canonical g., coalition-formation g., graph g.
tools: superadditivity, merge&split, etc.
solution: core, Shapley value, nucleolus, etc.

Discussion and perspectives

Back to the start: What is game theory?

Game theory (GT) is a flexible **design tool** to find solutions for a large number of problems in **wireless communications and networking**:

- *Scheduling* of processes in distributed computing systems
- *Routing* in communication networks, including *cooperative communications*
- *Resource allocation* in wireless communications (power, bitrate, bandwidth, etc.)
- *Bandwidth sharing* and *spectrum sensing* in cognitive wireless networks
- *Waveform adaptation* for multiple access
- *Source* and *channel* coding
- …

Mechanism design, a subfield of GT (also known as **reverse GT**) can be used to design effective protocols to address all these situations

Game theory and the WCN community

In the last **five years**, a consistent number of journal issues and conferences have been dedicated to GT for the broad area of communications, e.g.:

- *IEEE Signal Processing Magazine* (Sept. 2009)
- *IEEE Trans. Systems, Man, and Cybernetics* (June 2010)
- *IEEE J. Selected Areas Commun.* (Jan. 2012)
- *IEEE J. Selected Topics Signal Processing* (Apr. 2012)
- *Intl. Conf. Game Theory for Networks (GameNets)* (3 editions)
- *Intl. Work. Game Theory in Commun. Networks (GameComm)* (4 editions)

Bibliography/Webography (1/10)

Textbooks

[01] D. Fudenberg and J. Tirole, *Game Theory*. Cambridge, MA: MIT Press, 1991.

[02] R. Gibbons, *A Primer in Game Theory*. Harlow, UK: FT Prentice Hall, 1992.

[03] M. J. Osborne and A. Rubinstein, *A Course in Game Theory*. Cambridge, MA: MIT Press, 1994.

[04] S. Lasaulce and H. Tembine, *Game Theory and Learning for Wireless Networks*. Waltham, MA: Academic Press, 2011.

[05] Z. Han, D. Niyato, W. Saad, T. Başar, and A. Hjørungnes, *Game Theory in Wireless and Communication Networks. Theory, Models, and Applications*. Cambridge, UK: Cambridge University Press, 2012.

Non-technical readings

[06] L. Fisher, *Rock, Paper, Scissors. Game Theory in Everyday Life*. New York: Basic Books, 2008.

[07] P. Walker, "A chronology of game theory". [Online.] Available: http://www.econ.canterbury.ac.nz/personal_pages/paul_walker/gt/hist.htm

[08] *Game theory*, Wikipedia, the free encyclopedia. [Online.] Available: http://en.wikipedia.org/wiki/Game_theory

Bibliography/Webography (2/10)

Tutorial-style readings on wireless communications

[09] A. B. MacKenzie and S. B. Wicker, "Game theory in communications: Motivation, explanation, and application to power control," in *Proc. IEEE Global Telecommunications Conference (GLOBECOM)*, San Antonio, TX, Nov. 2001, pp. 821–826.

[10] V. Srivastava, J. Neel, A. B. MacKenzie, R. Menon, L. A. DaSilva, J. E. Hicks, J. H. Reed, and R. P. Gilles, "Using game theory to analyze wireless ad hoc networks," *IEEE Commun. Surveys & Tutorials*, vol. 7, no. 4, pp. 46–56, 4th Quarter 2005.

[11] A. B. MacKenzie and L. A. DaSilva, *Game Theory for Wireless Engineers*. San Rafael, CA: Morgan & Claypool, 2006.

[12] E. Altman, T. Boulogne, R. El-Azouzi, T. Jiménez, and L. Wynter, "A survey on networking games in telecommunications," *Computers & Operations Research*, vol. 33, no. 2, pp. 286-311, Feb. 2006.

[13] M. Félegyházi and J.-P. Hubaux, "Game theory in wireless networks: A tutorial," École Polytechnique Fédérale de Lausanne (EFPL), Tech. Rep. EPFL LCA-REPORT-2006-002, June 2007. [Online.] Available: http://infoscience.epfl.ch/record/79715/files/

Bibliography/Webography (3/10)

Tutorial-style readings on wireless communications (cont'd)

[14] G. Bacci, M. Luise, and H. V. Poor, "Game theory and power control in ultrawideband networks," *Physical Communication*, vol. 1, no. 1, pp. 21–39, March 2008.

[15] S. Lasaulce, M. Debbah, and E. Altman, "Methodologies for analyzing equilibria in wireless games," *IEEE Signal Process. Mag.*, vol. 26, no. 5, pp. 41-52, Sept. 2009.

[16] W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Başar, "Coalitional game theory for communication networks," *IEEE Signal Process. Mag.*, vol. 26, no. 5, pp. 77-97, Sept. 2009.

[17] K. Akkarajitsakul, E. Hossain, D. Niyato, and D. I. Kim, "Game theoretic approaches for multiple access in wireless networks: A survey," *IEEE Commun. Surveys & Tutorials*, vol. 13, no. 3, pp. 372–395, 3rd Quarter 2011.

[18] T. Alpcan, H. Boche, M.L. Honig, and H.V. Poor, *Mechanisms and Games for Dynamic Spectrum Access*. Cambridge, UK: Cambridge University Press, 2013.

Bibliography/Webography (4/10)

Noncooperative game theory

Static finite games

[19] M. Félegyházi and J.-P. Hubaux, "Game theory in wireless networks: A tutorial," École Polytechnique Fédérale de Lausanne (EFPL), Tech. Rep. EPFL LCA-REPORT-2006-002, June 2007.

[20] G. Bacci, M. Luise, and H. V. Poor, "Game theory and power control in ultrawideband networks," *Physical Communication*, vol. 1, no. 1, pp. 21–39, March 2008.

Static infinite games

[21] D. J. Goodman and N. B. Mandayam, "Power control for wireless data," *IEEE Personal Commun.*, vol. 7, pp. 48–54, Apr. 2000.

[22] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. Commun.*, vol. 50, no. 2, pp. 291-303, Feb. 2002.

Bibliography/Webography (5/10)

Noncooperative game theory (cont'd)

Supermodular games

[23] A. Altman and Z. Altman, "S-modular games and power control in wireless networks," *IEEE Trans. Automatic Control,* vol. 48, no. 5, pp. 839-842, May 2003.

[24] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. Commun.*, vol. 50, no. 2, pp. 291-303, Feb. 2002.

Potential games

[25] J. E. Hicks, A. B. MacKenzie, J. A. Neel, and J. J. Reed, "A game theory perspective on interference avoidance," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM)*, Dallas, TX, Nov.-Dec. 2004, pp. 257-261.

[26] G. Scutari, S. Barbarossa, and D. Palomar, "Potential games: A framework for vector power control problems with coupled constraints," in Proc. IEEE Intl. Conf. Acoustics, Speech and Signal Process. (ICASSP), Toulouse, France, May 2006.

Bibliography/Webography (6/10)

Noncooperative game theory (cont'd)

Generalized Nash games

[27] F. Facchinei and C. Kanzow, "Generalized Nash equilibrium problems," *Quarterly J. Operations Research*, vol. 5, no. 3, pp. 173–210, Sep. 2007.

[28] G. Bacci, L. Sanguinetti, M. Luise, and H.V. Poor, "A game-theoretic approach for energy-efficient contention-based synchronization in OFDMA systems," *IEEE Trans. Signal Processing*, vol. 61, no. 5, pp. 1258-1271, Mar. 2013.

Dynamic games

[29] M. Félegyházi and J.-P. Hubaux, "Game theory in wireless networks: A tutorial," École Polytechnique Fédérale de Lausanne (EFPL), Tech. Rep. EPFL LCA-REPORT-2006-002, June 2007.

[30] Z. Han, D. Niyato, W. Saad, T. Başar, and A. Hjørungnes, *Game Theory in Wireless and Communication Networks. Theory, Models, and Applications*. Cambridge, UK: Cambridge University Press, 2012.

Bibliography/Webography (7/10)

Noncooperative game theory (cont'd)

Stackelberg games

[31] S. Lasaulce, Y. Hayel, R. El Azouzi, and M. Debbah, "Introducing hierachy in energy games," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3833-3843, July 2009.

[32] S. Guruacharya, D. Niyato, E. Hossain, and D. I. Kim, "Hierarchical competition in femtocell-based cellular networks," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM)*, Miami, FL, Dec. 2010.

Repeated games

[33] M. Félegyházi and J.-P. Hubaux, "Game theory in wireless networks: A tutorial," École Polytechnique Fédérale de Lausanne (EFPL), Tech. Rep. EPFL LCA-REPORT-2006-002, June 2007.

[34] M. Le Treust and S. Lasaulce, "A repeated game formulation of energy-efficient decentralized power control," *IEEE Trans. Wireless Commun.*, vol. 9, no. 9, pp. 2860-2869, Sep. 2010.

Bibliography/Webography (8/10)

Noncooperative game theory (cont'd)

Bayesian games

[35] Z. Han, D. Niyato, W. Saad, T. Başar, and A. Hjørungnes, *Game Theory in Wireless and Communication Networks. Theory, Models, and Applications*. Cambridge, UK: Cambridge University Press, 2012.

[36] W. Wang, M. Chatterjee, and K. Kwiat, "Coexistence with malicious nodes: A game theoretic approach," in *Proc. Intl. Conf. Game Theory for Networks (GameNets)*, Istanbul, Turkey, May 2009, pp. 277-286.

Games with strict incomplete information

[37] G. Bacci and M. Luise, "A pre-Bayesian game for CDMA power control during network association," *IEEE J. Select. Topics Signal Process.*, vol. 6, no. 2, pp. 76-88, Apr. 2012.

[38] G. Bacci and M. Luise, "How to use a strategic game to optimize the performance of CDMA wireless network synchronization," in *Mechanisms and Games for Dynamic Spectrum Access*, T. Alpcan, H. Boche, M.L. Honig, and H.V. Poor, Eds. Cambridge, U.K.: Cambridge Univ. Press, 2013.

Bibliography/Webography (9/10)

Cooperative game theory

Bargaining problems

[39] Z. Han, D. Niyato, W. Saad, T. Başar, and A. Hjørungnes, *Game Theory in Wireless and Communication Networks. Theory, Models, and Applications*. Cambridge, UK: Cambridge University Press, 2012.

[40] E. G. Larsson and E. A. Jorswieck, "Competition versus cooperation on the MISO interference channel," *IEEE J. Select. Areas Commun.*, vol. 26, no. 7, pp. 1059-1069, Sep. 2008.

Coalitional game theory

[41] Z. Han, D. Niyato, W. Saad, T. Başar, and A. Hjørungnes, *Game Theory in Wireless and Communication Networks. Theory, Models, and Applications*. Cambridge, UK: Cambridge University Press, 2012.

[42] W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Başar, "Coalitional game theory for communication networks," *IEEE Signal Process. Mag.*, vol. 26, no. 5, pp. 77-97, Sept. 2009.

Bibliography/Webography (10/10)

Cooperative game theory (cont'd)

Canonical coalitional games

[43] R. J. La and V. Anantharam, "A game-theoretic look at the Gaussian multiaccess channel," in *Proc. Work. Network Information Theory*, Piscataway, NJ, pp. 87-106, Mar. 2003.

Coalition formation games

[44] W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Başar, "Coalitional games for distributed collaborative spectrum sensing in cognitive-radio networks," in *Proc. IEEE Conf. Computer Commun. (INFOCOM)*, Rio de Janeiro, Brazil, Apr. 2009, pp. 2114-2122.

Coalition graph games

[45] W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Başar, "A game-based self-organizing uplink tree for VoIP services in IEEE 802.16j networks," in *Proc. IEEE Intl. Conf. Commun. (ICC)*, Dresden, Germany, June 2009.

Questions???

Thank you!

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